Accurate positioning of a robot arm for radiotherapy of lung tissue tumour

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Tom Lefebvre
Ghent, 22 may 2015
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De auteur geeft de toelating deze masterproef voor consultatie beschikbaar te stellen en delen van de masterproef te kopiëren voor persoonlijk gebruik. Elk ander gebruik valt onder de bepalingen van het auteursrecht, in het bijzonder met betrekking tot de verplichting de bron uitdrukkelijk te vermelden bij het aanhalen van resultaten uit deze masterproef.

22 may 2015 Tom Lefebvre
REAL-TIME TUMOUR TRACKING (RTTT) is a recent addition to the Stereotactic Body Radiotherapy (SBRT) family, that caters to respiratory induced tumour movement. The technique entails active compensatory motion to counteract any wandering of the tumour, such that the relative position between radiation beam and target is kept constant. This motion is effectuated by accurate repositioning of the radiation source during the treatment as is found operational in the Cyberknife Robotic Radiosurgery system. In order to facilitate a sufficient manoeuvrability of the radiation source, it is mounted on a six degree of freedom robotic arm.

This study portrays the design of a supervisory control structure, based on a standard model based predictive control methodology, for a six degree of freedom robotic arm, aiming for effective synchronisation of the radiation focus to the movement of a lung tissue tumour. Such involved the elaboration of kinematic and dynamic models of the robot. Subsequently, the performance was validated by means of simulation on real patient tumour trajectories. Furthermore the influence of the control parameters, addressing tumour trajectory forecast horizon and accuracy, was explored and the optimal values identified.
Accurate positioning of a robot arm for radiotherapy of long tissue tumour

Tom Lefebvre
Supervisor(s): Clara Ionescu, Dirk Verellen

Abstract—This study portrays the design of a supervisory control structure, based on a standard model based predictive control methodology, aiming for effective synchronisation of the radiation focus to the movement of a lung tissue tumour. This is facilitated by accurate repositioning of a robotic arm manipulator as is found operational in the Cyberknife Robotic Radiosurgery system. Performance of the control structure is validated by means of simulation on an illustrative example.

Keywords—cyberknife robotic radiosurgery system, model based predictive control, real-time tumour tracking, stereotactic body radiosurgery

I. INTRODUCTION

STEREOTACTIC Body Radiotherapy (SBRT) is an overarching term addressing all cancer treating radiotherapies that exploit the principle of hyper-fractionation. This technique sequences a lethal radiation dose over multiple non-coplanar beams that only intersect at the cancerous target location, thereby creating a so-called localised radiation hot-spot. As such the adjacent healthy tissue is exposed only to non-lethal radiation doses.

Tumours nestled in lung tissue or near thoracic organs are often found to describe a spacial trajectory as a result of respiration induced lung movement. This phenomenon will substantially impede the application of SBRT since precise target localisation and accurate dose delivery can not be guaranteed, both being required for successful execution of SBRT.

Real-time tumour tracking (RTTT) is a recent addition to the SBRT family, that caters to the respiratory induced tumour movement. The technique entails active compensatory motion to counteract any wandering of the tumour, such that the relative position between radiation beam and target is kept constant. This motion is effectuated by accurate repositioning of the radiation source during the treatment. In order to facilitate a sufficient manoeuvrability, the source is mounted on a six degree of freedom robotic arm. A supervisory control structure was designed in this study and validated by means of simulation.

II. SET UP

This study emanates from the use of the KUKA, KR 120 R2700 extra HA series, as an adequate substitute for the robotic manipulator that is employed in the Cyberknife Robotic Radiosurgery system. The robotic arm consist of the concatenation of six rotational joints, and is as such capable of covering a six degree of freedom configuration space. Specific dimensions were found in the company catalogue [1], radiation source and beam were modelled respectively, as a cube (0.5 x 0.5 x 0.5 m) and a 1 m segment originating from the lower cube surface. (fig. 1).

Each robot joint was assumed to be equipped with a high accuracy lower-level closed-loop joint control unit. The closed loop joint dynamics could as such be modelled as second order transfer functions with a steady state gain of 1 and joint specific damping ratio and frequency [2]. This simplified representation is justifiable as these approximated dynamics will not influence the validity of the concept proof.

![Computer visualisation of the KUKA, KR 120 R2700 extra HA with mounted radiation source and beam.](image)

The designed control structure will operate at a higher hierarchical supervisory to the lower-level robot joint control units and is entrusted with the task of generating the lower-level joint reference signals. It is emphasised that the control will address the problem indirectly through the robot joint variable space, not directly through the Euclidean coordinate space.

The technical aspects associated with the acquisition of the real-time tumour coordinates with respect to a global Euclidean coordinate system is not addressed in this study and it was assumed that during control, the tumour positions were available up till time instant t.

III. ROBOT KINEMATICS

The robot state and therefore the radiation focus position and orientation can either be expressed in the robot joint variable space, captured by a six dimensional vector, $\bar{q}$, containing the robot joint values, or in an Euclidean framework, captured by a homogeneous matrix, $G_R$, containing the unit vectors (expressing spacial orientation, $E_R$) and position vector ($P_R$) of a coordinate frame linked to the radiation focus. (fig. 1, in green)

$$\bar{q} = [q_1 \ldots q_6]^T \iff G_R = \begin{bmatrix} E_R & P_R \\ 0 & 1 \end{bmatrix}$$ (1)

A. Kinematics

In order to facilitate a bridge between both expression forms, the transformation functions $K$ and $K^{-1}$ are introduced, respectively addressing the forward and inverse kinematic relation.

$$G_R = K(\bar{q}) \iff \bar{q} = K^{-1}(G_R)$$ (2)

The first was effectuated by an extended version of the Denavit-Hartenberg formalism [3], for the latter however there exists no general analytical solution.
B. Reduced inverse kinematics

For this study we opted not to address all six degrees of freedom contained in $G_R$, but only those three corresponding to the position vector $P_R$. It was found feasible to address the two last robot joints and additionally allow the radiation depth to vary, the remaining four are set to cover a reference state $G_{R}^{ref}$, covering the nominal angle of incidence and nominal tumour position. Perturbations, caused by the respiratory induced motion, are as such counteracted by varying the two last joints keeping the tumour on the line of radiation. The resulting perturbation in the angle of incidence will not affect the quality of the treatment.

$$\hat{q} = \begin{bmatrix} q_1^{ref} & \ldots & q_4^{ref} \end{bmatrix}^T$$

with $K^{-1}_R(G_R) = \begin{bmatrix} q_5 & q_6 \end{bmatrix}^T$ (3)

The function $K^{-1}_R$ than adopts an analytical solution.

IV. CONTROL STRUCTURE

The control embraces two consecutive steps. In a preparatory step the robot is manoeuvred to a reference state, $q^{ref}$, covering the mean tumour trajectory and desired angle of incidence. Subsequently, the respiratory motion is counteracted by adjustment of the two last robot joints ($q_5$ and $q_6$). The desired robot trajectory, is computed through application of the reduced inverse kinematics on the available tumour trajectory signal and serves as the reference signal for the supervisory control.

A. Control methodology

Given the nature of the problem, the presence of lag, inherent to less involved control methodologies, was intolerable. As such we resorted to an MBPC methodology. Moreover, the study will emanate from a linear context, both for process and disturbance model, consequently an EPSAC structure could be adopted [4].

B. Process model

The process model (fig. 2) was inspired on the generic process model found in EPSAC. Since the EPSAC control algorithm emanates from a known reference trajectory, the method had to be revised. We introduced a new control variable vector, $\Delta$, by subtraction of the tumour robot coordinates, $q_r$, from the robot coordinates, $q_p$. Consequently this new vector has to be controlled to the zero vector. The tumour trajectory is modelled as a coloured noise process, which is a standard for EPSAC, embracing an intelligent design [5]. Remark the absence of noise since the lower-level control loops are assumed to reject any.

$$e(t) = C \left( q_r(t) - q_p(t) \right)$$

$$P_R(t)$$

$$K_{red}^{-1}$$

$$q_R(t)$$

$$q_p(t)$$

$$\Delta(t)$$

$$\text{control vector}$$

Fig. 2. Process model with vector signals, generating the new control variable vector $\Delta(t)$

1If a certain $G_{R}^{ref}$ were to be desired, the vector $q^{ref}$, can only be computed by use of numerical methods approximating the general function $K^{-1}_R$. For this study however an illustrative reference state was postulated.

C. Final control structure

The finalised control structure is illustrated in fig. 3. It enfolds a clear lay-out of all the control related facets. For completeness, the lower-level closed-loop was depicted unambiguously.

V. PERFORMANCE VALIDATION

The proposed control design was validated by means of simulation. We postulated a reference vector, $q^{ref}$, and superposed real patient tumour movement on the corresponding radiation focus position, $P^{ref}_R$. Control was exerted on the two last robot joints. The corresponding radiation focus position was computed by utilisation of the forward kinematic function, $K$.

Effective synchronisation is demonstrated in fig. 4. The influence of the control parameters, addressing prediction horizon and accuracy, was explored and the optimal values identified.

VI. DISCUSSION

The simulations lead to encouraging results and proved the viability of the proposed control design. Although this study restricted the reduced inverse kinematic function to the two last joints, this number could be expanded without adapting the control structure, if a higher degree of freedom were to be desired.

However, further research should be performed, addressing the reliability of the tumour trajectory forecast and the development of more evolved prediction models.

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Chapter 1

Introduction

1.1 Radiotherapy in oncology

One speaks of cancer when confronted with a pathology characterised by abnormal and uncontrolled cell growth exhibiting the potential to spread to other parts of the body. It is the overarching term for a disease which appears in over a hundred guises. In 2012, approximately 14 million people were diagnosed with some form of cancer worldwide. If this trend of increasing incidences prosecutes according its current rate, the number of annual new cases is estimated to increase up to 70 \% over the next two decades, mounting up to 20 million cancer diagnoses in 2030 \[1\].

Fortunately, diverse as are its manifestation forms is the palette of existing treatments. Over the past few decades mesmerising advances have been achieved, concerning both the diagnostic as the therapeutic aspect. The latter, addressing surgical techniques, chemo- and not at least radiation therapy. Nowadays, radiation therapy is a major modality for treating cancer patients. The technique emanates from ionizing radiation; that is, radiation that contains sufficient energy to knock electrons out of their stable bound with the atom, thereby creating a so called ion. By this mean, considerable damage can be inflicted on sub-cellular levels even within the cells nucleus. In cancer orientated applications one aims for the damage to occur at the cells DNA (Deoxyribonucleic acid). Whether directly, inflicting damage to the DNA itself, or indirectly, ionizing the surrounding water molecules which then in their turn will chemically react with the DNA, subsequently achieving the goal that was aimed for. It is noted that ionizing radiation will interact equally with any cell type, independent whether a cancerous cell or a healthy cell was struck. The difference that allows the utilisation of radiation as a therapy, involves the biological mechanisms a cell can appeal to after the treatment was finalised \[2\].
Cells are equipped with mechanisms to repair DNA damage, however whether such repair is successful or leads to inevitable cell dead depends on the cell type. It was found that healthy tissues possess the capacity of superior repair for sub-lethal damage compared to cancerous cells. A technique that employs such behaviour in its favour is fractionation. By dividing the radiation dose in small fractions delivered over multiple sessions, it showed that the cumulative effect on healthy tissue was substantially less than on cancerous tissue, thereby ushering in the possibility to employ the principle as a treatment for cancer \[3\]. In subsequent development a new perspective on this principle was elaborated in a technique called stereotactic radiosurgery (SRS) \[4\]. The technique concentrates the delivery of the entire dose in a single session but exploits the principle of hypo-fractionation by sequencing the dose in a number of non-overlapping beams that only intersect at the target location thereby minimising the exposure of healthy tissue to the ionizing radiation. A steep dose fall-off is guaranteed in the tissue surrounding the target while at the radiation focus, a so called hot-spot is created. The high potent doses of a single treatment translate in a heavily quelled re-population of the cancerous cells resulting in an effectiveness unseen in the earlier fractionation.

The latter technique was originally developed to treat tumours located in the central nervous (CN) area, being the brain and spine. Stereotactic body radio therapy (SBRT) is a recent extension of this modality addressing other parts of the body. However, invoking either of both techniques to battle of cancer, will only be found possible if the following four main requirements are fulfilled, being: effective patient immobilization, precise target localisation, accurate dose delivery and the guaranty of a steep dose fall-off in the adjacent tissue.

1.2 Wandering tumours

An inevitable aspect of radiotherapy that will undermine these requirements if not properly accounted for in the therapy, is human breathing. The process of respiration includes the expansion of the thoracic region such that a vacuum is created in the lungs and air will be drawn in. This process inflicts lung- and surrounding tissue to describe a relative motion to parts of the body that remain fixed over time. Tumours that are nestled in the lung tissue or near thoracic organs often wander around as a result of this respiratory process. The phenomenon is referred to as respiratory (induced) tumour motion and the spacial path described by the tumour as the tumour trajectory.

As such, respiration will substantially impede the application of SBRT since both the requirements of a precise target localisation and an accurate dose de-
livery can not be guaranteed. A number of techniques have been elaborated over the past decade to cater to this respiratory induced tumour motion, addressing both the localisation as the specific delivery of the radiation.

1.3 State of art

Motion-adaptive radiotherapies explicitly account for and tackle the difficulties experienced during the radiation delivery to the moving tumour. Concerning such matter respiratory gating and tumour tracking are believed to be the most promising approaches, both are classified in the field of SBRT. Beyond doubt, for either technique to be feasible an accurate position of the target is of absolute essence.

In what follows we will introduce some terminology and concepts that are associated with the overarching principle of the SBRT technique. Subsequently, we will shortly address the problems accompanying the localisation together with a concise summary of existing approaches. We conclude this section with a discussion of the respiratory gating and tumour tracking techniques.

1.3.1 SBRT, the principle explained

The term stereotactic body radiotherapy contemplates every therapy whereby a lethal radiation dose is fractioned over a number of non-lethal beams, delivered during a single session. All of which are aimed directly at the target region however each individual beam is targeted from a different angle. The precise orientation of the radiation source with respect to the target or thus the angle from which the source is aimed, will be denoted as the angle of incidence. All beams will intersect at a single point being the target location thereby creating a so called local hot-spot. The principle is portrayed in figure 1.1 and ensures that only cancerous tissue will face the cumulative effect, guaranteeing a steep dose fall-off elsewhere.

![Figure 1.1: An SBRT beam arrangement, using multiple non-coplanar beams.](image-url)
Most importantly to the effectiveness of the technique is that each of the individual beams are directed precisely to the target during that time the patient is exposed to the radiation. The angle of incidence however allows variation over time as this will only scatter the exposure of healthy tissue even more.

### 1.3.2 Tumour localisation

The hands can’t hit what the eyes can’t see. As was formerly stated within the context of motion-adaptive radiotherapy it is of absolute essence to have access to an accurate and moreover up-to-date localisation of the tumour. Conventional methods, such as typical imaging techniques that are practised for diagnostic purposes, are founded on radiation principles themselves, to generate internal images of the body. As such they fall short as the imaging doses would mount up to unacceptable levels over the duration of a single radiation treatment.

In practise one resorts to other means all inspired by the same basic principle. A set of surrogate breathing signals that exhibit strong correlation to the actual tumour trajectory is measured, subsequently the set is fed to a correlation function which effectively generates the current tumour position.

Surrogate signals can be obtained by use of fiducial markers, however this requires an invasive implantation procedure which affect the patient’s comfort and physical involvement over the treatment substantially [6].

On the other hand, surrogates signals can be provided by external markers such as infrared reflectors interwoven into a vest the patient wears during treatment. Despite still ongoing perfecting, external surrogate techniques will intervene less with the patients comfort and are as such gaining importance over the years [7].

Most recently a new technique arose that completely circumvents any direct contact to the patient. Continuous-wave (CW) radar sensors measures the periodic motion of the body itself thereby minimizing the discomfort experienced by the patient. The technique however is yet in its infancy but shows great potential in motion-adaptive radiotherapy nevertheless [8].

Practising any of these surrogate techniques will heavily depend on the quality of the correlation function. They consist of patient specific mathematical algorithms sculpted ahead of treatment based on the surrogate signals and tumour trajectory signals obtained through conventional imaging. During treatment, the direction is altered and the surrogate signals are correlated to an accurate estimate of the tumour trajectory. Their accuracy is governed online by comparison of intermediate tumour images and its estimate. Whenever accuracy would fall short, treatment is interrupted and the function revised.
1.3. STATE OF ART

1.3.3 Dose delivery

Once the location of the target is known the treatment should act accordingly. A short discussion is devoted to two motion compensating techniques.

Respiratory gating

Respiratory gated radiotherapy (RGR) contemplates the static perspective on motion-adaptive therapy. The technique exploits the periodic nature of the breathing cycle and administrates a cyclic radiation exposure. Therefore the radiation source can remain fixed in space and is only turned on when the target moves into a predefined window or gate. Position and width of this gate depend on the specific behaviour of the target movement, the current angle of incidence and as such the exposed adjacent healthy tissue.

One can imagine that the treatment duration tends to mount up as only a part of the breathing cycle is involved with active radiation, hence the economical facet and comfortability remain questionable. Techniques are existing that effectively shorten the duration but all are at the expense of the patients discomfort.

Tumour tracking

The most recent member of the family of motion-adaptive radiotherapy is real-time tumour tracking (RTTT). As the dynamical counterpart of RGR, the technique entails active compensatory motion to counteract any wandering of the target. Hence the relative position between target and beam is kept constant during treatment enabling a continuous irradiation of the target.

The counteractive motion can be established by either repositioning of the patient or the beam. However feasible, the first lacks in comfort, surplus any movement of the patient could interfere with the motion pattern exhibited by the target. Therefore effectiveness remains questionable. Repositioning of the radiation source is to be preferred and is found operational in certain systems.

The latter technique can be practised with the Cyberknife Robotic Radiosurgery System. The establishment of the counteractive motion will be treated in this study, focusing on the accurate and moreover well-timed repositioning of the radiation source. Such study will demand a thorough understanding of the system and its dynamics as well as a profound insight in the interaction with the adopted control methodology.
CHAPTER 1. INTRODUCTION

1.4 Thesis outline

Tracking problems are one of the most discussed and investigated topics within the robotic research field. The common manifestation of this problem however is one for which the optimal trajectory must be planned taking into account the robot dynamics and possible obstacles along its path.

This scenario assumes the knowledge of a well considered goal that should be fulfilled within a reasonable timespan. The tracking of a tumour is in this sense essentially different from this common scenario. The trajectory is not to be planned but is imposed by circumstances.

The study portrays the design of a supervisory control structure for a six degree of freedom robotic arm, in the pursuit of accurate synchronisation of the radiation focus to a lung tissue tumour. The control structure was based on a standard model based predictive control methodology, since the presence of lag, as is inherent to less involved control methodologies, was intolerable. Basic mathematical filtering concepts were adopted and further elaborated in order to predict the tumour trajectory and added to the control.

We remark that the focus was solely directed to the repositioning of the radiation beam. Hence, this study does not address the processes and systems involved to obtain an accurate estimate of the tumour position. It was assumed that we had access to the tumour trajectory at every control step.

Once a feasible control structure was elaborated and the appropriate elements had been represented by mathematical models, a framework was constructed by extensive use of the MATLAB software which allowed to simulate the closed loop system for a diverse set of real patient cases such that the interplay of different prediction- and control levers would come forth. In some extent the observed results were discussed and elucidated.

The adopted approach may sometimes appear unwarly however one might bear in mind that this treatise does not put forth an off-the-shelf solution. The research must be approached as a proof of concept that will highlight possible directions for further research and improvement.
Chapter 2

Robotic system

Self-operating machines have dazzled the minds of great thinkers as long as written history has chronicled the coming and goings of mankind. Despite this long going fascination it is only since the second half of the previous century that the concept of a robot has eroded its current form in the industrial field. Together with the unravelling of modern technologies, during these last seventy years tremendous progress was made concerning all facets of robot-design and -control. Ever since the field of robotics has enrolled its tentacles in a large and diverse variety of applications. As the study of robotics has known this long history all aspects are well represented in scientific research and specialised literature is larded with mathematical models.

This chapter is devoted to a number of these topics in robotics that will be called upon in later chapters. Some of them will be approached from a general point of view whilst others will be applied directly on the specific problem addressed in this study, thereby circumventing a suffocating detour along digressive theory. This chapter starts with the provision of a short overview of the Cyberknife system together with an introduction of the industrial robot arm that was chosen as the robotic manipulator in this study. Hereafter the mathematical models that were utilised will be subjected to thorough consideration.

As the emphasis of this study is primarily on the design aspect of the supervisory control, the freedom was granted of simplifying some aspects of the robot model. Moreover since typical breathing patterns exhibit quite slow dynamics compared to dynamic behaviour achieved nowadays in robotics, these simplifications are justifiable and will not affect the validity of the obtained results.
2.1 The Cyberknife Robotic Radiosurgery System

Initially the Cyberknife Robotic Radiosurgery System by Accuray Incorporated (Sunnyvale, CA, USA) was designed as a medical tool capable of performing frameless stereotactic radiosurgery. It was only after, that one started to explore its employability concerning real-time tumour tracking cancer treatment.

The robotic system consist of a lightweight 6 MV X-band linear accelerator mounted on a high-accuracy multi-joined industrial robot arm. The arm can manoeuvre freely through a six-degree of freedom configuration space therefore capable of covering a large aiming area from which independently targeted (non-isocentric) and non-coplanar treatment beams can be delivered. Optimal dynamic performance is guaranteed by a proprietary lightweight design, reducing the weight of the beam-line up to 200 kg. The fully equipped system is portrayed below, in figure 2.1, the image was obtained from the company website [9].

![The Cyberknife Robotic Radiosurgery System](image)

**Figure 2.1:** The Cyberknife Robotic Radiosurgery System with off-board orthogonal kV imaging systems, robotic couch and stereo-camera.

Furthermore the system is equipped with two rigidly fixed off-board X-ray imaging systems orthogonally configured in the treatment room at 45° and 315° from the vertical axis (*image top right*). This system is used to acquire the internal target trajectory. A stereo-camera (*image top left*) configuration consisting
of three CCD cameras\(^1\) catches the light emitted by a set of LEDs\(^2\) which are most commonly fixed on a tightly fitting jacket worn by the patient during treatment, depicted in figure 2.2 the image was obtained from the company website \[^9\]. This systems provides the external trajectories, or the so called surrogate breathing signal, that are used as the input for the correlation model \[^10\] \[^11\].

![Figure 2.2: A synchrony tracking vest as used in real-time tumour tracking treatment by the Cyberknife System.](image)

As proclaimed above, the generation of the internal signal, or the real-time tumour trajectory, was taken for granted in this study. This implies the assumption that the described sensory systems, are performing flawless such that any further conclusions are related to the control rather than that they would simply reflect imperfections embedded in the correlation model.

### 2.2 Industrial robot arm

The industrial robot arm embedded in the Cyberknife system is provided by the German company, KUKA Robotics, a world leading industrial robot manufacture. While the Cyberknife is well addressed in literature as a treatment for cancer the reader is most often spared from technical details concerning the robotic system. Given this lack of specialised information, one could resort to an adequate substitute for the robotic manipulator.

\(^1\) Charge-Couple Device, an electrical component capable of converting light in an electrical signal. It is found in digital cameras as part of the system that captures the light of the image and converts it to a digital signal.

\(^2\) Light-Emitting Diode
This study emanates from the use of the KUKA, KR 120 R2700 extra HA series, see figure 2.3. Designed for high-precision specifications, this series features particularly high degrees of accuracy and fast performing wrist axes, making it specifically suitable given the nature of application. Just as the Cyberknife System, the industrial arm exhibits 6 rotational joints.

All robot information that was used in the mathematical models explicated further on can be found in the electrical catalogue published on the companies website [12] [13].

2.3 Kinematics

A mathematical description of a robotic arm might be that of an open-loop articulated chain consisting from a set of rigid bodies, so called links, connected in series by actuator driven joints. One end is attached to a supporting base while the other part is free and equipped with a tool, the so called end-effector.

Robot arm kinematics addresses to the study restricted to motion without regarding the torques and forces whereat they sprout. One studies the spatial trajectories described by the links with respect to a reference frame typically fixed to the base. Of particular interest is the analytical relation between the joint-variable space and the position and orientation of the end-effector.

Systematically we speak of a serial-link manipulator if a chain of links is
2.3. **KINEMATICS**

connected by a set of joints. Each joint holding one degree of freedom, either rotational or translational. A robot consisting of \( n \) joints, numbered from 1 to \( n \), compromises \( n + 1 \) links, 0 and \( n + 1 \) respectively corresponding to the robot base and the end-effector \([14] [15]\).

The state of a robot can thus be captured by a generalised joint variable vector \( \vec{q}(t) = [q_1(t) \ldots q_n(t)]^T \) containing the joint variables. For a rotational joint this accords to the rotation angle about the joints rotation axis with reference to the ground state. For a translational joint this accords to the translation about the joints translation axis. The arm considered for this study has 6 rotational joints.

To facilitate the mathematical description of the robot kinematics a coordinate frame is attached to each individual link. As proposed by Denavit and Hartenberg \([16]\) the transition between consecutive link frames is effectuated by a homogeneous transformation matrix. Their ideas have been somewhat extended in this treatise to circumvent a cumbersome description of the robot addressed in this study.

Each coordinate frame fixed to a link can thus be described by the homogeneous matrix \( jG_i \), containing the base vectors and base origin of coordinate frame \( i \) with respect to coordinate frame \( j \). Equation 2.1 illustrated the specific matrix representation of such coordinate frame. The application of this methodology on the robot addressed in this study is depicted in figure 2.4. Notice that a seventh coordinate frame is attached to the radiation focus point as it is the latter’s position and orientation that are persecuted.

\[
jG_i = \begin{bmatrix} jE_i & jP_i \\ 3 \times 3 & 3 \times 1 \\ 0 & 1 \times 3 \\ 1 & 1 \times 3 \end{bmatrix} \quad jE_i = \begin{bmatrix} j e_1^i & j e_2^i & j e_3^i \end{bmatrix} \quad jP_i = \begin{bmatrix} j p_1^i \\ j p_2^i \\ j p_3^i \end{bmatrix} \quad \text{(2.1)}
\]

Each transformation matrix consist of a rotation matrix \( i^{-1}R_i \) and a translation vector \( i^{-1}T_i \). Assembled into one transformation matrix \( i^{-1}DH_i \), concatenating frame \( i - 1 \) to frame \( i \).

\[
i^{-1}DH_i = \begin{bmatrix} i^{-1}R_i & i^{-1}T_i \\ 3 \times 3 & 3 \times 1 \\ 0 & 1 \times 3 \\ 1 & 1 \times 3 \end{bmatrix} \quad \text{(2.2)}
\]

The transformation matrix is constructed as such that between two consecutive links the frame is first translated and subsequently rotated along the axis of rotation according the coordinate frame of the first link, therefore the dimensional configuration of the the first link is compressed in the translation vector
while the rotational effect of the second is found in the rotation matrix. The specific form adopted by the Denavit-Hartenberg transformation matrices, applied on the KUKA R2700 is included in table 2.1.

This formulation obeys the transition law presented in equation 2.3, which demonstrates its ease of use when describing the transformation between consecutive coordinate frames.

\[ \dot{^jG_i} = \dot{^jG_{i-1}} i^{-1} DH_i \]  

(2.3)

Figure 2.4: Visualisation of the coordinate frames fixed to each link body on the KUKA, KR 120 R2700. Frames were attributed directly after a joint, orienting the frame as such that the rotation axis of the following joint coincides with one of the base vectors and the next frame is reached translating over the others.
### 2.3. KINEMATICS

Table 2.1: Specific Denavit-Hartenberg transformation matrices form as adopted for the KUKA R 2700. The subdivision of the very last translation emphasises the part corresponding to the linear accelerator and the part subscribed to the radiation beam length.

<table>
<thead>
<tr>
<th>$i \rightarrow i+1$</th>
<th>$i^{-1}R_i$</th>
<th>$i^{-1}T_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \rightarrow 1$</td>
<td>$\begin{bmatrix} \cos(q_1) &amp; \sin(q_1) &amp; 0 \ -\sin(q_1) &amp; \cos(q_1) &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0 \ 0.24 \text{ m} \end{bmatrix}$</td>
</tr>
<tr>
<td>$1 \rightarrow 2$</td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; \cos(q_2) &amp; -\sin(q_2) \ 0 &amp; \sin(q_2) &amp; \cos(q_2) \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0.35 \text{ m} \ 0.435 \text{ m} \end{bmatrix}$</td>
</tr>
<tr>
<td>$2 \rightarrow 3$</td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; \cos(q_3) &amp; -\sin(q_3) \ 0 &amp; \sin(q_3) &amp; \cos(q_3) \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0 \ 1.109 \text{ m} \end{bmatrix}$</td>
</tr>
<tr>
<td>$3 \rightarrow 4$</td>
<td>$\begin{bmatrix} \cos(q_4) &amp; 0 &amp; \sin(q_4) \ 0 &amp; 1 &amp; 0 \ -\sin(q_4) &amp; 0 &amp; \cos(q_4) \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>$4 \rightarrow 5$</td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; \cos(q_5) &amp; -\sin(q_5) \ 0 &amp; \sin(q_5) &amp; \cos(q_5) \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0.215 \text{ m} \end{bmatrix}$</td>
</tr>
<tr>
<td>$5 \rightarrow 6$</td>
<td>$\begin{bmatrix} \cos(q_6) &amp; 0 &amp; \sin(q_6) \ 0 &amp; 1 &amp; 0 \ -\sin(q_6) &amp; 0 &amp; \cos(q_6) \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0.215 \text{ m} \end{bmatrix}$</td>
</tr>
<tr>
<td>$6 \rightarrow R$</td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0.25 \text{ m} \ -0.25 - 1 \text{ m} \end{bmatrix}$</td>
</tr>
</tbody>
</table>
2.3.1 Forward Kinematics

The computation of the position and orientation of the end-effector with respect to an arbitrary reference coordinate system, given the joint variable vector $\vec{q}(t)$, is referred to in literature by the term *forward kinematics*.

The prior can be obtained by a serial application of the transition law of equation 2.3. For the sake of generality the end-effector of the serial link is chosen to be at the mounting plate such that the problem specific end-effector can be modelled in a final transformation matrix $^nDHR$, the latter contains the dimensional configuration of the chosen end-effector and is therefore by definition constant. In this study the matrix $^nDHR$ specifies the position of the part of the beam that is to be synchronised to the tumour, with respect to the mounting plate. The linear accelerator was modelled as a cube with dimensions of $0.5 \times 0.5 \times 0.5\,\text{m}$, the beam itself was said to have the illustrative length of $1\,\text{m}$.

\[
^0G_R(\vec{q}) = ^0DH_1(q_1)^1DH_2(q_2)\ldots^n-1DH_n(q_n)^nDHR \tag{2.4}
\]
\[
= \begin{bmatrix}
^0E_R(\vec{q}) & ^0PR(\vec{q}) \\
\vec{0} & 1
\end{bmatrix} \tag{2.5}
\]
\[
= K(\vec{q}) \tag{2.6}
\]

Three degrees of freedom are found in the end-effector position $^0PR$ as well as in the rotation matrix $^0E_R$, therefore exhibiting a six dimensional nature. The pre-subscript $0$ refers to the frame in which respect the vector is expressed. This will be the base frame unless specified, allowing us to drop the explicit notation.

For an illustrative example, we refer to appendix A.

2.3.2 Inverse Kinematics

The inverse kinematics problem contemplates the reverse relation between the joint variable vector $\vec{q}$ and arbitrary $G_R$.

\[
\vec{q} = K^{-1}(G_R) \tag{2.7}
\]

With the latter exhibiting six degrees of freedom an exact solution to this problem will be determinable only if the joint variable space is six dimensional as well. An analytical solution of the six dimensional problem is generally none existing, only iterative and numerical solutions are addressed in literature and exhibit high complexity. Moreover a solution is not guaranteed and a singularity might be observed due to the alignment of rotation axes reducing the effective
degree of freedom\textsuperscript{3} In this study however will be assumed that such singularities are avoided.

**Reduced Inverse Kinematics**

In such case that only small perturbations about a certain reference end-effector state $G_{R}^{ref}$ should be compensated the number of degrees and thereby the complexity of this question can be slimmed down by narrowing down the amount of joints that are considered. This strategy will be named the *reduced inverse kinematic problem*. Needles to say that such an approach involves the loss of a certain degrees of freedom of the end-effector reference state. Only the degrees of freedom that are addressed by the reduced inverse kinematics will be met accurately. However since we work in the context of small perturbations about the reference state, the joints that were chosen variable will only vary little and the effect of these small changes will not be reflected prominently in the degrees of freedom that are not considered in this reduced inverse kinematic approach.

We introduce a new notation for the joint variable vector, which is the cumulative result of a reference vector and a perturbation vector

$$\vec{q} = \vec{q}^{ref} + \delta \vec{q} \quad (2.8)$$

The reference vector $\vec{q}^{ref}$ is defined in equation \textsuperscript{2.9} and can be computed by application of numerical methods well represented in literature.

$$\vec{q}^{ref} = \mathcal{K}^{-1}(G_{R}^{ref}) \quad (2.9)$$

The perturbation vector determines the amount of joints that are considered. In general this could be any set of joints, however it will show dynamically beneficial and kinematically feasible to consider a set of ending joints. The perturbation vector is constructed as in equation \textsuperscript{2.10}

$$\delta \vec{q} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \delta q_k \\ \vdots \\ \delta q_n \end{bmatrix} \quad (2.10)$$

\textsuperscript{3}For example in figure\textsuperscript{2.4} rotation axis $y_4$ and $y_6$ align when the rotation about $x_5$ is zero. For this case a change of joint variables $q_4$ and $q_5$ will only result in a cumulative rotation about the mutual axis
Employing these freshly introduced notations we can write out a mathematical definition of the reduced inverse kinematics problem. Remember that we are working in the context of small perturbations about a reference end-effector state, such that \( G_R \approx G_R^{ref} \). Mathematically this is,

\[
\bar{q} = [q_1^{ref} \ldots q_{k-1}^{ref} \ K^{-1}_{red}(G_R)^T]
\]

(2.11)

with

\[
K^{-1}_{red}(G_R) = [q_k \ldots q_n]^T = [q_k^{ref} \ldots q_n^{ref}] + [\delta q_k \ldots \delta q_n]
\]

(2.12)

with \( K^{-1}_{red} \) considering \( G_R \) only partially, it is noted that if the element \( G_R(i, j) \), corresponding to a degree of freedom, is considered in the reduced inverse kinematics than the couple \( (i, j) \) is part of the set \( \xi_{red} \). Furthermore we introduce the matrix \( \delta G_{R}\) being the perturbation caused with respect to the desired end-effector state \( G_{R} \), due to the fact not all joints are considered variable.

Given these notations we can ascribe one explicit and one implicit property to the function \( K^{-1}_{red} \):

1. \( K^{-1}_{red}(G_R^{ref}) = \bar{0} \)
2. \( K(\bar{q}^{ref} + \delta \bar{q}) = K([q_1^{ref} \ldots q_{k-1}^{ref} \ K^{-1}_{red}(G_R)^T]) = G_R + \delta G_R \)

for which \( \delta G_R(i, j) = 0 \) if \( (i, j) \in \xi_{red} \).

Problem specific reduced inverse kinematics

Since the objective of this study is to accurately synchronise the radiation focus with the tumour trajectory the latter will determine the end-effector state containing the orientation and position of the radiation source and therefore the radiation focus. From the perspective of the treatment we will deduce a reference end-effector state and the number of degrees of freedom that should be addressed by the reduced inverse kinematic solution such that we can apply the theory of the reduced inverse kinematic problem.

- As the trajectory described by the tumour does not extent a large spacial area, the reference end-effector state can be adjusted to the trajectory center and the desired reference angle of incidence.
- The mathematical description of the orientation of the ray and so the angle of incidence coincides with the base vectors of the coordinate frame fixed to the end-effector. From the perspective of the treatment it was not required to maintain a precise angle of incidence therefore the corresponding degrees of freedom can be dropped.

\(^4\)Despite the notation, the matrix \( \delta G_{R} \) should not be mistaken for the effect resulting from \( \delta \bar{q} \) with respect to the reference end-effector state \( G_{R}^{ref} \).
The position of the radiation focus is described by the position of the end-effector. Since the objective is to synchronise the tumour trajectory and the radiation focus, it is of essence to address the corresponding degrees of freedom in the reduced inverse kinematics.

Consequently, we opted not to address all six degrees of freedom contained in \( G_R \), but only those three corresponding to the position vector \( P_R \). The reference end-effector state \( G_R^{ref} \) was said to cover the nominal angle of incidence and nominal tumour position.

It was found feasible to address the two last robot joints and additionally allow the radiation depth to vary. Perturbations, caused by the respiratory induced motion, are as such counteracted by varying the two last joints keeping the tumour on the line of radiation.

The resulting perturbations in the angle of incidence will not affect the treatment as it was stated in chapter 1 some variation is acceptable. The perturbation caused with respect to the desired \( G_R \), \( \delta G_R \), will by definition adapt the following form, if \( E_R \) was stated to be constant and equal to \( E_R^{eff} \):

\[
\delta G_R = \begin{bmatrix}
\delta E_R \\
0 \\
0
\end{bmatrix}
\]  

(2.13)

The reduced inverse problem is visualised in figure 2.5.

Figure 2.5: Geometrical visualisation of the reduced inverse kinematics problem with respect to \( \theta G_4 \), side to side with a depiction of the last two robot joints and mounted radiation source.
CHAPTER 2. ROBOTIC SYSTEM

The desired coordinates, $x_T$, $y_T$ and $z_T$, are first transformed to the fourth coordinate frame, see figure 2.4. As the first four joints are non-variable this coordinate frame is constant as well as the corresponding transformation matrix. This transformation will be considered as a part of the reduced inverse kinematics problem and is easily performed, as is demonstrated in the next equation:

$$
\begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
= ^0G_4(\vec{q}_{ref})
\begin{bmatrix}
4x \\
4y \\
4z \\
1
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
4x \\
4y \\
4z \\
1
\end{bmatrix}
= ^0G_4^{-1}(\vec{q}_{ref})
\begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
$$

(2.14)

The inverse kinematics problem can now be solved by writing out the desired coordinates in function of the two last joint variables and the beam length $X$.

$$
\begin{align*}
4x_T &= -X \sin(q_6) \\
4y_T &= b + a \cos(q_5) + X \sin(q_5) \cos(q_6) \\
4z_T &= a \sin(q_5) - X \cos(q_5) \cos(q_6)
\end{align*}
$$

(2.15)

After manipulation off the system, the problem can be solved analytically.

$$
K_{\text{red}}^{-1}\begin{pmatrix}
x_T \\
y_T \\
z_T
\end{pmatrix}
= \begin{cases}
q_5 = \tan^{-1}\left(\frac{4z_T a \mp 4y_T (4y_T^2 + 4z_T^2 - a^2)^{1/2}}{4y_T a \pm 4z_T (4y_T^2 + 4z_T^2 - a^2)^{1/2}}\right) \\
q_6 = \tan^{-1}\left(\pm \frac{4x_T^2}{(4y_T^2 + 4z_T^2 - a^2)^{1/2}}\right) \\
X = (4y_T^2 + 4z_T^2 - a^2)^{1/2}
\end{cases}
$$

(2.16)

The parameters $a$ and $b$ are constant and depend on the robot. Their values amount 46.5 cm and 21.5 cm respectively.

It should be noted that the variations that will occur in the radiation beam length, and consequent penetration depth variation, should not affect the quality of the treatment. Based on these expression we can make a statement about the maximal radiation depth deviation, $\Delta X$. This deviation should stay within the limits for which a qualitative treatment can be guaranteed. If this demand could not be fulfilled, the degree of freedom covered by the beam length $X$ should be replaced by addition of an extra variable joint angle.

For an illustrative example of the reduced inverse kinematic function, we refer to appendix A.
2.3.3 Transformational aspect

We end this section by noting that the functions $K$ and $K^{-1}$ as well as the function $K_{red}^{-1}$ can be seen as non-linear transformations between the Euclidean space, where spacial entities are represented in a homogeneous matrix form, and the joint variable space, represented by the joint variable vector. Both expression forms are therefore equal and can be translated to one another by application of the corresponding transformation.

![Transformational aspect of the kinematic functions $K$ and $K^{-1}$](figure)

2.4 Dynamics

A thorough description of the robot link dynamics was not included in this study. Such study would involve a complete mathematical discussion of the robot inertial-, Coriolis-, centrifugal- and frictional forces, the influence of gravity, the robot actuator dynamics and more. An expedition into this jungle would only distract from the objective. For a sophisticated and profound contemplation of this matter we advise the reader to consult specialised literature.

It was assumed that the robot is equipped with accurate high-speed local control loops driving each individual joint. The closed loop joint dynamics were modelled as second order transfer functions with a steady state gain of 1 and joint specific damping ratio, $\zeta_i$, and natural frequency, $\omega_i$. Therefore our supervisory control will operate at a higher control level and generate the setpoint signals imposed to the local control loops. The precise values are listed in chapter 4.

$$P_i(s) = \frac{\omega_i^2}{s^2 + 2\zeta_i\omega_i + \omega_i^2} \text{ with } i \in \bar{q}$$  \hspace{1cm} (2.17)

Such dynamic behaviour is not rarely seen in modern applications and with the rise of technologies as gravity-compensation, multi-variable predictive control and visual servoing, extraordinary performance is to be expected [15] [17] [18].
2.5 Visualisation

All visualisations of the robot were executed with the MATLAB function Cyberknife3D. This function was written in the early beginning of the development of this thesis as a helpful tool for visualising the robot and calculating the Denavit-Hartenberg transformation matrices as well as the homogeneous matrix describing the end effector's state.

Interested readers that have opened and examined the file might have struck upon a rather unconventional exploitation of the Denavit-Hartenberg formalism. The sole purpose of this function was however to visualize the geometry of the robot, and as the reader will notice, it has served its aim. As the use of this function was completely limited to visualisation and kinematic computation purposes it did not feel mandatory to obey the conventional application of the formalism.

Moreover, the code is developed as such that it can be employed to visualise almost any serial linked robotic manipulator. It should be added that the user then might want to change the geometry of the end effector to a more suitable design.
Chapter 3

Control

In chapter 1 the anticipated objective of this study was introduced in some extend from a biological perspective. The emphasis of this chapter is put on the implementation of a feasible supervisory control, effectively synchronising the radiation focus to the spacial trajectory described by the tumour. Therefore making use of the dynamical models of the robot joints together with the deployment of a mathematical model capable of accurately predicting the tumour trajectory.

3.1 Control problem

The pursued control objective is the accurate synchronisation of the radiation focus to the tumour position, the latter is approached as the cumulative result of the trajectory center and small respiratory induced perturbations about it. The anticipated objective will be governed by a supervisory control unit which will operate at a higher hierarchical level and will generate the reference signals for the lower level joint control.

The adopted approach embraces two consecutive steps:

- In a first step, the position of the robot is adjusted to the trajectory mean and the desired angle of incidence \( \bar{q}^{ref} = K_{-1}(G^{ref}) \). The execution of this step is not addressed in this study.

- In a second step, small perturbations about the reference state are compensated by adjusting the two last joints as discussed in 2.3.2. The desired joint values are computed through application of the reduced inverse kinematics on the available tumour coordinate signal.

As the nature of the problem does not allow the presence of lag, as is inherent to less involved control methodologies, a model based predictive control methodology (MBPC) was applied on the control problem.
CHAPTER 3. CONTROL

3.2 Control methodology

The MBPC methodology is based on a model of the process, which is used for calculating the prediction of the controlled variables. It is characterised by explicit on-line use of the process model to forecast the process output at future time instants and the calculation of an optimal control strategy based on the minimization of an involved cost function.

Typically such methodologies emanate from linear process models, likewise does the EPSAC. In the following subsection a brief summary to the latter was handed, the part is strongly inspired by the paper [19].

3.2.1 EPSAC

This methodology is based on the generic process model, which emanates from a completely linear context. The model is illustrated in figure 3.1.

\[ y(t) = x(t) + n(t) \] (3.1)

\( y(t) \): (measured) process output
\( u(t) \): process output
\( x(t) \): model output
\( n(t) \): (calculated) process disturbance
\( e(t) \): uncorrelated (white) noise

The disturbance \( n(t) \) includes the effects in the measured output \( y(t) \) which are not resulting from the input \( u(t) \) imposed to the process model. The disturbances are stochastic by nature and have a non-zero average value. As implied by the figure, this is modelled by a coloured noise process fuelled by a white noise zero-average signal, \( e(t) \).

Both process model as noise model are interpreted as linear filters and represented by monic polynomials in the backwards shift operator. While \( A(q^{-1})/B(q^{-1}) \) represents a discretisation of the (possibly linearised) continuous process model, the filter \( C(q^{-1})/D(q^{-1}) \) is considered to be a design parameter [20].
3.2. CONTROL METHODOLOGY

Prediction algorithm

The fundamental step in EPSAC consist in the prediction of the future process output values $y(t + k|t)$ at time instant $t$ over a prediction horizon $N_2$ indicated by the parameter $k$, based on:

- the measurements available at time instant $t$:
  $$\begin{bmatrix} y(t) & y(t-1) & \cdots & u(t-1) & u(t-2) & \cdots \end{bmatrix}$$

- the postulated future values:
  $$\begin{bmatrix} u(t|t) & u(t+1|t) & \cdots & u(t+N_2|t) \end{bmatrix}$$

Using the generic process model 3.1, the forecasting step reduces to

$$y(t + k|t) = x(t + k|t) + n(t + k|t) \quad (3.2)$$

in which $x(t + k|t)$ and $n(t + k|t)$ are computed by use of filtering techniques on the process- and noise model [19].

Control Algorithm

The control algorithm leading to the optimal choice for the postulated future control variables and therefore the corresponding control scenario is portrayed briefly in the following.

The future response is treated as the cumulative result of two components:

$$y(t + k|t) = y_{\text{base}}(t + k|t) + y_{\text{optimize}}(t + k|t) \quad (3.3)$$

We ascribe the following factors to each of the contributing effects:

$y_{\text{base}}(t + k|t)$:
- effect of past control $[u(t-1) \ u(t-2) \ \cdots]$
- effect of a base future control scenario, called $u_{\text{base}}(t + k|t), k \geq 0$, which is defined a priori. For linear systems however the choice is irrelevant, making the simple choice $u_{\text{base}}(t + k|t) \equiv 0$ appropriate
- effect of future (predicted) disturbances $n(t + k|t)$

$y_{\text{optimize}}(t + k|t)$:
- effect of the optimizing future control actions
  $$[ \delta u(t|t) \ \delta u(t+1|t) \ \cdots \ \delta u(t+N_u-1|t)]$$
  with $\delta u(t + k|t) = u(t + k|t) - u_{\text{base}}(t + k|t)$ and $N_u$ a design parameter.
The manifestation of the component \( y_{\text{optimize}}(t + k|t) \) is interpreted as the cumulative result of a series of impulse inputs and a step input.

As such the effect of impulse with amplitude \( \delta u(t + l|t) \) occurring at time instant \( t + l \) is compromised in a contribution \( h_{k-1}\delta u(t + l|t) \) to the process output at time instant \( t + k \). A final step input \( \delta u(t + N_u - 1|t) \) at time \( t + N_u - 1 \) results in a contribution \( g_{k-N_u+1}\delta u(t + N_u - 1|t) \) to the predicted process output at time instant \( t + k \).

All contributions attribute to the cumulative effect:

\[
y_{\text{optimize}}(t + k|t) = h_k\delta u(t|t) + \ldots + g_{k-N_u+1}\delta u(t + N_u - 1|t) \quad (3.4)
\]

The coefficients \( h_1, h_2, \ldots, h_{N_2} \) and \( g_1, g_2, \ldots, g_{N_2} \), respectively the impulse- and steps response of the system, are easily calculated given the process model.

Using the following concise notation, equations 3.3 and 3.4 can be summarised in the following key equation:

\[
Y = \bar{Y} + GU \quad (3.5)
\]

With

\[
Y = [y(t + N_1) \ldots y(t + N_2t)]^T
\]
\[
\bar{Y} = [y_{\text{base}}(t + N_1|t) \ldots y_{\text{base}}(t + N_2|t)]^T
\]
\[
U = [\delta u(t|t) \ldots \delta u(t + N_u - 1|t)]^T
\]
\[
G = \begin{bmatrix}
h_{N_1} & h_{N_1-1} & \ldots & g_{N_1-N_u+1} \\
h_{N_1+1} & h_{N_2} & \ldots & g_{N_1-N_u+2} \\
\vdots & \vdots & \ddots & \vdots \\
h_{N_2} & h_{N_2-1} & \ldots & g_{N_2-N_u+1}
\end{bmatrix}
\]

**Optimization**

With the objective being the achievement of a desired reference trajectory \( r(t) \) for the process output \( y(t) \) a cost function can be constructed, compromising the control objective in a mathematical cost function \( \mathcal{V} \), which should be minimized over \( U \) upon optimization:

\[
\mathcal{V}(U) = \sum_{k=N_1}^{N_2} (r(t + k|t) - y(t + k|t))^2 
\]

(3.6)

With \( N_1 \) and \( N_2 \), being design parameters. This cost function is of quadratic form and reviving the matrix notation of equation 3.5:

\[
\mathcal{V}(U) = [R - \bar{Y} - GU]^T[R - \bar{Y} - GU] 
\]

(3.7)
3.2. CONTROL METHODOLOGY

The optimal solution is obtained for:

\[ U^* = [G^T G]^{-1} G^T (R - \bar{Y}) \]  (3.8)

The actual control input \( u(t) \) can thus be computed and applied to the process.

\[ u(t) = u_{base}(t) + U^*(1) \]  (3.9)

3.2.2 Disturbance filter design

The disturbance \( n(t) \) exists of all the effects found in the measured process output \( y(t) \) which are not originated in the model output \( x(t) \). For obvious reasons, this bundled effect of all unknown disturbances can not be modelled by a physical process, moreover this is a non-measurable signal and has to be calculated by subtracting the measured process output and the modelled output \[20\].

Nevertheless, we wish to forecast this signal based on a mathematical model. This net effect is stochastic by nature with a non-zero average value and is modelled by the coloured noise process, where \( C(q^{-1})/D(q^{-1}) \) is named the disturbance model:

\[ n(t) = \frac{C(q^{-1})}{D(q^{-1})} e(t) \]  (3.10)

Within the context of MBPC it is common practice to consider this filter as a designable tool that helps the controller to handle the disturbance, thereby improving the control performance. This description induces the question how this filter should be designed. We refer to table 3.1 for the different design topologies:

- The simplest choice being design 1, such design emanates from completely uncorrelated disturbance, consequently this controller will not be able to eliminate non-zero average terms.
- A better choice being design 2. The latter results in a non-zero average disturbance signal \( n(t) \) and will therefore be able to eliminate steady-state errors in a similar way as the integrator does for a PID-methodology. This design is usually the default setting.
- Yet, no additional information about the nature of the disturbance is incorporated in the design. The former is just a mathematical construction to eliminate steady-state errors, incapable of conceiving the presence of

\[ \text{Such include process disturbances, effects of un-modelled process inputs, measurement noise, model errors, etc.} \]
any underlying mechanisms. In practical application it proves useful to subject the disturbance signal \( u(t) \) to a PDS (power density spectrum) analysis to distil the most energetic frequency component. Such knowledge can be incorporated in the design by addition of poles close to the unity circle around frequency \( f_0 \). In [20] the form adopted in design 3 was suggested. The parameter \( \alpha = 2\pi f_0 T_s \) is a discretized expression of the filter frequency, \( f_0 \), with \( T_s \) being the sampling period. The parameter \( a \leq 1 \) is a design lever governing the proximity of the poles to the unity circle and therefore the accuracy of the model\(^2\). It is noted that for \( a = 0 \), design 2 and design 3 coincide.

<table>
<thead>
<tr>
<th>design</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C(q^{-1}) )</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( D(q^{-1}) )</td>
<td>( 1 - q^{-1} )</td>
<td>( (1 - q^{-1})(1 - ae^{j\alpha}q^{-1})(1 - ae^{-j\alpha}q^{-1}) )</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Enlisting of designs adoptable for the EPSAC methodology.

### 3.3 Problem specific EPSAC application

This section contemplates the application of the concepts as they were conveyed in the previous to the specific problem addressed in this study. Since the problem exhibits quite a few signals it seemed mandatory to provide a clear introduction to all of the appearing signals before we venture deeper into the problem, such that readers on the edge of suffocation have something to regain their grip:

**Robot related signals**:

- \( P_R(t) = [x_R(t) \ y_R(t) \ z_R(t)]^T \): Radiation focus or end-effector position, expressed in Euclidean coordinates.
- \( q_R(t) = [q_5(t) \ q_6(t)]^T \): Radiation focus or end-effector position, expressed in the robot joint variable space.
- \( \dot{q}_R(t) = [\dot{q}_5(t) \ \dot{q}_6(t)]^T \): Reference signals for the lower level robot joint control.

\(^2\)The parameter \( a \) is directly related to the distinctiveness of the peak at frequency \( f_0 \) in the frequency response and therefore tightly interwoven with the accuracy of the forecast. If the disturbance frequency is well known, a value close to 1 is recommendable however if the disturbance frequency is subject to some uncertainty a lower value is advisable at the cost of frequency sensitiveness. An appropriate choice for \( a \) will therefore entail a balancing act between the knowledge of the disturbance frequency and the accuracy of the forecast.
3.3. PROBLEM SPECIFIC EPSAC APPLICATION

Tumour related signals:

- $P_T(t) = [x_T(t) \ y_T(t) \ z_T(t)]^T$: Tumour position, expressed in Euclidean coordinates. This signal is the result of the online tumour tracking system and is known up to time $t$.
- $q_T(t) = [q_5,T(t) \ q_6,T(t)]^T$: Tumour position, expressed in the robot joint variable space. Computed through application of the reduced inverse kinematics on $P_T(t)$.

The control objective being the accurate synchronisation of $P_R(t)$ to $P_T(t)$, or expressed equivalently in the robot joint variable space, $q_R(t)$ to $q_T(t)$ (2.3.3). The supervisory control unit it entrusted with the accomplishment of the latter by providing the appropriate reference signals, $\hat{q}_R(t)$, to the robot lower-level control.

3.3.1 First approach

If we were to allocate each of these signals to concepts that were conveyed in the previous section, the most natural approach would be that, suggested hereafter:

$$r(t) = K_{\text{red}}^{-1}(P_T(t))_i = q_i,T(t), \ y(t) = q_i,R(t), \ u(t) = \hat{q}_i,R(t) \quad i \in \{5,6\}$$ (3.11)

Constellating two parallel single-input single-output (SISO) control loops, each addressing one robot joint.

As is depicted in figure 3.2a there is no addition of a disturbance term as was the case for the generic process model. This assumption is justified since we can assume that the lower level control loop executes an excellent job, thereby eliminating any error. In this case the presence of an EPSAC controller is merely to avoid lag on the signal.

Nevertheless, this approach overlooks an important requirement being the knowledge of future reference trajectory values, $r(t + k|t)$, as they occur in the
cost function of the optimization step. This subtlety undermines the fundamental idea of EPSAC. This problem could be catered by expanding the EPSAC algorithm with a prediction step for the reference trajectory. It was however chosen to revise the approach, avoiding any changes to the algorithm.

### 3.3.2 Rebased control variable

Suggested by the author, is a well-considered choice of the base to which respect the control variable is expressed. As such we can successfully adopt a new control variable which reference trajectory will be known a priori by definition.

We introduce therefore the rebased control vector:

\[
\Delta_i(t) = q_{i,R}(t) - q_{i,T}(t) \Rightarrow \Delta(t) = [\Delta_5(t) \ \Delta_6(t)]^T \quad (3.12)
\]

Consequently rebasing the reference trajectories over the same vector, thereby creating a new reference trajectory vector:

\[
\hat{\Delta}_i(t) = q_{i,T}(t) - q_{i,T}(t) \equiv 0 \Rightarrow \hat{\Delta}(t) = [\hat{\Delta}_5(t) \ \hat{\Delta}_6(t)]^T \quad (3.13)
\]

#### Process model

The process model than should be revised as well and is portrayed in the figure 3.3. It is noted that this model displays quite the similarities with the generic model of figure 3.1.

As such we can interpret the reference trajectory \(q_{i,T}(t)\) as we did the model disturbance \(n(t)\). Only in this set-up, the disturbance is measurable and the process output has to be computed, despite this subtlety both are equal. The problem we did encounter with the first approach is circumvented and the forecast of the future tumor positions is inherent to the algorithm. Note that this trick would not be applicable if the assumption of a disturbance free model could not be justified\(^3\).

\[\hat{q}_{i,R}(t) \quad \text{lower level closed loop} \quad q_{i,R}(t) + \Delta_i(t) \quad q_{i,T}(t)\]

**Figure 3.3:** Process model with subtraction of the reference signal, creating the new control variable \(\Delta_i\).

---

\(^3\)It is in fact assumed that the model process output \(q_{i,R}\) does not differ from the real process output, therefore making a measurement irrelevant. We do not make a difference between model process output and real process output, and both are denoted by \(q_{i,R}\).
Disturbance prediction

As was mentioned in the previous, the forecast of the tumour trajectory will be similar to that of the disturbance \( n(t) \), emanating from a coloured noise model. Since \( q_{i,T}(t) \) is still computed by application of the reduced inverse kinematics as was portrayed by figure 3.2a, the question arises on which signal - \( P_T(t) \) - the prediction step should be applied. Due to the severe non-linear character of the reduced inverse kinematics, both will lead to different results.

Based on the filtering aspect of the prediction, we made a clear choice for the signal expected to display the strongest sinusoidal resemblance, being the tumour position expressed in the Euclidean coordinates.

The completed process model, incorporating the filtering processes resulting in the tumour position coordinates \( P_T(t) \) and the subsequent application of reduced inverse kinematics, was portrayed in figures 3.4a and 3.4b.

**Figure 3.4:** Problem specific process models
With the following filter designs for each coordinate signal:

$$\frac{C_i(q^{-1})}{D_i(q^{-1})} = \frac{1}{(1 - q^{-1})(1 - a_i e^{j\alpha_i}q^{-1})(1 - a_i e^{-j\alpha_i}q^{-1})}, \quad i \in \{x, y, z\}$$

and the lower level closed loop joint dynamics:

$$\frac{A_j(q^{-1})}{B_j(q^{-1})} = \frac{\omega_j^2}{s^2 + 2\zeta_j \omega_j + \omega_j^2}, \quad j \in \{q_5, q_6\}$$

### 3.4 Finalised control structure

In this concluding section the finalised control structure is illustrated in figure 3.5. It enforces a clear lay-out of all the occurring control related facets. Moreover, the constituting EPSAC prediction steps were displayed explicitly to emphasize the succession of control steps. For the sake of completeness we opted to depict the lower-level closed-loop unambiguously.

The scheme includes:

- Online computation of the tumour position $P_T(t)$, by detection of the surrogate breathing signals fed to the correlation model. Encapsulated in the detector block.
- Forecast of the tumour position by the prediction model block, following the coloured noise methodology. The block includes the disturbance models $C_i(q^{-1})/D_i(q^{-1})$ (equation 3.12).
- Computation of the future tumour trajectory $q_T(t + k|t)$ in robot coordinates by application of the reduced inverse kinematics on the tumour position forecast (figure 3.2a), as well as a computation of the future robot joint values $q_R(t + k|t)$ based on the postulated input vector $\hat{q}_R(t + k|t)$ and robot model (equation 3.2).
- Creation of the rebased control vector $\Delta(t)$ by subtraction upon optimizing the current model input $\hat{q}_R(t)$ (equation 3.12).
\[ \hat{\Delta}_q = 0 \]

Reference

\[ \Delta_q = q_R - q_T \]

\[ \dot{q}_R(t) \]

Robot model

\[ q_R(t + k|t) \]

Robot joints

\[ q_T(t + k|t) \]

Robot tumor coordinates

Inverse kinematics

\[ P_T(t + k|t) \]

Prediction model

Euclidean tumor coordinates

\[ P_T(t) \]


tumor

detector

Figure 3.5: Fullsize feedback loop. The EPSAC prediction steps are explicitly displayed to emphasize the control structure.
Chapter 4
Simulation & Results

The performance of the control sequence, as it was thoroughly elaborated in the previous chapter, was validated by use of the software package MATLAB on real patient tumour trajectories. Based on simulation results, the influence of various control parameters was investigated and the optimal identified. Preliminary, the tumour trajectories were submitted to profound analysis in order to distil the dominant frequencies such that, based on this information, the disturbance filters could be tuned appropriately.

As the inspiration for this study sprouted from the collaboration between research groups from the ‘Vrije Universiteit Brussel’ and the ‘Univeristeit Gent’, the possibility existed to obtain real patient tumour trajectories.

The data was acquired from the oncology department from UZ Jette and contains tumour position signals, each lasting about 20s. These signals were constructed by surrogate breathing signals that were correlated to the tumour position using a correlation model. As it has been widely cited in previous chapters, the technical aspect associated with the acquirement of these signals is not addressed in this study and it was just assumed that during control, the tumour positions were available up till time $t$.

The signals were measured with a sampling time ($T_s$) of 80 ms, this sampling time was consequently enforced to the control sequence as well.

This chapter will fixate on one patient’s trajectory to guide the reader to the entire design- and control process. As such, all steps explicated in previous chapters will be demonstrated on the considered patient signal. A total of five patient cases was made available by the UZ Jette, the results of the remaining four are included in appendix B to secure the readability of this text but still cater to the need for sufficient sample material.
4.1 Patient in the loop

With the intention of introducing the trajectory unique to the considered patient, a brief discussion was devoted to its physical features. The trajectory was portrayed in figure 4.1. Depicted are the relative tumour position coordinates with respect to the tumour trajectory center, expressed in the coordinate frame of the base. It is noted that the amplitude in the $y$ direction is about three times as large as the amplitude in $x$ and $z$ direction, though this spacial unbalance is merely a consequence of the adopted coordinate frame it will practise some effect on the trajectory forecast. The patients tumour describes an overall smooth and periodic trajectory and is therefore an ideal test case for the control framework that was elaborated in the previous chapter. Around 4 s and 18 s some irregularities arise, subsequently creating a challenging bit for the control. Some arduousness is not to be excluded.

Of primary importance is the observation that inhaling is more time consuming than exhaling, here resulting in a steeper descent than rise for the dominant $y$-signal. This observation suggest that the presence of two dominant frequencies is to be expected in the PDS, corresponding to the in- and exhale movement. It is insurmountable that this dynamic duality will have its implications for the control step addressing the forecast of the tumour trajectory. Yet, given an adequate design heuristic this duality becomes irrelevant.

![Figure 4.1: Relative tumour trajectory with respect to the trajectory center, expressed in the coordinate frame fixed to the robot base.](image)

To illustrate the spacial manifestation of the trajectory, figure 4.2 depicts in green a fading sequence of tumour positions, the current position marked by the blue asterisk. The black curves mark the trajectory projection onto the $XY$-, $XZ$- and $YZ$-plane. One could observe that the paths corresponding in- and exhale do not coincide and despite being equal in length they are not consuming equal amounts of time as was noted before.
4.2 Disturbance filter

Section 3.2.2 described how an intelligent disturbance filter could be designed, incorporating an inherent sensitivity to the expected frequency. The filter design included the choice of two signature parameters being $f_0$, the expected frequency, and $a$, the parameter governing the degree of sensitivity. This section guides the reader through the design methodology that was adopted for this study.

4.2.1 Preliminary remark

In real life application the signal analysis should be performed online, considering a moving time window to obtain and maintain an accurate estimation of the breathing trajectory frequency. However, as we had limited access to real patient data, the time signals were not of adequate length to investigate the performance of real time frequency estimation performed parallel to the control sequence and was therefore not addressed in this study. Consequently, the analysis was
executed off-line and prior to the control, resulting in a disturbed chronology of the design methodology. Nevertheless, since this study was construed as a proof for the applicability of the concept, the consequences of this contradiction were assessed acceptable and did not practise significant influence on the obtained results.

4.2.2 Frequency analysis

The presence of dominant frequencies in disturbance signals can be brought to light by a PDS analysis. Such analysis was executed on the relative tumour coordinate signals. Each of them was first zero-padded by element-wise multiplication with a Blackman function of equal length before the PDS was determined by use of the Fourier transform. The resulting power density spectra are portrayed in figure 4.3.

![PDS analysis graphs](image)

**Figure 4.3:** Power density spectrum of the relative tumour coordinate signals.
Neglecting the mutual shift in amplitudes, it is observed that each of the coordinate signals displays a similar PDS. The two reoccurring bumps indicate the presence of two dominant frequencies, as was expected. These latter, arranged by increasing order, correspond to the movement associated with the in- and exhalation respectively. In an attempt to cover both frequencies in the design frequency, $f_0$, one could resort to a design heuristic whereby the design frequency is equated with the geometrical mean of the in- and exhale frequencies. The adopted approach is discusses in the following section.

### 4.2.3 Filter design

**Design heuristic**

It can be expected, when either the frequency of in- or exhaling would be adopted in the design, the trajectory forecast will be more accurate on the corresponding trajectory sections. It should be added that the tumour will be longer exposed to inhale than exhale movements and that, from the perspective of treatment, an overall better performance will be obtained if the forecast is more accurate over longer periods of time. Given this reasoning, we opt to assimilate the design frequency to the inhale and thus slower frequency. Patient specific values are depicted in table 4.1.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$f_{i}^{in}$ [Hz]</th>
<th>$f_{i}^{ex}$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$, $y$, $z$</td>
<td>0.2783</td>
<td>0.5313</td>
</tr>
</tbody>
</table>

*Table 4.1: Patient specific breathing frequencies for each of the coordinate signals.*

**Tuning the accuracy**

The design frequency, resulting from the PDS analysis and the opted design heuristic, can now be incorporated in the filter, leaving the question which value for $a$ should be adopted. By definition the choice is limited to values in between 0 and 1. It seemed expedient to get a taste of the behaviour close to these extrema, in order to argue an appropriate choice for the value of $a$.

A value of $a$ close to 0 will lean towards the default design, therefore not able to pick up the dominant frequency. In the time domain this corresponds to a forecast that is equal to the last known value. A value of $a$ close to 1 however will fixate to strongly to the design frequency and will choke in the presence of secondary frequencies. The manifestation of this behaviour in the frequency domain is illustrated in figure 4.4 where the bode plot of the filter is plotted.
over the PDS of the corresponding signals. Since both contain a different energy the bode plot of the filter was shifted such that the similarity in shape could be verified by the eye. By mathematical construction only for a value of $a$ close to 1 there is a visible manifestation of the design frequency.

As these filters will be utilised to forecast the evolution of the tumour trajectory at a certain time $t$, the influence of $a$ should be investigated within this context as well. The quality of the forecast, thereby focusing on the influence of the parameter $a$, was verified by isolating the trajectory forecast from the control sequence. The influence of $a$ in the time domain is illustrated in the figures 4.5, 4.6 and 4.7 on each of the coordinate signals. The real trajectory is compared with two trajectory forecasts utilising different filters. Both are based on the tumour trajectory known at time instant $t - k$, therefore making a forecast of $k$ timesteps. The trajectories were predicted utilising the default filter (design 2) and the intelligent filter (design 3), once for a value of $a$ of 0.65 and once for $a$ equal to 0.90.

We observe that as the forecast horizon rises, the prediction becomes less accurate for both scenarios, implicating an inherent limitation of the disturbance model. Furthermore it is noted that for a larger value of $a$ the prediction is subject to a more fierce presence of noise. As this seemingly pleads the choice of a smaller value of $a$, we remark that for the $y$- and $z$-coordinate signal, a larger value leads to a stronger absence of lag.

The obtained results and applicability of a coloured noise process model in order to provide an accurate forecast of the tumour trajectory is threatened to become overshadowed by two imperfections. On the one hand, the $x$-coordinate exhibits substantially less accurate forecast for both small or large values of $a$ if compared to the other two coordinate signals. Such was to be expected given the PDS analysis of the $x$-coordinate signal revealed the presence of more dominant sub-frequencies. On the other hand, it is noted that for all of the predicted signals accuracy is rapidly diminishing if the prediction horizon rises.

These visual results strongly suggest that an intermediate value should be adopted if optimal performance is pursued. Hence the parameter $a$ was interpreted as a control lever wherefofe the search for an optimal value will be addressed in the optimization of the control sequence.
Figure 4.4: Overlay plot of the PDS of the relative coordinate signals and the bode frequency answer of the disturbance filters, for varying values of the parameter \(a\).
Figure 4.5: Isolated trajectory forecasts by utilisation of the intelligent disturbance filter for different values of $a$. Portrayed are the predicted trajectory values of the $x$-coordinate signal based on the measured coordinate values $k$ timesteps ahead compared to the actual coordinate signal and the result of the default filter.
Figure 4.6: Isolated trajectory forecasts by utilisation of the intelligent disturbance filter for different values of $a$. Portrayed are the predicted trajectory values of the $y$-coordinate signal based on the measured coordinate values $k$ timesteps ahead compared to the actual coordinate signal and the result of the default filter.
Figure 4.7: Isolated trajectory forecasts by utilisation of the intelligent disturbance filter for different values of $a$. Portrayed are the predicted trajectory values of the $z$-coordinate signal based on the measured coordinate values $k$ timesteps ahead compared to the actual coordinate signal and the result of the default filter.
4.3 Simulator

Once the filter design frequency was tuned to the patient tumour trajectory, a software framework was established within the MATLAB environment to simulate the control sequence. SIMULINK continuous models of the robot joint lower-level closed-loop dynamics were coupled to the governing software framework, generating a zero-order hold control scenario, based on real time measurements of the tumour trajectory. Within this governing software framework the control structure of [3.3] was adopted. Before the actual control sequence can be launched and the synchronisation takes place, this governing framework runs through a number of initialisation steps wherefore still two more questions, remain unanswered.

4.3.1 Simulator initialisation

Reference end-effector state

The first addresses the reference end-effector state. Hitherto only relative tumour coordinates with respect to a certain tumour trajectory center were addressed, the specific trajectory center was not yet specified as well as the desired angle of incidence.

It was explained in section [2.3.2] that both are compromised in the reference end-effector state, \( G_{R}^{ref} \), or so in the corresponding reference joint variable vector, \( \tilde{q}^{ref} \). In real application this reference state would depend on the position of the patient and the current phase of the treatment. However, since this is only an illustration of the elaborated methodologies, we are granted the freedom of choosing one ourselves.

\[
\tilde{q}^{ref} = \begin{bmatrix}
\frac{\pi}{4} & \frac{\pi}{8} & \frac{\pi}{8} & \frac{\pi}{8} & -\frac{\pi}{4} & 0
\end{bmatrix}^T
\]  

(4.1)

The relative trajectory was then superposed on the corresponding trajectory center in order to generate a feasible trajectory for the control. The according robot configuration is portrayed in figure [4.8]
Robot joint dynamics

The second open question concerns the robot joint lower-level closed-loop dynamics of the two ending joints. They were modelled by two continuous second order transfer function whom general form was introduced in section 2.4.

The specific joint parameters are enlisted in table 4.2 and are an approximation of responses that can be expected in high-accuracy applications [15] [17] [18]. In reality they should either be obtained by extensive modelling of the robot actuators and lower level control or by the means of identifications methods as are abundantly available in literature.

\[
\begin{align*}
\hat{q}_5 \rightarrow q_5 & : 3\pi & \zeta_5 = 0.9 & \frac{\omega_i}{s+5.4\pi s+9\pi^2} & \frac{P_i(s)}{P_i(q^{-1})} = \frac{3\pi}{s^2+5.4\pi s+9\pi^2} + 0.1823q^{-1} + 0.1156q^{-2} \\
\hat{q}_6 \rightarrow q_6 & : 4\pi & \zeta_6 = 1 & \frac{\omega_i}{s+8\pi s+16\pi^2} & \frac{P_i(s)}{P_i(q^{-1})} = \frac{4\pi}{s^2+8\pi s+16\pi^2} + 0.267q^{-1} + 0.1361q^{-2} \\
\end{align*}
\]

Table 4.2: Robot joint closed-loop transferfunction parameters.

Furthermore, it is noted that the imposed sampling time of 80 ms practises a limiting effect on the robot dynamics. A settling time of about 1 s may be considered as slow for high-accuracy applications. However we resorted to such hebetude dynamics since the discretized step response of the models are incorporated in the control algorithm (section 3.2.1). If it were to be that the dynamics would reach the step setpoint within a single time step, optimization would become redundant.

Yet another aspect that was aimed to be covered is the increased inertial and gravitational forces experienced by \(q_5\) with respect to \(q_6\). As the latter is only
subject to the mass of the radiation source, \( q_5 \) will be subject to both radiation source as previous robot link. Therefore slower dynamics were attributed to it.

Figure [4.9] portrays an overlay plot of the continuous and discrete stepresponses of the robot joint closed-loop transferfunctions. The discretized transferfunctions were used as the polynomial models of the real process, the continuous transferfunctions were used as the real processes and were implemented in the SIMULINK program.

![Figure 4.9: Stepresponses of the robot joint closed-loop transferfunctions.](image-url)
4.3.2 Results - robot joint variable space

With both questions answered we can pursue to simulate the control sequence. An illustrative set of control parameters \((N_1 = 1, N_2 = 3, N_u = 1\) and \(a = 0.75\)) was implemented in the governing program and the synchronisation was launched. The results are summarised in the figure 4.10.

Control variable

Figure 4.10a portrays the joint vector \(q_R = [q_{5,R}, q_{6,R}]^T\), the tumour position expressed in the robot joint variable space, \(q_T = [q_{5,T}, q_{6,T}]^T\), and the actual control variable vector \(\Delta = [\Delta_5, \Delta_6]^T\), with the latter shifted to the corresponding reference joint value for clarity. It can be visually verified that the control sequence has successfully synchronized the radiation focus and tumour position both expressed in the robot joint variable space, over a period of 20 sec. Consequently, aside the foreseeable noise, the rebased control variables \(\Delta_5\) and \(\Delta_6\) maintain an approximated value of 0. Even though the effect of the adopted frequency design heuristic practices little influence on the result, it is of visually verifiable presence. As one can notice in the slightly inferior accuracy on the descending sections corresponding the exhale motion and thus the exhale frequency. Due to the small amplitude of the perturbation about the center the inverse kinematic function exhibits almost linear behaviour. Furthermore one should notice the clear dominance of the \(y\)-coordinate signal in the reduced inverse kinematic problem. Appendix B provides more balanced examples.

Control input

Portrayed in figure 4.10b are the lower-level joint reference signals \(\hat{q}_5\) and \(\hat{q}_6\), side by side with the tumour position expressed in the robot joint variable space. As we resorted to the EPSAC control methodology to avoid the lag inherent to less sophisticated approaches, a positive phase shift is present in the reference signals. Hence, the control undertakes action before the actual disturbances had occurred thereby successfully circumventing the manifestation of lag between the radiation focus and the tumour position. The ill conditioned reference values at the very beginning of the control are a consequence of the sudden commutation from standstill to synchronisation. In real life application the consequences of such are simply avoided by starting the treatment only after the radiation focus has locked on the tumour position.
4.3. SIMULATOR

Radiation depth

The very last figure 4.11 depicts the radiation depth variation which is limited to the order of 1 cm. However it does depend on the chosen and illustrative radiation beam length and therefore will be less profound in real life application.

4.3.3 Results - Euclidean coordinate space

In order to obtain a visual indication of performance in the euclidean space, we resort to the forward kinematics, as introduced in chapter 2. The joint signals from the previous paragraph are now fed to the forward kinematics, as such the position of the radiation focus can be uncovered. We note that the radiation length is varied accordingly, superposing the radiation depth variation on the nominal beam length. Results are portrayed in figure 4.12.

Again inferior accuracy is observed in the section corresponding to inhalation as a direct result of the adopted disturbance filter design heuristic. Furthermore it is observed that the $x$-coordinate signal is displaying more profound deviations than either the $y$- or $z$-coordinate signal. Such is an immediate manifestation of the trajectory prediction model falling short for the $x$-signal.
Figure 4.10: Simulation results of the synchronisation of the radiation focus to the real time tumour position. Control parameters were set to $N_2 = 3$, $a_i = 0.75$ and $f_i = 0.2783$ Hz with $i \in \{x, y, z\}$

Figure 4.11: Radiation depth variation. ($N_2 = 3$, $a_i = 0.75$ and $f_i = 0.2783$ Hz with $i \in \{x, y, z\}$)
Figure 4.12: Radiation focus and tumour position in Euclidean coordinates. ($N_2 = 3, a_i = 0.75$ and $f_i = 0.2783$ Hz with $i \in \{x, y, z\}$)
4.4 Optimization

4.4.1 Performance indicator

In an attempt to quantify the performance associated with a set of control parameters, we resorted to a performance indicator more objective than the human eye. This indicator emulates to compress the accuracy of the synchronisation associated with one control setting over an entire patient trajectory into a single number. We postulate the root mean square (RMS) value of the control variables, \( \Delta_5 \) and \( \Delta_6 \), with respect to the 0 RMS value that ought to be obtained with perfect synchronisation. The smaller the value, the better is performance.

\[
RMS(t_0 \rightarrow t_N) = \frac{\sum_{k=t_0}^{t_N} \Delta_i^2(k)}{t_N - t_0} \tag{4.2}
\]

4.4.2 Optimal parameters

The parameters that were labelled as control levers are the control horizons \( N_1 \), \( N_2 \) and \( N_u \) and the sensitivity parameter \( a \).

However it is not relevant to address the parameter \( N_1 \) as it specifies the first future input that is considered to vary, obviously the sooner the better. This parameter was frozen to a value of 1. As well was the parameter \( N_u \) kept to the constant value of 1 in order to suppress the otherwise negative effect of a poor forecast of the sections in the tumour trajectory which are displaying irregularities. As was demonstrated in section 4.2.3 the forecast for such sections became unreliable, especially for increasing values of \( N_2 \). Since this might undermine the overall accuracy of an otherwise well performing set this parameter value was said to be constant as well.

The interplay of the parameters \( a \) and \( N_2 \) can now be mapped, moreover we would like to investigate for which couples of \( a \) and \( N_2 \) the most accurate synchronisation is achieved. Figure 4.13 illustrates such a mapping, it contains the RMS values for the control variables for couples of \( N_2 \) and \( a \) ranging from 3 to 11 and 0.4 to 0.8 respectively. All the contained couples were implemented in the main program and simulated on the patient tumour trajectory.

One observes that the surfaces exhibit valley like shapes that slope down for decreasing values of \( N_2 \) and an intermediate value of \( a \). It is found that a minima occurs on both planes for \( N_2 = 3 \), which is a direct translation of the diminishing prediction accuracy for rising prediction horizons. The corresponding value of \( a \) differs for each control variable. In figure 4.14, the curves resulting from the intersection of the performance map and associated \( N_2 \)-planes are depicted.
4.4.3 Online performance optimization

We end this section with the remark that these performance planes will be highly patient- and therefore case-specific. Even in between two apart treatments these will vary as a result of a variety of casualties as the possible progression or retrogression of the patient, the patient emotional status and many more.

Hence, again a certain duality is present since again we computed these performance parameter surfaces after the treatment therefore in this form they are of no use during treatment. Yet we can distil a general shape which is expected to be reoccurring in all performance surfaces.

Moreover during real life treatment a framework could be constructed, that simulates the behaviour for different sets of parameters than the one currently incorporated in the control. Whenever it occurs that a simulated set would exhibit superior accuracy than the current set, a transition should be made to the superior one. Not to overload the simulation a set searching algorithm can be adopted that emanates from the assumption that the current set is in fact the minima of the valley such that only neighbouring sets should be investigated instead of the whole parameter plane.
Figure 4.13: Performance mapping with respect to the parameters $N_2$ and $a$, ranging from 3 to 11 and 0.4 to 0.8 respectively. ($N_1 = 1, N_a = 1$)

(a) Performance map for the control variable $\Delta_5$

(b) Performance map for the control variable $\Delta_6$
Figure 4.14: Parameter performance curves resulting from the intersection of the performance plane with corresponding \( N_2 \) planes.
Chapter 5

Discussion & Conclusion

The emphasis of this study was directed to the design of a feasible control structure inspired on a standard predictive control methodology. Consequently some of the elements interfering with the problem statement were subjected to substantial simplification. It is duly noted that the author is aware of such and has by all means acted to prevent any conflict of over simplification that would affect the validity of the proof.

This chapter will address all aspects that are open for reconsideration and bring to light what possible enhancements could be undertaken. In a final section we state formally our conclusions and plead the way for future work.

5.1 Open for reconsideration & Future work

5.1.1 Tumour prediction model

This study emanates from the utilisation of mathematical filtering techniques in order to forecast the tumour trajectory. It is remarked that such might be a more naive approach even so it has been tried to include the nature of the disturbance. Throughout the result, its shortcoming were brought to light.

Despite these filtering techniques are adjusted to expect a certain disturbance frequency, they will never be able to mimic a real breathing pattern. Therefore they lack the understanding of the biological mechanisms that lie at the very origin of tumour movement.

Further research should be performed, addressing the reliability of the tumour trajectory forecast and the development of more evolved prediction models.
5.1.2 Dynamic robot model

A secondary aspect are the dynamic robot joint models, as they were suggested in this study. Despite the choice was justified, both the speed and supposed linearity are open for reconsideration.

The choice embraced rather slow dynamic behaviour. On the one hand, such was imposed by the sampling time of the tumour position measurement, forcing the dynamics to larger settling times as can be obtained nowadays. Since the discretized stepresponse is incorporated within the adopted control methodology’s optimization step, a settling time displaying only a few timesteps was unacceptable. On the other hand, one could strive that such could be a deliberate choice to keep the robot dynamics in pace with the disturbance dynamics.

Robot dynamics are by nature heavily non-linear, therefore the assumption that the closed lower-level control loops will be linear can only be an approximation. Possibly this could be accounted for by incorporating a realistic robot joint control structure in the SIMULINK model. However, it is noted that we are in fact working in a small area around a reference state, implying that the dynamics could be seen as a local linearisation. Moreover, in the design of the lower-level control one could strive to approximate such linear behaviour at the cost of slower dynamics.

5.1.3 Angle of incidence

This work did not spend proper thought to the angle of incidence. While this is an aspect of the treatment that evokes further exploration. The spacial configuration of the tumour trajectory as portrayed in figure 4.2 could be exploited in favour of the treatment. If the arrangement would be as such that the angle of incidence is aligned with the overall direction vector of the tumour substantially less corrections should be executed to ensure the synchronisation of tumour and radiation focus, resulting in a more economical treatment.

Moreover this study restricted the reduced inverse kinematic function to the two last joints, this number could be expanded without adapting the control structure, if a higher degree of freedom were to be desired.
5.2 Conclusion

5.2.1 Thesis Overview

We seize the opportunity to revise the work that has been done in this study.

In chapter 1 the problem was formulated from a biological perspective, preceded by an introduction to some of the existing techniques coping with inherent tumour movement during radiotherapeutic treatment. Focus was put on the real-time tumour tracking (RTTT) approach as employed by the Cyberknife Robotic Radiosurgery System. The concluding sections presented an outline of the objectives that were to be pursued during this study. Namely, to explore the applicability of a standard model based predictive control methodology to manage accurate synchronisation between radiation focus position and tumour and to construct a software framework wherein performance could be validated.

Chapter 2 subjected the robotic system to thorough consideration, thereby elaborating kinematical and dynamical models. A substantial reduction of the problem complexity was found feasible based on biological justification. A solution for this reduced problem was suggested and elaborated by the author.

Subsequently, the configuration was stripped down from its physical shell such that a feasible control problem could be distilled. Chapter 3 elaborated a customized supervisory control algorithm based on a standard model based predictive control methodology. By means of mathematical filtering techniques, a prediction model was suggested to forecast the tumour trajectory.

In chapter 4 the control structure from chapter 3 was linked to the robot models elaborated in chapter 2. As such the effect of various control levers could be explored. The chapter concluded with a suggestion to incorporate the simulation aspect in real life treatment while aiming for therapy optimisation.

5.2.2 Allocation of study

We have been successful in customizing a general control structure to the specific problem addressed in this study. The validity of the concept has been demonstrated by simulations executed on realistic patient tumour movement.

The main accomplishment is attributed to the fact the customized control structure has been proven to be feasible. As such we plead for future work to further elaborate the simulation framework at the many fronts that we mentioned above. A realistic simulation framework would seize the possibility to further explore and improve the utilisation of radiation within the strife against cancer.


Appendix A

Demonstration of kinematics

A.1 Forward kinematics

The forward kinematic function $K$ demonstrated by means of an illustrative example. The robotic system is visualised over a path described by:

$$\bar{q}(t) = \underbrace{\bar{q}_{\text{end}} - \bar{q}_{\text{ini}}} + \frac{\bar{q}_{\text{ini}}}{2} - \cos(t)$$  \hspace{1cm} (A.1)

Wherefore

$$\bar{q}_{\text{end}} = \begin{bmatrix} \pi/4 \\ \pi/8 \\ \pi/8 \\ \pi/8 \\ -\pi/4 \\ 0 \end{bmatrix}^T$$  \hspace{1cm} (A.2a)

and

$$\bar{q}_{\text{ini}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$  \hspace{1cm} (A.2b)

The computer visualisations of the robot configuration were obtained by application of the MATLAB function CYBERKNIFE3D.
A.2 Reduced inverse kinematics

The reduced inverse kinematics, compromised in the function $K^{-1}_{\text{red}}$, is demonstrated in figure A.2. The premised reference state, $\bar{q}_{\text{ref}}$, is the ending state, $\bar{q}_{\text{end}}$, from the previous example. On the according trajectory center, an illustrative signal was superposed.

$$G_R(t) = G_{R}^{\text{end}} + \begin{bmatrix} 0 & \Delta P(t) \end{bmatrix} \Delta P(t) = \begin{bmatrix} \frac{1}{4} \sin(t) \cos(t) \\ \frac{1}{4} \sin(t) \\ \frac{1}{4} \sin(t) \end{bmatrix}$$  \hspace{1cm} (A.3)

As was explained in section 2.3.2, it is was found kinematically feasible only to adjust the two last joint values in order to track the desired spacial path, if in addition the radiation depth was allowed to vary. It is noted that this was emphasised in the figure by a green segment covering the according variation.

The computer visualisations of the robot configuration were obtained by application of the Matlab function Cyberknife3D.
Figure A.1: Demonstration of the forwards kinematics, visualised by the MATLAB function `CYBERKNIFE3D`, covering the trajectory $\bar{q}(t)$. 
Figure A.2: Demonstration of the reduced inverse kinematics, visualised by the MATLAB function Cyberknife3D, covering the path $\Delta P(t)$.
Appendix B

Patient log files

Here we include results as they were obtained for different patient tumour trajectories in order to cater to the need for sufficient sample material. The parameter values are chosen equal to those of the example in chapter 4 in order to maintain consistency ($N_u = 1$, $N_1 = 1$, $N_2 = 3$, $a = 0.75$). As such the performance of the proposed control structure can be compared unambiguously for different trajectories.

UZ Jette provided a total of five signals, three measured with patient 1 and two with patient 2. The second signal of patient 1, was used as the example in chapter 4. The others are included hereafter. In addition a rescaling of the example trajectory was included, governing a more balanced, in sense of coordinate amplitudes, and space consuming trajectory. The latter is referred to as trajectory X. The $x$-, $y$- and $z$-coordinate signal were multiplied with a factor 20.5 and 20 respectively.

This appendix is structured as follows. First each patient trajectory with respect to the trajectory center, is portrayed. The trajectories were each submitted to a PDS analysis, dominant frequencies are included in a table. Subsequently we portray the corresponding simulation results.
B.1 Tumour trajectories

Figure B.1: Patient 1. Trajectory 1 is displaying larger irregularities than both trajectory 2 (fig. 4.1) and 3. The $z$-coordinate of trajectory $X$ is now dominant, moreover the amplitudes are of a magnitude larger. This will result in a more emphasized manifestation of the non-linearity inherent to the inverse kinematics.

Figure B.2: Patient 2. Note the fierce inconsistency between both trajectories, strongly indicating that patient specific parameters should be revised each treatment. In addition the truncated peaks might cause problem for the prediction model.
B.2 PDS analysis

We have solely included the results of the PDS analysis, they are summarised in table B.1. Note that the trajectories of patient 2 were not found to contain a second dominant frequency allocated to the exhale motion. Such was to be expected given the trajectories portrayed in figure B.2.

<table>
<thead>
<tr>
<th>Patient</th>
<th>Trajectory</th>
<th>$f^{in}_i$</th>
<th>$f^{ex}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.2784</td>
<td>0.5315</td>
</tr>
<tr>
<td>3</td>
<td>0.2531</td>
<td>0.5316</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>0.2783</td>
<td>0.5313</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.3287</td>
<td>/</td>
</tr>
<tr>
<td>2</td>
<td>0.2269</td>
<td>/</td>
<td></td>
</tr>
</tbody>
</table>

Table B.1: PDS results
B.3 Simulation results - Robot joint variable space

B.3.1 Control variables

Figure B.3: Patient 1. Remark the dominance of the $y$-coordinate signal for trajectory 1 and 3 in the inverse kinematic for both $q_5$ and $q_6$, as was the case for trajectory 2. Trajectory X however displays quite a different robot trajectory, given that the $z$-coordinate is now dominant for the $q_6$ joint variable as was the purpose. In addition, note the overshoots at the irregularities, they are damped however by the robot dynamics and quickly corrected by the prediction model itself.

Figure B.4: Patient 2. Visual results are good. The truncated peaks did not cause trouble for the trajectory prediction and are well synchronised. Moreover it seems that the prediction model had no problem with the constant sections.
B.3.2 Control input

Figure B.5: Patient 1.

(a) trajectory 1

(b) trajectory 3

(c) trajectory X

Figure B.6: Patient 2.
B.4 Simulation results - Euclidean coordinate space

![Graphs of coordinate signals for Patient 1 and 2 with trajectory 1, 2, 3, X and Y.

Figure B.7: Patient 1.

Figure B.8: Patient 2. The error on both all coordinate signals is equally large, however due to the fact the $x$-coordinate amplitudes is of a magnitude smaller than the others, the error is seemingly more pronounced.
B.5 Performance

B.6 Performance maps

Figure B.9: Patient 1.
Figure B.10: Patient 2.
B.7 Performance curves

Figure B.11: Patient 1.
Figure B.12: Patient 2.