Computational modeling of surface roughness effects in copper

Niels Lambrecht

Supervisors: Prof. dr. ir. Daniël De Zutter, Prof. dr. ir. Dries Vande Ginste
Counsellor: Ir. Dieter Dobbeelaere

Master's dissertation submitted in order to obtain the academic degree of Master of Science in Electrical Engineering

Department of Information Technology
Chairman: Prof. dr. ir. Daniël De Zutter
Faculty of Engineering and Architecture
Academic year 2013-2014
Computational modeling of surface roughness effects in copper

Niels Lambrecht

Supervisors: Prof. dr. ir. Daniël De Zutter, Prof. dr. ir. Dries Vande Ginste
Counsellor: Ir. Dieter Dobbelaere

Master's dissertation submitted in order to obtain the academic degree of
Master of Science in Electrical Engineering

Department of Information Technology
Chairman: Prof. dr. ir. Daniël De Zutter
Faculty of Engineering and Architecture
Academic year 2013-2014
Preamble

First and foremost, I want to thank Prof. De Zutter and Prof. Vande Ginste to give me the opportunity and resources to study this subject. The realization of this thesis was a special learning experience, which I am very pleased to be able to look back at. It was a great honor and an even greater pleasure to do my thesis with them as my supervisors.

I also want to thank my counselor Dieter Dobbelaeere in particular. Without his valuable advice and his instructive discussions during the year, this thesis would not have been the same. I also want to thank him for revising and improving this master thesis.

My fellow thesis students, in no particular order, Olivier Caytan, Irven Aelbrecht, Martijn Huynen, Erica Debels and my new Italian friend Lorenzo Silvestri earn a recognition for the pleasant company in the thesis room.

I also wish to thank my parents and friends for the support they gave me.

Niels Lambrecht, May 2014
Admission to loan

"The author(s) gives (give) permission to make this master dissertation available for consultation and to copy parts of this master dissertation for personal use. In the case of any other use, the limitations of the copyright have to be respected, in particular with regard to the obligation to state expressly the source when quoting results from this master dissertation."

Niels Lambrecht, May 2014
Computational Modeling of Surface Roughness Effects in Copper

by

Niels LAMBRECHT

Master’s dissertation submitted in order to obtain the academic degree of Master of Science in Electrical Engineering

Academic year 2013-2014

Supervisors: Prof. dr. ir. D. DE ZUTTER, Prof. dr. ir. D. VANDE GINSTE
Counsellor: ir. D. DOBBELAERE
Faculty of Engineering and Architecture
Ghent University
Department of Information Technology
Chairman: Prof. dr. ir. D. DE ZUTTER

Summary

This master’s thesis aims to extend our knowledge and to improve our intuition in the effects of surface roughness in copper. For this, we solve a 2D electromagnetic problem using a boundary element method, together with the method of moments. We make use of a parallel-plate waveguide to observe the behavior of the fields inside the waveguide, as well as the current distribution in and around the surface roughness of the copper walls, by means of simulations. We also implement a method to calculate the fields in a conductive medium, from which the current density can be deduced.

Keywords

Computational electromagnetism, surface roughness, method of moments
Computational Modeling of Surface Roughness Effects in Copper
Niels Lambrecht

Abstract—This master’s thesis aims to extend our knowledge and to improve our intuition on the effects of surface roughness in copper. For this we solve a 2D electromagnetic problem using a boundary element method, together with the method of moments. We make use of a parallel-plate waveguide to observe the behavior of the fields inside the waveguide, as well as the current distribution in and around the surface roughness of the copper walls, by means of simulations.

Keywords—Computational electromagnetics, surface roughness, method of moments

I. INTRODUCTION

In the past years electronic devices, integrated circuits, packaging and assembly have shown a trend towards greater miniaturization. Also the speed of electronic signals has increased with data rates approaching 100 Gb/s. At high frequencies, the skin effect causes current crowding in the conductors, and in combination with the surface roughness profiles of the conductors, the skin effect leads to signal integrity problems. Some [1] [2] research has been done to study the effects of surface roughness, but none of them gave insight into the mechanism of the current density inside the surface roughness. The origin of surface roughness comes from the need to have a better trace-to-substrate adhesion on printed circuit boards [2]. To calculate the fields from Maxwell’s equations, a 2D solver is used. In this master’s thesis we used Nero2d, the 2D Maxwell solver which is developed at INTEC, Ghent university [3]. The purpose of this master’s thesis is to get more insight in the effects due to surface roughness. For this we will make use of a parallel-plate waveguide in which we will insert an isolated surface roughness element and from which we will analyze the fields inside the waveguide. We will also analyze the current density in and around the surface roughness of the copper walls. To accomplish this, Nero2d is extended to be able to calculate the fields inside a conductive medium from which the current density can be deduced. To the authors’ best knowledge, no results have been published about the current density behavior in a rough surface. This is the main purpose of this master’s thesis. The used geometry and methodology for calculating the fields inside a conductive medium is presented in Section II. The simulations of the field behavior inside the waveguide and the current density in the surface roughness is presented in Section III. Section IV presents a conclusion of the results of this master’s thesis.

II. GEOMETRY AND METHODOLOGY

A. Geometry

For analyzing the fields we use a parallel-plate waveguide with an inserted surface roughness element (Fig. 1). The parallel-plate waveguide is terminated at one side to avoid unwanted reflection on the edges of the waveguide and to have full control over the behavior of the fields inside the waveguide, so that we have a clear view of what the influence of the surface roughness is.

![Fig. 1. The geometry of the terminated waveguide with surface roughness.](image)

We only want the fundamental quasi-TEM mode to propagate inside the waveguide, which is the case if the distance between the plates d is less than half a wavelength. In this master’s thesis, we always choose the distance, d, between the plates as a quarter of a wavelength. The thickness of the plates, a, is chosen to be several skin depths large. The operating frequency of the simulation is 300 MHz. The length of the waveguide, L, is chosen to be 20 m, such that all the unwanted higher order modes have died out away from the roughness element and waveguide edges.

B. Methodology

As we are dealing with a 2D electromagnetic problem the Green’s function of the scalar Helmholtz equation is given by:

\[ G(\rho|\rho') = \frac{j}{4} H_0^{(2)}(\gamma|\rho - \rho'|), \]

where \( H_0^{(2)} \) is the Hankel function of the second kind and order 0, \( \gamma \) is the propagation constant of the medium and \( \rho \) is the position vector in the xy plane with \( \rho = xu_x + yu_y \). In a highly conductive medium the wavenumber becomes \( \gamma = \frac{1}{\delta} \), with \( \delta \) the skin depth. The large imaginary part of the propagation constant causes a strong exponential decay of the Green’s function. Moreover, we will encounter rapid oscillations of the Green’s function. This gives rise to inaccurate evaluation of the interaction integrals. In Dobbelaere et al. [4] a solution was found to solve this problem. To solve this problem, the numerical quadrature points are placed only in those regions where the Green’s function has non-negligible value. So one can then approximate the Green’s function by:

\[ G(\rho|\rho') = \frac{j}{4} H_0^{(2)}(\gamma|\rho - \rho'|) \Theta(r_{cut} - |\rho - \rho'|), \]
where $H$ is the Heaviside step function, and where $r_{cut}$ is the cut-off distance as the distance above which the modulus of the Green’s function drops beyond a given threshold $\Delta_{cut}$. More details are given in [4].

III. SIMULATIONS

A. Fields inside the waveguide

In this master’s thesis we have first simulated the fields inside the waveguide along the x-axis in the middle of the waveguide. In the simulations we see a clear attenuation over the total waveguide due to the surface roughness and an extra reflection due to the surface roughness, as expected. The higher the conductivity of the surface roughness the more losses and the less the reflection from the surface roughness.

B. Current density in and around the surface roughness

The effect of surface roughness on the current density is mostly determined by the ratio of the height of the surface roughness ($H_{SR}$) to the skin depth. Table I shows this ratio, as a function of the conductivity, at the frequency of 300 MHz.

![Fig. 2. $|J|$ in and around the surface roughness at a conductivity of 1 S/m.](image1)

![Fig. 3. $\angle J$ in and around the surface roughness at a conductivity of 1 S/m.](image2)

<table>
<thead>
<tr>
<th>conductivity [S/m]</th>
<th>Skin depth ($\delta$) [m]</th>
<th>$\frac{H_{SR}}{\Delta_{cut}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.0038</td>
<td>0.1501</td>
</tr>
<tr>
<td>4</td>
<td>0.0145</td>
<td>0.580</td>
</tr>
<tr>
<td>3</td>
<td>0.0268</td>
<td>0.670</td>
</tr>
<tr>
<td>1</td>
<td>0.0291</td>
<td>1.164</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0411</td>
<td>1.644</td>
</tr>
</tbody>
</table>

TABLE I

Conductivity with $H_{SR}=0.025$ m at 300 MHz.

Although we want to study the behavior of the surface roughness in copper, the same effects will happen when we are dealing with a medium of lower conductivity. The ratio $\frac{H_{SR}}{\Delta_{cut}}$ is the determining factor. The height of the surface roughness is now chosen as 2.5 cm, which is way bigger than the surface roughness in real life (where we are speaking of several $\mu$m). One can now apply these simulation results at higher frequencies, at higher conductivity and at smaller spatial dimensions by scaling Maxwell’s equations. When we scale the dimensions with a constant $\alpha < 1$ this leads to: $\omega' = \frac{\omega}{\alpha}$, $\sigma' = \frac{\sigma}{\alpha}$ and $dx' = \alpha dx$, for the same electric and magnetic fields. In the simulation we approached the surface roughness as a half disk and as a trapezoid. In both cases, the field behavior is very similar, but with the trapezium we had a 6 times better condition number as in the case of the half disk. The behavior of the current density as a function of the skin depth is as follows:

When the ratio $\frac{H_{SR}}{\Delta_{cut}}$ is less than 20 % the current density follows the border very tightly. When the skin depth is larger than the height of the surface roughness, we see that the current density becomes more homogeneous in and around the surface. When the skin depth is everything in between, the current density is somewhat higher on the left side of the surface roughness. This is due to the fact that the wave in impinge on the left side of the surface roughness.

In figures 2 and 3 two typical results are given. One can see the current density magnitude and the direction of the current density (angle w.r.t. the x-axis in absolute value) at a conductivity of 1 S/m.

IV. CONCLUSION

In this masters’ thesis we have extended the Nero2d program with a function to calculate the fields inside a conductive medium. Further, we made use of a parallel-plate waveguide to observe the behavior of the fields inside the waveguide, as well as the current density distribution in and around the copper walls, by means of simulations. From these simulations, we have gained more insight in the field behavior due to surface roughness.

REFERENCES

Computationele modellering van oppervlakteruwheideffecten in koper

Niels Lambrecht

Abstract—Deze masterproef heeft als doel om zowel onze kennis als onze intuïtie uit te breiden inzake de effecten van oppervlakteruweheid in koper. Hiervoor lossen we een elektromagnetisch 2D probleem op m.b.v. een randelementenmethode, tezamen met de momentenmethode. We maken gebruik van een parallelle plaat golfgeleider, waarbij zowel het gedrag van de velden in de golfgeleider, als de stroomverdeling in en rond de oppervlakteruweheid van de koperen wanden worden geobserveerd door middel van simulaties.

Keywords—Computationeel elektromagnetisch, oppervlakteruweheid, momentenmethode

I. INLEIDING

In de afgelopen jaren hebben elektronische apparaten, geïntegreerde schakelingen, verpakking en montage een trend laten zien naar een grotere miniaturisering. Ook de snelheid van elektronische signalen is toegenomen met datasnelheden tot bijna 100 Gb/s. Op deze hoge frequenties zal de skin-diepte stroomverdringing veroorzaken in de geleiders. Dit kan tot bijna 100 Gb/s leiden, terwijl de skin-diepte stroomverdringing veroorzaakt een sterk exponentieel verval van de Greense functie. Bovendien, hebben we in een goede geleider moeilijkheden met een enkel de fundamentele quasi-TEM mode kan propageren binnenin de golfgeleider, dit kan enkel gebeuren als de afstand tussen de platen kleiner is dan een halve golflengte. In deze masterproef stellen we de afstand, d, steeds gelijk aan een kwart golflengte. Voor de rek van de platen, a, kiezen we steeds een dikte van een aantal golflengtes. De frequentie waarop we al onze simulaties laten lopen is 300 MHz. De lengte van de golfgeleider is, L, hiervoor hebben we een waarde van 20 m gekozen, zodat alle ongewenste hogere modi uitgestorven zijn tegen dat we aan het oppervlakteruweheidelement toekomen.

II. GEOMETRIE EN GEBRUIKTE METHODE

A. Geometrie

In de analyse van de velden in de golfgeleider willen we dat enkel de fundamentele quasi-TEM mode kan propageren binnenin de golfgeleider, dit kan enkel gebeuren als de afstand tussen de platen kleiner is dan een halve golflengte. In deze masterproef stellen we de afstand, d, steeds gelijk aan een kwart golflengte. Voor de dikte van de platen, a, kiezen we steeds een dikte van een aantal golflengtes. De frequentie waarop we al onze simulaties laten lopen is 300 MHz. De lengte van de golfgeleider is, L, hiervoor hebben we een waarde van 20 m gekozen, zodat alle ongewenste hogere modi uitgestorven zijn tegen dat we aan het oppervlakteruwheidelement toekomen.

B. Gebruikte methode

Doordat we te maken hebben met een 2D elektromagnetisch probleem zal de Greense functie van de scalaire Helmholtz vergelijking gegeven worden door:

$$G(\rho|\rho') = \frac{2}{\pi} H_0^{(2)}(\gamma|\rho - \rho'|),$$  (1)

waar $H_0^{(2)}$ de Hankel functie van de tweede soort en orde 0 is, $\gamma$ is de propagatieconstante van het medium en $\rho$ is de positie vector in het $xy$ vlak met $\rho = xu_x + yu_y$. In een goede geleider is de propagatieconstante gelijk aan $\gamma = \frac{1}{\sqrt{\varepsilon}}$, waarbij $\varepsilon$ de diepte is. Het grote imaginairdeel van de nieuwe propagatieconstante veroorzaakt een sterk exponentieel verval van de Greense functie. Bovendien, hebben we in een goede geleider
ook te maken met de snelle oscillaties van de Greense functie. Al deze factoren leiden tot een onnauwkeurige evaluatie van de interactie integralen. In Dobbelaere et al. [4] is er een oplossing gevonden om dit probleem te verhelpen, de oplossing bestaat eruit dat de numerieke kwadratuurpunten alleen worden geplaatst in de regio’s waar de Greense functie een niet te verwaarlozen waarde heeft. Aldus kan men dan de Greense functie voorstellen als:

\[
G_1(\rho|\rho') = \frac{i}{4} H_0^{(2)}(\gamma|\rho - \rho'|) H(r_{\text{cut}} - |\rho - \rho'|),
\]

waar \( H \) de Heaviside stap functie is, en waar \( r_{\text{cut}} \) de cut-off afstand is. De \( r_{\text{cut}} \) afstand is de afstand waarbij de modulus van de Greense functie kleiner is dan de gegeven threshold waarde. De details van deze werkwijze vindt men in [4].

III. SIMULATIES

A. Velden binnenin de golfgeleider

In deze masterproef hebben we eerst de velden binnenin de golfgeleider volgens de \( x \)-as in het midden van de golfgeleider gesimuleerd. Uit deze simulaties zien we een duidelijke attenuatie over de totale golfgeleider en een extra reflectie t.o.v. de oppervlakteruwheid.

B. Stroomdichtheid in en rond de oppervlakteruwheid

Het effect van de oppervlakteruwheid op de stroomdichtheid is vooral bepaald door de verhouding tussen de hoogte van de oppervlakteruwheid \( (H_{SR}) \) t.o.v. de skindiepte. Tabel I toont deze verhouding als functie van de geleidbaarheid, bij een frequentie van 300 MHz.

<table>
<thead>
<tr>
<th>geleidbaarheid [S/m]</th>
<th>skindiepte (( \delta )) [m]</th>
<th>( \frac{\delta}{H_{SR}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.0038</td>
<td>0.1501</td>
</tr>
<tr>
<td>4</td>
<td>0.0145</td>
<td>0.580</td>
</tr>
<tr>
<td>3</td>
<td>0.0268</td>
<td>0.670</td>
</tr>
<tr>
<td>1</td>
<td>0.0291</td>
<td>1.164</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0411</td>
<td>1.644</td>
</tr>
</tbody>
</table>

**TABLE I**

**GELEIDBAARHEID MET \( H_{SR}=0.025 \) M BIJ 300 MHZ.

Hoewel we de effecten van oppervlakteruwheid in koper willen bestuderen zullen dezelfde effecten ook optreden in een geleidbaar medium met een lagere geleidbaarheid. De hoogte van de oppervlakteruwheid is in onze simulaties gekozen als 2.5 cm, welke veel groter is dan deze bij oppervlakteruwheid in werkelijkheid (waar we dan spreken over hoogtes van een aantal \( \mu \)m). D.m.v. het schalen van de ruimtelijke dimensies in de Maxwell vergelijkingen kunnen we onze simulatieresultaten toepassen op hogere frequenties, bij hogere geleidbaarheid en bij kleinere ruimtelijke dimensies. Wanneer we de ruimtelijke dimensies schalen met een constante \( \alpha < 1 \) zal dit leiden tot: \( \omega' = \frac{\omega}{\alpha} \), \( \sigma' = \frac{\sigma}{\alpha} \) en \( dx' = \alpha dx \), voor dezelfde elektrische en magnetische veldsterktes. In de simulaties hebben we de oppervlakteuwheid benaderd door een halve schijf en door een trapezium. In beide gevallen hebben we het zelfde gedrag van de velden, maar in het geval van de trapezium hebben we een conditiegetal dat 6 keer kleiner is dan in het geval van de halve schijf. Het gedrag van de stroomdichtheid als functie van de skindiepte is als volgt:

Wanneer de verhouding \( \frac{\delta}{H_{SR}} \) kleiner is dan 20 %, dan zal de stroomdichtheid de vorm van de oppervlakteruwheid zeer goed volgen. Wanneer de skindiepte groter is dan de hoogte van de oppervlakteruwheid zien we dat de stroomdichtheid meer homogeen wordt in en rond de oppervlakteruwheid. Wanneer de skindiepte alles tussen deze 2 laatste gevallen is, dan zien we in onze simulaties een mooi overgangsgebied. We merken op dat de stroomdichtheid aan de linkerkant van het oppervlakteruwheidelement steeds wat groter is dan aan de andere zijde, dit komt doordat de invallende golf invalt op de linkerkant van het oppervlakteruwheidelement.

\[ |J| \text{ in en rond de oppervlakteruwheid bij een geleidbaarheid van 1 S/m.} \]

\[ \angle J \text{ in en rond de oppervlakteruwheid bij een geleidbaarheid van 1 S/m.} \]
IV. CONCLUSIE

In deze masterproef hebben we het programma Nero2d uitgebreid met een functionaliteit om de velden in een geleidbaar medium te berekenen. Verder, hebben we gebruik gemaakt van een parallelle plaat golfgeleider om zowel de velden in de golfgeleider als de stroomdichtheidsverdeling in en rond de koperen wanden te simuleren. Via deze simulaties, zijn we erin geslaagd om meer inzicht te krijgen in het gedrag van de velden ten gevolge van oppervlakteruwheid.

REFERENCES


# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>vi</td>
</tr>
<tr>
<td>Abstract in Dutch</td>
<td>ix</td>
</tr>
<tr>
<td>1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Goal</td>
<td>4</td>
</tr>
<tr>
<td>2 Method of Moments</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Integral Equations</td>
<td>6</td>
</tr>
<tr>
<td>2.2.1 PMCHWT Formulation</td>
<td>9</td>
</tr>
<tr>
<td>2.3 The Method of Moments</td>
<td>11</td>
</tr>
<tr>
<td>2.4 Basis and Test Functions</td>
<td>12</td>
</tr>
<tr>
<td>3 Parallel-Plate Waveguide</td>
<td>14</td>
</tr>
<tr>
<td>3.1 Parallel-Plate Waveguide</td>
<td>14</td>
</tr>
<tr>
<td>3.1.1 Geometry</td>
<td>15</td>
</tr>
<tr>
<td>3.1.2 Simulations</td>
<td>16</td>
</tr>
<tr>
<td>3.2 Terminated Parallel-Plate Waveguide</td>
<td>16</td>
</tr>
<tr>
<td>3.2.1 Geometry</td>
<td>16</td>
</tr>
<tr>
<td>3.2.2 Simulations</td>
<td>18</td>
</tr>
<tr>
<td>3.3 Surface Roughness in Terminated Waveguide</td>
<td>21</td>
</tr>
<tr>
<td>4 Fields in a Highly Conductive Medium</td>
<td>26</td>
</tr>
<tr>
<td>4.1 Problems Occurring in a Highly Conductive Medium</td>
<td>26</td>
</tr>
</tbody>
</table>

xii
CONTENTS

4.2 Geometry Problem ...................................................... 28
4.3 Calculating the Fields and Current Density ...................... 29
4.4 Testing the algorithm .................................................. 32

5 Analysis of the Current Density ........................................ 36
  5.1 Surface Roughness as a Half Disk ................................. 37
  5.2 Surface Roughness as a Trapezoid ............................... 45
  5.3 Relation between the Fields and Current Density Inside the Waveguide ................................................. 51
  5.4 Scaling the Dimensions ............................................. 51
  5.5 Influence of Surface Roughness on a PCB Transmission Line Model ......................................................... 52

6 Analysis of the Fields on the Border ................................... 57
  6.1 The Anomalous Effect .............................................. 57
  6.2 Fields on the Border with a Terminated Parallel-Plate Waveguide ......................................................... 59
    6.2.1 Segmentation with $\lambda_{10}$ .................................. 59
    6.2.2 Segmentation with $\lambda_{20}$ .................................. 63

7 Conclusions .................................................................. 66

A Appendix A .................................................................. 68
  A.1 Derivation of Wavenumber and Wavelength in a Good Conductor ......................................................... 68
  A.2 Derivation of the $r_{cut}$ Distance ................................. 69

Bibliography ..................................................................... 71

List of Figures .................................................................. 73

List of Tables .................................................................... 77
Abbreviations

2D 2 dimensional
3D 3 dimensional
PMCHWT Poggio-Miller-Chew-Harrington-Wu-Tsai
TM Transversal Magnetic
TE Transversal Electric
PEC Perfect Electric Conductor
MoM Method of Moments
rms root mean square
PCB printed circuit board
RLGC resistance inductance admittance capacitance
p.u.l per-unit of-length
1

Introduction

"For, usually and fitly, the presence of an introduction is held to imply that there is something of consequence and importance to be introduced."

– Arthur Machen (1863 - 1947)

In the past years electronic devices, integrated circuits, packages and assemblies have shown a trend towards greater miniaturization. Also the speed of electronic signals has increased with data rates approaching 100Gb/s. At high frequencies, the skin effect causes current crowding in the conductors, and in combination with the surface roughness profiles of the conductors, the skin effect leads to signal integrity problems.

(a) Pictorial transmission line cross section showing rough copper and relation to skin depth [1].

(b) Printed coplanar-waveguide sample [2].

Fig. 1.1b shows a coplanar waveguide consisting of a metal signal trace, and two metal reference conductors on a dielectric medium. On printed circuit boards (PCBs), the metal is typically copper, and the dielectric is typically FR4. The copper foils used in high-volume low-cost PCBs are often roughened for better trace-to-substrate adhesion (see Fig. 1.1a). At multi-GHz frequencies, the surface roughness is of the same order as the skin depth, and as a result, the resistance increases faster than skin effect loss due to current flowing on non-smooth surfaces. The surface
roughness has a random character (as can be seen in Fig. 1.1). Furthermore, FR4 materials exhibit noticeable frequency dependent material properties. However in this thesis we will not further go into detail on the dielectric properties.

![Surface profile measurement of a rough copper foil](image)

Figure 1.1: Surface profile measurement of a rough copper foil [1].

We need to investigate several scenarios of the surface roughness in relation to the skin depth. A first one concerns the case where the height of the surface roughness is much smaller than the skin depth. In that case we usually neglect the surface roughness and apply the classical calculations. A second case is when the skin depth is much smaller than the height of the surface roughness. In that case the current follows the boundary of the surface, and we may apply the traditional skin effect calculations. In the third case, however, when the skin depth is of the same order as the roughness’ dimensions, it is still unknown how the current is exactly behaving. Because the curvature radius of the surface roughness is not large w.r.t. the skin depth, one can no longer calculate the fields by relying on the surface impedance [3]. Consequently, the attenuation due to the surface roughness is hard to determine and one also does not know the current density distribution inside the conductor.

Surface roughness effects were first modeled by Hammerstad and Jensen [4]. They introduced a correction factor that can be applied to the conductor attenuation or surface resistance to model the effect of surface roughness. Another approach to understand the effect of surface roughness is to model the resistive losses in planar transmission line which suffers from surface roughness. In [2] modelling is done by means of a filament model which divides the cross section of a conductor into elementary volume cells. These conductors are small enough such that the current distribution of each of these filaments can be approximated as uniform. In this paper [2] one also discusses the effect of the current crowding on both the resistance and the inductance of the conductor, which then leads to an effect on the delay of
Figure 1.2: Surface profile measurement of rough copper foil [2].

a transmission line.

Paper [1] also discussing the resistance and inductance effect due to the surface roughness but now by modeling the surface by means of using hemispheres with the same rms volume as the measured surface profile. In another paper [5], the authors purely look at the effects of the surface roughness on the RLGC elements and try to apply these results to a transmission line model which suffers under the surface roughness behavior. In [6] and [7] a statistical model is adopted.

To our best knowledge, no accurate description has been given of the the current density behavior in and around a rough copper surface. Only some intuitive attempts are available (see Fig. 1.2).
1.1 Goal

The goal of this thesis is to study and describe the effects of the surface roughness. More specifically, we aim to determine the behavior of the current density in and around the rough copper surface. This to gain more inside in the skin effect mechanism in rough copper and also to understand the effects of surface roughness in even more general and complex cases.

To accomplish this we will extend the program Nero2d, to allow calculations of the fields in conductive media. The program Nero2d is a fast 2D problem solver for large electromagnetic problems which is developed at INTEC Ghent university [8]. For 2D configurations, the distribution of matter and sources is uniform in a certain direction. In this thesis the z-direction will be the direction of invariance. Furthermore, all sources and fields are assumed time harmonic with angular frequency $\omega$ and time dependencies $e^{j\omega t}$ are suppressed.
2

Method of Moments

"It's of no use whatsoever...this is just an experiment that proves Maestro Maxwell was right - we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there."

– Heinrich Rudolf Hertz (1887)

2.1 Introduction

In a linear, homogeneous, isotropic medium, Maxwell’s equations in time-harmonic regime are given by:

\[
\nabla \times \mathbf{E}(\mathbf{r}) = -j\omega \mu \mathbf{H}(\mathbf{r}), \quad (2.1)
\]

\[
\nabla \times \mathbf{H}(\mathbf{r}) = j\omega \varepsilon \mathbf{E}(\mathbf{r}) + \mathbf{J}(\mathbf{r}), \quad (2.2)
\]

\[
\nabla \cdot \varepsilon \mathbf{E}(\mathbf{r}) = \rho(\mathbf{r}), \quad (2.3)
\]

\[
\nabla \cdot \mu \mathbf{H}(\mathbf{r}) = 0, \quad (2.4)
\]

where bold symbols denote vectors, where \( \mu \) is the permeability in \( \frac{H}{m} \), \( \varepsilon \) is the permittivity in \( \frac{F}{m} \), \( \mathbf{r} \) is the place vector in 3D, \( \mathbf{E}(\mathbf{r}) \) is the electric field vector in \( \frac{V}{m} \), \( \mathbf{H}(\mathbf{r}) \) is the magnetic field vector in \( \frac{A}{m} \), \( \mathbf{J}(\mathbf{r}) \) is the electric current density vector in \( \frac{A}{m^2} \), \( \rho(\mathbf{r}) \) is the the electric charge density in \( \frac{C}{m^3} \) and \( \nabla = \nabla_t + \frac{\partial}{\partial z} u_z \), with \( \nabla_t = \frac{\partial}{\partial x} u_x + \frac{\partial}{\partial y} u_y \).

Only for a few problems an analytic solution exist, so in most cases we need a numerical algorithm to solve electromagnetic problems. In this dissertation we use a boundary element method know as the method of moments (MoM) \[9\]. The MoM solution procedure is as follows: first, we formulate Maxwell’s equations as a system
of coupled boundary integral equations, leveraging the pertinent Green’s functions. Second, we discretize the material boundaries over N segments. Then we define a finite number (N) basis functions over the discretized material boundaries and N test functions to test the continuity equations, so we have a system of linear equations. Once the linear equations are solved we can calculate the fields inside the medium by calculating the contributions from the boundary fields to the observation point of interest via representation formulas.

2.2 Integral Equations

As mentioned in the introduction, we are facing a 2D-problem (like in Fig. 2.1), allowing us to split Maxwell’s equations into a transversal and a longitudinal part:

\begin{align*}
\nabla \times E_t(\rho) &= -j\omega \mu H_z(\rho), \\
\nabla \times E_z(\rho) &= -j\omega \mu H_t(\rho), \\
\nabla \times H_t(\rho) &= j\omega \varepsilon E_z(\rho) + J_z(\rho), \\
\nabla \times H_z(\rho) &= j\omega \varepsilon E_t(\rho) + J_t(\rho),
\end{align*}

\begin{equation}
(2.5)
\end{equation}

\begin{equation}
(2.6)
\end{equation}

\begin{equation}
(2.7)
\end{equation}

\begin{equation}
(2.8)
\end{equation}

where \( \rho \) is the place vector in 2D with \( \rho = xu_x + yu_y \). The z-axis is the axis of invariance and it is the longitudinal direction. \( E_t(\rho) \) is the transversal electric field vector, \( H_t(\rho) \) is the transversal magnetic field vector, \( E_z(\rho) \) is the electric field vector in the z-direction, \( H_z(\rho) \) is the magnetic field vector in the z-direction, \( J_t(\rho) \) is the transversal electric current vector and \( J_z(\rho) \) is the electric current vector in the z-direction. If we are dealing with a TM-problem, then \( E_z(\rho) \) (with \( E_z(\rho) = E_z(\rho)u_z \)) and \( H_t(\rho) \) are unknown. If we are dealing with a TE-problem, then \( E_t(\rho) \) and \( H_z(\rho) \) (with \( H_z(\rho) = H_z(\rho)u_z \)) are unknown. Equations (2.6) and (2.7) constitute TM, which are decoupled from the TE problem, described by (2.5) and (2.8).

In this thesis we are dealing with a TE-problem, so we can derive from (2.5) and (2.8) the Helmholtz equation for \( H_z \). Applying the transversal curl operator to (2.8) yields:
2.2. INTEGRAL EQUATIONS

\[ \nabla_t \times (\nabla_t \times H_z(\rho)) = j\omega\varepsilon(\nabla_t \times E_t(\rho)) + \nabla_t \times J_t(\rho), \quad (2.9) \]

Using the vector triple product identity \( \nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A \) yields:

\[ \nabla_t(\nabla_t \cdot H_z) - \nabla_t^2 H_z = \omega^2 \varepsilon \mu H_z + \nabla_t \times J_t. \quad (2.10) \]

With \( \gamma = \omega \sqrt{\varepsilon \mu} \), we get the Helmholtz equation:

\[ \nabla_t^2 H_z + \gamma^2 H_z = - (\nabla_t \times J_t). \quad (2.11) \]

In general, a scalar Helmholtz equation may be written as:

\[ \nabla_t^2 F(\rho) + \gamma^2 F(\rho) = K(\rho), \quad (2.12) \]

Here, \( K(\rho) \) is the source term.

We now define the Green’s function as the function \( G(\rho|\rho') \) that satisfies:

\[ \nabla_t^2 G(\rho|\rho') + \gamma^2 G(\rho|\rho') = \delta(\rho - \rho'), \quad (2.13) \]

together with the Sommerfeld radiation condition at \( \infty \), leading to \( G_i(\rho|\rho') = \frac{j}{4} H_0^{(2)}(\gamma_i|\rho - \rho'|). \quad (2.14) \]

For a general derivation of the fields in a medium, we multiply (2.12) with \( G(\rho|\rho') \) and integrate over the total surface \( S \) (which is the surface over the material area), we get:

\[ \int_S \left( G(\rho|\rho') \nabla_t^2 F(\rho') + G(\rho|\rho') \nabla^2 F(\rho') \right) dS = \int_S G(\rho|\rho') K(\rho') dS. \quad (2.15) \]

If we substitute \( k^2 G(\rho|\rho') = -\nabla_t^2 G(\rho|\rho') + \delta(\rho - \rho') \), we get:

\[ \int_S \left( G(\rho|\rho') \nabla_t^2 F(\rho') - F(\rho') \nabla^2 G(\rho|\rho') \right) dS = \int_S \left( G(\rho|\rho') K(\rho') - \delta(\rho - \rho') F(\rho') \right) dS. \quad (2.16) \]
Applying the theorem of Green leads to:

\[
\oint_C \left( G(\rho|\rho') \frac{\partial}{\partial n} F(\rho') - F(\rho') \frac{\partial}{\partial n} G(\rho|\rho') \right) dC = \\
\int_S G(\rho|\rho') K(\rho') dS - F(\rho),
\]

(2.17)

where \( C \) stands for the boundary contour of \( S \).

Consider now figure (2.1). The field \( F_i \) inside sourceless medium \( i \), is given by:

\[
F_i(\rho) = \oint_{C_i} \left( -G_i(\rho|\rho') \frac{\partial}{\partial n_i} F_i(\rho') + F_i(\rho') \frac{\partial}{\partial n_i} G_i(\rho|\rho') \right) dC_i.
\]

(2.18)

Where \( G_i \) is the Green’s function inside the medium \( i \). When we want to calculate the fields outside the medium \( i \) then we run the border in the opposite direction (hence negative sign). In the background medium 0, the field is given by

\[
F_0(\rho) = \int_{S_0} G_0(\rho|\rho') K(\rho') dS_0 - \\
\sum_j \oint_{C_j} \left( -G_i(\rho|\rho') \frac{\partial}{\partial n_i} F_i(\rho') + F_i(\rho') \frac{\partial}{\partial n_i} G_i(\rho|\rho') \right) dC_j.
\]

(2.19)

Where the first integral stems from the sources present in the background medium.

In the case of figure (2.1), the source is an incident plane wave, so we can rewrite (2.18) as follows:

\[
F_i(\rho) = F_0^{inc}(\rho) - \\
\sum_j \oint_{C_j} \left( \oint_{C_i} \left( -G_i(\rho|\rho') \frac{\partial}{\partial n_i} F_i(\rho') + F_i(\rho') \frac{\partial}{\partial n_i} G_i(\rho|\rho') \right) \right) dC_j.
\]

(2.20)

From Maxwell’s equations, we can derive that

\[
\frac{\partial}{\partial n} H_z(\rho) = -j\omega \varepsilon E_i(\rho),
\]

(2.21)

\[
\frac{\partial}{\partial n} E_z(\rho) = j\omega \mu H_i(\rho),
\]

(2.22)
2.2. INTEGRAL EQUATIONS

Figure 2.1: Geometry and incoming field.

where \( \frac{\partial}{\partial n} \) denotes the normal derivative w.r.t. contour and the subscript \( t \) denotes the tangential component of the field w.r.t. this contour. Replacing \( F \) in (2.17) by the general fields of interest (\( E \) and \( H \)) and using \((2.21)\) and \((2.22)\) yields

\[
E_{z,i}(\rho') = \oint_{C_i} \left( E_i(\rho) \frac{\partial G_i(\rho|\rho')}{\partial n_i} - j \omega \mu_i H_i(\rho) G_i(\rho|\rho') \right) dC_i, \tag{2.23}
\]

\[
H_{t,i}(\rho) = \oint_{C_i} \left( \frac{1}{j \omega \mu_i} E_z(\rho') \frac{\partial^2 G_i(\rho|\rho')}{\partial n_i \partial n_i'} - H_i(\rho) \frac{\partial G_i(\rho|\rho')}{\partial n_i} \right), \tag{2.24}
\]

\[
H_{z,i}(\rho) = \oint_{C_i} \left( H_z(\rho) \frac{\partial G_i(\rho|\rho')}{\partial n_i} + j \omega \varepsilon_i E_i(\rho) G_i(\rho|\rho') \right) dC_i, \tag{2.25}
\]

\[
E_{t,i}(\rho) = \oint_{C_i} \left( \frac{j}{\omega \varepsilon_i} H_z(\rho') \frac{\partial^2 G_i(\rho|\rho')}{\partial n_i \partial n_i'} - E_i(\rho) \frac{\partial G_i(\rho|\rho')}{\partial n_i} \right). \tag{2.26}
\]

2.2.1 PMCHWT Formulation

All the previous formulas are useful when we want to know the field inside a medium if the fields on the material boundary are already known. Now we will explain which method is used in Nero2d to calculate the fields on the material boundary.
In the Nero2d program there has been made use of the fact that we demand continuity of the tangential electric and magnetic fields on the material boundary. This leads to the PMCHWT (Poggio-Miller-Chang-Harrington-Wu-Tsai) formulation \[10\] [11] [12]:

\[
\begin{align*}
\lim_{\rho \to \rho_i} E_{z,i}(\rho) &= \lim_{\rho \to \rho_i} E_{z,0}(\rho), \\
\lim_{\rho \to \rho_i} H_{t,i}(\rho) &= \lim_{\rho \to \rho_i} H_{t,0}(\rho), \\
\lim_{\rho \to \rho_i} E_{t,i}(\rho) &= \lim_{\rho \to \rho_i} E_{t,0}(\rho), \\
\lim_{\rho \to \rho_i} H_{z,i}(\rho) &= \lim_{\rho \to \rho_i} H_{z,0}(\rho),
\end{align*}
\]

where \(\rho_i\) is the point on the border of the object \(i\). At the interfaces between dielectric media, this reduces to:

\[
\begin{align*}
\oint_{C_i} \left( E_z(\rho') \frac{\partial G_i(\rho'|\rho')}{\partial n_i} - j \omega \mu_i H_t(\rho')G_i(\rho'|\rho') \right) dC_i &= E_z^{inc}(\rho) - \sum_j \oint_{C_j} \left( E_z(\rho') \frac{\partial G_j(\rho'|\rho')}{\partial n_j} - j \omega \mu_j H_t(\rho')G_j(\rho'|\rho') \right) dC_j, \\
\oint_{C_i} \left( \frac{1}{j \omega \mu_i} E_z(\rho') \frac{\partial^2 G_i(\rho'|\rho')}{\partial n_i \partial n_i'} - H_t(\rho') \frac{\partial G_i(\rho'|\rho')}{\partial n_i'} \right) dC_i &= H_t^{inc}(\rho) - \sum_j \oint_{C_j} \left( \frac{1}{j \omega \mu_j} E_z(\rho') \frac{\partial^2 G_j(\rho'|\rho')}{\partial n_j \partial n_j'} - H_t(\rho') \frac{\partial G_j(\rho'|\rho')}{\partial n_j'} \right) dC_j, \\
\oint_{C_i} \left( H_z(\rho') \frac{\partial G_i(\rho'|\rho')}{\partial n_i} + j \omega \epsilon_i E_t(\rho')G_i(\rho'|\rho') \right) dC_i &= H_z^{inc}(\rho) - \sum_j \oint_{C_j} \left( H_z(\rho') \frac{\partial G_j(\rho'|\rho')}{\partial n_j} + j \omega \epsilon_j E_t(\rho')G_j(\rho'|\rho') \right) dC_j, \\
\oint_{C_i} \left( \frac{j}{\omega \epsilon_i} H_z(\rho') \frac{\partial^2 G_i(\rho'|\rho')}{\partial n_i \partial n_i'} - E_t(\rho') \frac{\partial G_i(\rho'|\rho')}{\partial n_i'} \right) &= E_t^{inc}(\rho) - \sum_j \oint_{C_j} \left( \frac{j}{\omega \epsilon_j} H_z(\rho') \frac{\partial^2 G_j(\rho'|\rho')}{\partial n_j \partial n_j'} - E_t(\rho') \frac{\partial G_j(\rho'|\rho')}{\partial n_j'} \right) dC_j,
\end{align*}
\]
leading to 4 unknowns and 4 equations.

On the other hand, when we are dealing with a PEC then the electric field on the border becomes zero.

\[
\lim_{\rho \to \rho_i} E_{z,0}(\rho) = 0, \quad (2.35)
\]

\[
\lim_{\rho \to \rho_i} E_{t,0}(\rho) = 0, \quad (2.36)
\]

so we get the following equations:

\[
E_{i}^{inc}(\rho) - \sum_j \oint_{C_j} \left( \frac{j}{\omega \epsilon_j} H_z(\rho') \frac{\partial^2 G_j(\rho|\rho')}{\partial n_j \partial n'_j} - E_i(\rho') \frac{\partial G_j(\rho|\rho')}{\partial n'_j} \right) = 0, \quad (2.37)
\]

\[
E_{z}^{inc}(\rho) - \sum_j \oint_{C_j} \left( E_z(\rho') \frac{\partial G_j(\rho|\rho')}{\partial n_j} - j \omega \mu_j H_t(\rho') G_j(\rho|\rho') \right) dC_j = 0. \quad (2.38)
\]

Because \( E_i \) and \( E_z \) are zero inside the PEC, we have 2 unknowns and 2 equations, sufficient to yield a unique solution.

### 2.3 The Method of Moments

In this section we will convert the integral equations to a matrix system, which will then be solved numerically.

In general the method of moments (also called the Method of Weighted Residuals) works as follows [13]:

We want to solve a linear equation of the form

\[
\mathcal{L}(\phi(\tau)) = f(\tau), \quad (2.39)
\]

where \( \phi(\tau) \) is the unknown function, \( \mathcal{L} \) is a linear operator and \( f(\tau) \) is a known excitation or forcing function. To solve this problem, we start by expressing the unknown solution as a series of basis or expansion functions, \( v_n(\tau) \),

\[
\phi(\tau) = \sum_{i=1}^{N} a_n v_n(\tau), \quad (2.40)
\]
where the \( a_n \in \mathbb{C} \) are unknown coefficients. Thus we have written the unknown \( \phi(\tau) \) as a sum of \( N \) terms. This stems from the fact that we have discretized the continuous problem. So, the unknown is approximated by a linear combination of a set of basis function \( v_n(\tau) \).

To solve for the values of \( a_n \), we need \( N \) linearly independent equations. So, a weighting with \( N \) different weighting or testing functions, \( w_n(\tau) \), is applied. For the \( m \)-th testing function, we get:

\[
\sum_{i=1}^{N} a_n \int_m L(v(\tau')) w_m(\tau') d\tau' = \int_m f(\tau') w_m(\tau') d\tau'.
\] (2.41)

In conclusion, the procedure results in the matrix system:

\[
Z \cdot X = B.
\] (2.42)

With

\[
Z_{mn} = \int_m L(v_n(\tau)) w_n(\tau) d\tau,
\] (2.43)

\[
X_n = a_n,
\] (2.44)

\[
B_n = \int_m f(\tau) w_m(\tau) d\tau.
\] (2.45)

The system is solved for the unknown expansion coefficients \( a_n \), which are then substituted back into (2.40), finally yields \( \phi(\tau) \).

### 2.4 Basis and Test Functions

In the previous section we have seen that we need a set of basis functions and test functions to obtain the matrix system. In general there are 4 unknowns on the boundary: \( E_t, E_z, H_z \) and \( H_t \). We choose a hat function as basis function for \( E_z \) and \( H_z \) (see Fig. 2.2).

\[
H_n(\tau) = \begin{cases} \frac{\tau - \tau_{n-1}}{l_n} & \tau_{n-1} \leq \tau \leq \tau_n \\ \frac{\tau_{n+1} - \tau}{l_n} & \tau_n \leq \tau \leq \tau_{n+1} \\ 0 & \text{otherwise} \end{cases}
\] (2.46)

With \( \tau_n \) as endpoint of the \( n \)-th segment, and \( l_n \) as the length of the segment. For the \( E_t \) and \( H_t \) we use a pulse functions as basis function (see Fig. 2.3).
\[
P_n(\tau) = \begin{cases} 
\frac{1}{l_n} & \tau_{n-1} \leq \tau \leq \tau_n \\
0 & \text{otherwise}
\end{cases}
\] (2.47)

Figure 2.2: Hat function as basis and test function

Figure 2.3: Pulse function as basis and test function
"Nothing is too wonderful to be true, if it be consistent with the laws of nature"

– Michael Faraday

As it is necessary to have a well-defined structure where we are able to calculate the fields outside and inside the copper walls, we will use a parallel-plate waveguide which allows us to do accurate calculations.

### 3.1 Parallel-Plate Waveguide

First we consider a parallel-plate waveguide of finite length, with copper plates embedded in free space (Fig. 3.1). In the parallel-plate waveguide, we only want that the lowest order quasi-TEM mode propagates. To ensure that this will happen, the distance $d$ needs to be carefully chosen, such that all higher order modes are evanescent, and only the quasi-TEM mode can propagates in the waveguide \[3\]. Let us denote $\beta_n$ as the propagation constant of the $n^{th}$ mode and $\lambda$ is the wavelength in vacuum,

$$\beta_n = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{n\pi}{d}\right)^2}.$$  \hspace{1cm} (3.1)

The first higher mode has propagation constant:

$$\beta_1 = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{d}\right)^2}.$$  \hspace{1cm} (3.2)

Imposing $\beta_1 < 0$ ensures that only the quasi-TEM mode propagates in the waveguide, leading to
3.1. PARALLEL-PLATE WAVEGUIDE

\[ d < \frac{\lambda}{2}, \quad (3.3) \]

When a plane wave impinges upon the waveguide there will be still a region where higher modes haven’t died out yet. Let’s assume that the wave amplitude of the first higher order mode has decayed a factor \( e^{-\alpha} \) over a distance \( l_\alpha \), with \( \alpha > 0 \). Then for a certain value of \( \alpha \) we can calculate the associated distance \( l_\alpha \) as:

\[
\left(\frac{\pi}{d}\right)^2 = \left(\frac{\alpha}{l_\alpha}\right)^2 + \left(\frac{2\pi}{\lambda}\right)^2,
\]

\[
\rightarrow l_\alpha = \frac{\alpha}{\sqrt{\left(\frac{\pi}{d}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2}}.
\]

(3.4)

(3.5)

As a conclusion, at a distance of at least \( l_\alpha \) away from waveguide discontinuities, higher order modes have decayed at least by a factor of \( e^\alpha \).

3.1.1 Geometry

![The open waveguide with length L.](image)

Figure 3.1: The open waveguide with length \( L \).

We only want the fundamental quasi-TEM mode to propagate inside the waveguide, which is the case if the distance between the plates, \( d \), is less than half a wavelength. In this master’s thesis, we always choose the distance, \( d \), as a quarter of a wavelength. The thickness of the plates, \( a \), is chosen to be several skin depths large. The length of the waveguide, \( L \), is chosen to be 20 wavelengths long, such that all the unwanted higher order modes have died out away from the edges of the waveguide. In our simulations we will observe the fields in the waveguide on the observation line (in the middle of the waveguide at \( y = \frac{d}{2} \) along the x-axis) as is marked in Fig. 3.1.
3.1.2 Simulations

Although we let just only one wave impinge on the waveguide, we see a standing wave pattern. This is due to the reflections on the edges of the side walls at $x = L$ and $x = 0$. This results in a standing wave pattern. To tackle this problem we imposed a second plane wave excitation to compensate the unwanted reflections. The total excitation thus comprises two plane waves with different amplitude, propagating in the positive and negative x-direction, such that only the fundamental mode in the positive x-direction is excited.

3.2 Terminated Parallel-Plate Waveguide

The disadvantage of the geometry in fig. 3.1 is the excitation of higher order modes at both ports at x=-L and x=0. We can however eliminate unwanted higher order reflections by terminating the waveguide at x=0.

3.2.1 Geometry

The new geometry of the waveguide is presented in figure 3.2.

The total electric field ($E_{\text{tot}}$) inside the waveguide, far enough form $x = -L$ (i.e. at a distance larger than $l_\alpha$ in (3.5)) is the summation of the inserted wave (going in the positive x-direction) and the reflected wave (going in the negative x-direction). We assume that the waves are in time-harmonic regime with $\alpha, \beta > 0$, so that the total electric field is as follows:
\[ E_{\text{tot}} = A e^{-\alpha x} e^{-j\beta x} + B e^{\alpha x} e^{j\beta x}, \]  

(3.6)

\[ E_{\text{tot}} = A e^{-\alpha x} \cos(\beta x) + B e^{\alpha x} \cos(\beta x) - j \left[ A e^{-\alpha x} \sin(\beta x) + j B e^{\alpha x} \sin(\beta x) \right]. \]  

(3.7)

We now calculate the magnitude square of the total electric field:

\[
|E_{\text{tot}}|^2 = (|A| e^{-\alpha x} \cos(\beta x) + |B| e^{\alpha x} \cos(\beta x))^2 + 
(-|A| e^{-\alpha x} \sin(\beta x) + |B| e^{\alpha x} \sin(\beta x))^2.
\]

(3.8)

When we write everything out then this leads to:

\[
|E_{\text{tot}}|^2 = |A|^2 e^{-2\alpha x} \cos(\beta x)^2 + 2|A||B| \cos(\beta x) \cos(\beta x) + 
|B|^2 e^{2\alpha x} \cos(\beta x)^2 + |A|^2 e^{-2\alpha x} \sin(\beta x)^2 - 
2|A||B| \sin(\beta x) \sin(\beta x) + |B|^2 e^{2\alpha x} \sin(\beta x)^2,
\]

(3.9)

with \( \sin(\beta x)^2 = \frac{1-\cos(\beta x)}{2} \) and with \( \cos(\beta x)^2 = \frac{\cos(\beta x) + 1}{2} \) we get:

\[
|E_{\text{tot}}|^2 = |A|^2 e^{-2\alpha x} + |B|^2 e^{2\alpha x} + 2|A||B| \cos(2\beta x).
\]

(3.10)

As the \( \cos(2\beta x) \) is at maximum +1 or -1, the envelope of the magnitude square of the total electric field equals to:

\[
|E_{\text{tot,env}}|^2 = |A|^2 e^{-2\alpha x} + |B|^2 e^{2\alpha x} \pm 2|A||B|.
\]

(3.11)

If we assume that everything is reflected (\( |A| = |B| \)) then we get the formula:

\[
|E_{\text{tot}}|^2 = |A|^2 e^{-2\alpha x} + |A|^2 e^{2\alpha x} \pm 2|A|^2,
\]

(3.12)

which can be written as:

\[
|E_{\text{tot,env}}| = |A|(e^{\alpha x} \pm e^{-\alpha x}).
\]

(3.13)

If we only want to see the field envelope below then we obtain the formula:

\[
|E_{\text{tot,env}}| = |A|(e^{\alpha x} - e^{-\alpha x}).
\]

(3.14)

We choose the field envelope below, as this is the one on which we will apply the fitting, because the lower part of the envelope has the largest slope.

In our simulations we will observe the fields on the observation line (in the middle of the waveguide) as is marked in Fig. 3.2.
3.2. TERMINATED PARALLEL-PLATE WAVEGUIDE

3.2.2 Simulations

If we now look at the simulations in case of a waveguide made out of a PEC (see Fig. 3.3) and when we look at the simulations when we have a waveguide made out of copper walls (see Fig. 3.4 and Fig. 3.5) then we see what one would expect in a shorted waveguide.

To see if the geometry is indeed the one that we can use, we will do some calculations on it to obtain the attenuation factor. Then we will compare the simulated attenuation factor of the waveguide with the theoretical results from our calculations.

![Electric field inside the shorted waveguide with PEC.](image)

Figure 3.3: Electric field inside the shorted waveguide with PEC.

If we now want to extract the attenuation constant out of our simulations, then we can do this by fitting on one side of the envelope of the total electric field. We choose to fit on the lower side of the envelope, as this side has a higher slope than the upper side of the envelope.

For the theoretical attenuation constant we use the formula (under the condition
3.2. TERMINATED PARALLEL-PLATE WAVEGUIDE

that $\sigma >> \omega \epsilon$):

$$\alpha = \sqrt{\frac{\epsilon \omega}{\sigma d}} \quad (3.15)$$

where $\sigma$ is the conductivity in $\frac{S}{m}$. For the fitting we use GNUPLOT, because other programs do not give a good fit for a combination of exponential functions. This program works in an iterative way, to fit the parameter space. With the following results:

- Shorted waveguide with losses ($\sigma = 5.8 \cdot 10^3 \text{ S/m}$) at 300 MHz
  $\alpha_{\text{theory}} = 0.0270 \text{ 1/m}$, $\alpha_{\text{simulated}} = 0.0271344 \text{ 1/m}$

- Shorted waveguide with losses ($\sigma = 5.8 \cdot 10^6 \text{ S/m}$) at 300 MHz
  $\alpha_{\text{theory}} = 0.0008556 \text{ 1/m}$, $\alpha_{\text{simulated}} = 0.000812742 \text{ 1/m}$

- Shorted waveguide with losses ($\sigma = 5.8 \cdot 10^7 \text{ S/m}$) at 300 MHz
  $\alpha_{\text{theory}} = 0.00010726 \text{ 1/m}$, $\alpha_{\text{simulated}} = 0.0000966036 \text{ 1/m}$

We can conclude that our simulated results are very close to the theoretical results, so we can use this geometry further for simulating the effects of surface roughness in the waveguide. Another advantage of the terminated waveguide is that the con-
3.2. TERMINATED PARALLEL-PLATE WAVEGUIDE

Figure 3.5: Electric field inside the shorted waveguide with losses (detail).

Figure 3.6: Fitting the data of the lower envelope to the theoretical function.

The condition number of this geometry is much smaller than the condition number of the matrix.\footnote{Note that the condition number of a matrix is a measure for the precision at which the matrix can be inverted by means of a numerical procedure. In other terms, the solution of a linear system}
open ended waveguide.

3.3 Surface Roughness in Terminated Waveguide

To observe the effect of a single surface roughness element, we insert an isolated surface roughness element in the middle of the waveguide (Fig. 3.7).

![Figure 3.7: The terminated waveguide with length of 20 m and with a surface roughness element in the middle of the length (10 m).](image)

We did simulations on this structure for two different conductivity values ($\sigma=5.8 \cdot 10^6$ S/m and $\sigma=5.8 \cdot 10^7$ S/m).

From our simulation at $\sigma=5.8 \cdot 10^6$ S/m (see Fig. 3.8) we can already clearly see the effect of the surface roughness. Due to the surface roughness we have an extra reflection (reflection coefficient less than 1), this can easily be seen in the area between the open end of the waveguide and the surface roughness. As we have there an extra reflection we have also in total a larger attenuation over the total waveguide because of the surface roughness. In the area between the terminated end of the waveguide and the surface roughness we see the same shape of pattern of equations that is described by an ill-conditioned matrix, i.e. with a large condition number, is prone to numerical errors. Additionally, solving the linear system in a iterative way takes a long time, as the required number of iterations to reach a desired precision is large for ill-conditioned matrices.
3.3. SURFACE ROUGHNESS IN TERMINATED WAVEGUIDE

Figure 3.8: Detail of the lower part of the envelope for the terminated waveguide with length of 20 m and with a surface roughness element in the middle of the length (10 m) with \( \sigma = 5.8 \cdot 10^6 \) S/m.

as we would have in a waveguide with losses without surface roughness. This is to be expected because after the surface roughness we have the same situation as in figure 3.2 (although we have already suffered from attenuation).

A simulation for a copper waveguide \((\sigma=5.8 \cdot 10^7 \text{ S/m})\), reveals that the dip in the pattern has a shift to the right (see Fig. 3.9).

We can verify the behavior analytically of the wave pattern between the surface roughness element and the open end by means of doing the calculations in the waveguide with two waves of which one has a shift (place of the surface roughness).

The calculations for this are as follows:

\[
E_{\text{tot}} = |E_1|e^{-ax}e^{-j\beta x} + |E_2|e^{\alpha(x-d)}e^{j\beta(x-d)},
\]

where \( d \) is the position of the surface roughness. Next we will take the modulus
Figure 3.9: Detail of the the lower part of the envelope for the terminated waveguide with length of 20 m and with a surface roughness element in the middle of the length (10 m) with $\sigma = 5.8 \cdot 10^7$ S/m.

The result is shown in Fig. 3.10. Here we did the calculation for a surface roughness element placed at 20 m. As can be seen in Fig. 3.10, we indeed have the same behavior in the graph as in our simulations.

We can now take a look at the behavior of the surface roughness. For this, we use $|E_1|$ as the amplitude of the impinge wave (which is propagating in the positive x-direction) and $|E_2|$ as the amplitude of the reflected wave (which is propagating in the negative x-direction).

The square of this expression

$$|E_{tot}|^2 = |E_1|^2 e^{-2\alpha x} \cos(\beta x)^2 + |E_2|^2 e^{-2\alpha(x-d)} \cos(\beta(x-d))^2 +$$

$$|E_1||E_2| e^{-\alpha d} \cos(\beta x) \cos(\beta(x-d)) + |E_1|^2 e^{-2\alpha x} \sin(\beta x)^2 +$$

$$|E_2|^2 e^{-2\alpha(x-d)} \sin(\beta(x-d))^2 +$$

$$|E_1||E_2| e^{-\alpha d} \sin(\beta x) \sin(\beta(x-d)) .$$

(3.17)
3.3. SURFACE ROUGHNESS IN TERMINATED WAVEGUIDE

Figure 3.10: Plot of the field behavior between the surface roughness and the open end of the waveguide by means of calculation.

We see that the reflected wave is becoming smaller, due to the absorption of the surface roughness, the dip will move to the right (Fig. 3.11 and Fig. 3.12). This is conform with our previous simulations (see Fig. 3.8 and Fig. 3.9).

Of course also the position of the surface roughness element will determine the shape of the fields inside the waveguide. One would have a different pattern as the place of the surface roughness would be on a whole number of wavelengths, than for an odd number of half wavelengths.
3.3. SURFACE ROUGHNESS IN TERMINATED WAVEGUIDE

Figure 3.11: Field behavior between the surface roughness and the open end of the waveguide when $|E_1|$ equals $|E_2|$.

Figure 3.12: Field behavior between the surface roughness and the open end of the waveguide when $|E_2|$ is 99% of $|E_1|$.
Fields in a Highly Conductive Medium

"Engineers like to solve problems. If there are no problems handily available, they will create their own problems."

– Scott Adams

As we want to know the behavior of the current density in copper (which is a highly conductive medium), we have to solve some numerical problems. We dedicate this entire chapter to describe how we will do this and by means of which method we will try to accomplish this [4].

4.1 Problems Occurring in a Highly Conductive Medium

In a conductive medium the permittivity is given by $\epsilon = \epsilon_0 - j\frac{\sigma}{\omega}$, where $\epsilon_0$ is the permittivity of vacuum which approximately equals $8.85 \cdot 10^{-12}$ F/m, and where $\mu_0$ is the permeability of vacuum which equals $4\pi \cdot 10^{-7}$ H/m, so the wavenumber becomes

$$\gamma = \omega \sqrt{\mu_0 \epsilon_0 \left(1 + \frac{\sigma}{j\omega \epsilon_0}\right)}.$$  \hspace{1cm} (4.1)

As we can see in the appendix the propagating factor in a good conductor becomes

$$\gamma = \frac{1 - j}{\delta},$$  \hspace{1cm} (4.2)

with skin depth $\delta$ equal to

$$\delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}}.$$  \hspace{1cm} (4.3)
4.1. PROBLEMS OCCURRING IN A HIGHLY CONDUCTIVE MEDIUM

This form of the wavenumber leads to difficulties in evaluating the interaction integrals in a conductive medium. The Green’s function now becomes:

$$G(\rho|\rho') = \frac{j}{4} H_0^{(2)} \left( \frac{1 - j}{\delta} |\rho - \rho'| \right), \hspace{1cm} (4.4)$$

for large $|\gamma r|$ with $r = |\rho - \rho'|$. For large arguments, the Hankel function of the second kind and order $\nu$ behaves as,

$$H_\nu^{(2)} \approx \left( \frac{2}{\pi \gamma r} \right)^{1/2} e^{-j\gamma r + j \frac{\pi}{4} (2\nu + 1)}.$$  \hspace{1cm} (4.5)

The large imaginary part of the propagation factor will cause a strong exponential decay of the Green’s function and its derivatives. Moreover, the wavelength is small w.r.t. the free space wavelength in a conductive medium $\lambda = 2\pi \delta$ (see appendix for derivation), which leads to a lot of oscillations of the Green’s function and its derivatives over one boundary segment.

As we use Gauss-Legendre quadrature rule for numerical integration, we experience inaccuracy in the evaluation of the interaction integrals due to the exponential decay of the Green’s function. The inaccuracy is even that large that we get not a number (nan) as a solution for the fields in a conductive medium.

To solve this problem, the numerical quadrature points are placed only in those regions where the Green’s function has a non-negligible value. The value from which we say it has a negligible value is called $\Delta_{cut}$. This value than determines the so-called cut-off distance $r_{cut}$. Where the cut-off distance $r_{cut}$ is defined as the distance above which $|G|$ (modulus of the Green’s function) drops beyond the threshold $\Delta_{cut}$ (Theorem 5.1 [14]). At a sufficiently large $r_{cut}$ (small $\Delta_{cut}$), the asymptotic expression [4.5] holds and imposing $|G(\gamma r_{cut})| = (8\pi |\gamma| r_{cut})^{-1/2} e^{\gamma r_{cut}^3 / (8)}$

For good conductors this leads to (Definition 1 [14]) (see derivation in the appendix):

$$r_{cut} = \frac{1}{2} \mathcal{W} \left( \frac{1}{4\pi \sqrt{2\Delta_{cut}^2}} \right) \delta. \hspace{1cm} (4.6)$$

In this way, interactions between points that are separated further than $r_{cut}$ are neglected. So the approximate Green’s function becomes:

$$G((\gamma|\rho - \rho'|)) = \frac{j}{4} H_0^{(2)}(\gamma|\rho - \rho'|)H(r_{cut} - |\rho - \rho'|), \hspace{1cm} (4.7)$$
with $H$ the Heaviside step function. To get a visual idea of the solution, see Fig. 4.1.

Figure 4.1: Visual image of the solution to limit the integration area

Form Fig. 4.1 one can clearly see that the interactions between points that are separated further than $r_{cut}$ are neglected.

4.2 Geometry Problem

Now that we have discussed a solution for the problems that occur in highly conductive medium, we will discuss how we implemented this in Nero2d. To explain our implementation, we refer to Fig. 4.2.

Figure 4.2: Visual image of the algorithm

The first step in the algorithm is to check if there is any segment that lies in the region of the circle with center the observation point $\rho$ and radius $r_{cut}$. For this
we just look if the perpendicular distance between the segment and the observation point is smaller than $r_{cut}$ and if there are any points on the segment that cross the circle of interest. If so, we can conclude that the segment lies in the circle of interest. If the segment does not lie fully in the circle of interest we will calculate the intersection point(s) $(P_{c,x}, P_{c,y})$ of the circle with the segment. To calculate an intersection point we rotate the coordinate coordinate system in that way that the new x-axis is parallel with the segment of interest. Now that the x-axis is parallel with the segment of interest we can easily calculate the perpendicular distance between the observation point and the segment (because we know the coordinates of the nodes of the segment). Once this distance is know we use the following formula to calculate the coordinates of the cross point:

$$\Delta X^2 + \Delta Y^2 = r_{cut}^2, \quad (4.8)$$

$$\Rightarrow \Delta Y^2 + |\rho_{x,rot} - P_{c,y,rot}|^2 = r_{cut}^2. \quad (4.9)$$

Once this is done, we rotate the calculated coordinates back to the original coordinate system.

For the rotation we use the relationship:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad (4.10)$$

where $(x, y)$ are the coordinates of the original coordinate system and $(x', y')$ are the coordinates of the rotated coordinates system and where $\theta$ is the angle between the segment and the original x-axis (See Fig. 4.3).

### 4.3 Calculating the Fields and Current Density

At first sight we would just calculate the current density as:

$$J = \sigma E. \quad (4.11)$$

The formulas to calculate the electric field from the know tangential boundary fields with representation formulas are given by [8]:
4.3. CALCULATING THE FIELDS AND CURRENT DENSITY

Figure 4.3: Visual image of the solution

\[ E_z(\rho) = \oint_{C_i} d\mathbf{c}' \left[ E_z(\rho') \frac{\partial G_i(\rho|\rho')}{\partial n'} - \left( \frac{j\gamma_i^2}{\omega \epsilon_i} H_i(\rho') - \frac{\beta}{\omega \epsilon_i} \frac{\partial H_z(\rho')}{\partial t'} \right) G_i(\rho|\rho') \right], \]

(4.12)

\[ E_t(\rho) = \oint_{C_i} d\mathbf{c}' \left[ \frac{j\omega \mu_i}{\gamma_i^2} H_z(\rho') \frac{\partial^2 G_i(\rho|\rho')}{\partial n \partial n'} \right. \]

\[ + \left. \frac{j\omega \mu_i}{\gamma_i^2} \left( \frac{j\gamma_i^2}{\omega \mu_i} E_i(\rho') - \frac{\beta}{\omega \epsilon_i} \frac{\partial E_z(\rho')}{\partial t'} \right) \frac{\partial G_i(\rho|\rho')}{\partial n} \right] \]

\[ + \oint_{C_i} d\mathbf{c}' \left[ -\frac{j\beta}{\gamma_i^2} E_z(\rho') \frac{\partial^2 G_i(\rho|\rho')}{\partial t \partial n'} \right. \]

\[ + \left. \frac{j\beta}{\gamma_i^2} \left( \frac{j\gamma_i^2}{\omega \epsilon_i} H_i(\rho') - \frac{\beta}{\omega \epsilon_i} \frac{\partial H_z(\rho')}{\partial t'} \right) \frac{\partial G_i(\rho|\rho')}{\partial t} \right]. \]

(4.13)

Although we are dealing with a TE problem (so only the \( E_t \) and \( H_z \) field are unknown), we are still dealing with second order derivatives of the Green’s functions. This is quite complicated to implement especially if we find ourselves in a highly conductive medium like copper, so we will try to calculate the current density in another way. We will use Faraday’s law in a conductive medium with \( \sigma >> \omega \epsilon \), and hence:
\( \nabla \times \mathbf{H}(\mathbf{r}) = \mathbf{J}(\mathbf{r}). \)  

(4.14)

So then we find that:

\[
\frac{\partial H_z}{\partial y} u_x - \frac{\partial H_z}{\partial x} u_y = J_x u_x + J_y u_y,
\]

(4.15)

where we can calculate the field \( H_z \) as:

\[
H_{z,i}(\rho) = \oint_{C_i} \left( H_z(\rho') \frac{\partial G_i(\rho|\rho')}{\partial n_i} + \frac{\partial H_z(\rho')}{\partial n_i} G_i(\rho|\rho') \right) dC_i, \tag{4.16}
\]

\[
H_{z,i}(\rho) = \oint_{C_i} \left( H_z(\rho') \frac{\partial G_i(\rho|\rho')}{\partial n_i} + j \omega \epsilon_i E_t(\rho') G_i(\rho|\rho') \right) dC_i. \tag{4.17}
\]

Here we do not have to calculate second order derivatives of the Green’s function, we also do not have to take tangential derivatives of the fields, which makes it easier to implement. However, the drawback is that the derivatives in (4.15) still need to be implemented through a finite difference technique.

**The programming aspect:**

In the end the fields are calculated by means of use of the basis function (with the right coefficients)

\[
e = \sum_{i=0}^{N_e} a_i b_i, \tag{4.18}
\]

\[
h = \sum_{i=0}^{N_h} c_i d_i, \tag{4.19}
\]

where \( b_i \) and \( d_i \) are basis functions and \( a_i \) and \( c_i \) are the unknown coefficients.

Now we briefly sketch the implemented extension of Nero2d, to allow for the calculation of magnetic field inside a conductive region.

In the calculations we take only the basis functions of the segments into account which are inside the circle. If the basis function is a pulse and the segment over which the pulse is defined is completely inside the circle than we do not need to
change anything to the current implementation and we can calculate the segment to observation point interaction. However, if the later segment is crossing the circle, than we only take the piece of the segment inside the circle into account, and we will redefine the right node (nodes) to the intersection point(s) of the circle and the segment for the calculation of the segment to observation point interaction. When we are dealing with a pulse function than we only have to redefine the nodes. The height of the pulse is always one, so the scaling of the pulse will happen with the use of the basis functions coefficients.

When we are dealing with a hat basis function the situation is different. The first difference is that the hat function is defined over two segments instead of one segment, as was the case with the pulse function. On one segment we have the part of the hat function with positive slope, on the second segment we have the part of the hat function with negative slope. In case both the segments are completely falling into the circle, than we can reuse the existing implementation and just can calculate the segment to observation point interaction (taking into account if we are dealing with the positive slope or negative slope part). If the segments are crossing the circle, than we only take the part inside the circle into account. If we are dealing with the positive slope part and the start node is falling into the circle, while the end node is falling outside the circle than we just have to maintain the slope end just change the end node coordinates to the edge of the circle (Fig. 4.4). If we are in the case where we are dealing with the positive slope part but now the end node is in the circle and the start node is outside the circle. Now we can divide the remaining piece in a combination of a pulse and hat basis function (as can be seen in Fig. 4.5). Now the height of the pulse is not longer equal to one so we have to calculate this height first so that we are still integrating with the right coefficients. These later cases are analogues when we are dealing with the negative slope part.

4.4 Testing the algorithm

In a good conductor the current magnitude decays exponentially away from its boundary. If the depth of the copper is in the x- direction then the current density can be described by:

\[ |J| = |E_A| e^{-\alpha_c x} \sigma, \]  

(4.20)
where $\alpha_c$=attenuation constant in the conductor. The penetration depth of the current density (d) is then equal to $d = \frac{1}{\alpha_c} = \delta$ where $\delta$ equals the skin depth. The current density in copper will be simulated with help of a plane wave, impinging on an rectangular piece of copper (Fig. 4.6).

When we now take a cross section in the middle of the block copper we can see the current density along the x-axis (see Fig. 4.7). As can be seen in the figure we have a nice exponential decay of the current density as we move along the x-axis in the copper. This simulation is done at a frequency of 300 Hz, so the skin depth in
copper at that frequency is 3.8 mm, this can be verified in Fig. 4.7 as one knows that the skin depth is defined as the depth below the surface of the conductor at which the current density has fallen to \( \frac{1}{e} \) of the current density at the surface.

![Current density graph](image)

Figure 4.7: Current density \( |J_y| \) in a copper rectangular piece with \( \sigma = 5.8 \times 10^7 \) S/m with a plane wave impinging at 300 Hz.

Another test can be done in the terminated waveguide with losses which we used in chapter 3, here we take a waveguide with \( \sigma = 4 \) S/m at 300 MHz. We see how the current density behaves itself in the lower copper wall of the waveguide, the result can be seen in Fig. 4.8. If we calculate the skin depth theoretically then we obtain a skin depth of 0.0145 m, which agrees with the results in Fig. 4.8.
4.4. TESTING THE ALGORITHM

Figure 4.8: Current density $|J|$ in a terminated waveguide with losses with $\sigma=4 \text{ S/m}$ at 300 MHz.

Figure 4.8: Current density $|J|$ in a terminated waveguide with losses with $\sigma=4 \text{ S/m}$ at 300 MHz.
Analysis of the Current Density

"Just because something doesn’t do what you planned it to do doesn’t mean it’s useless."

– Thomas Alva Edison

Now that we have reached the point that we can calculate the fields inside the copper, we will study the behavior of the current density in and around the surface roughness. We will vary the skin depth by changing the conductivity of the medium. The applied frequency is always 300 MHz, the corresponding skin depth and the ratio of the skin depth to the height of the surface roughness ($H_{SR}$) can be seen in table 5.1.

For analyzing the current density in the parallel-plate waveguide, we have to take several things into account. First we need to see that we have still a configuration with a well-conditioned system matrix. For this, the surface roughness should not be too small, as otherwise the segmentation of the material boundaries becomes non-uniform which lead to a bad conditioned system and hence a high condition number.

Another parameter in the setup of our simulations is the frequency. For a given geometry, we choose the frequency as high as possible as this leads to a good seg-

<table>
<thead>
<tr>
<th>conductivity [S/m]</th>
<th>Skin depth ($\delta$) [m]</th>
<th>$\frac{\delta}{H_{SR}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.0038</td>
<td>0.1501</td>
</tr>
<tr>
<td>4</td>
<td>0.0145</td>
<td>0.580</td>
</tr>
<tr>
<td>3</td>
<td>0.0168</td>
<td>0.670</td>
</tr>
<tr>
<td>1</td>
<td>0.0291</td>
<td>1.164</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0411</td>
<td>1.644</td>
</tr>
</tbody>
</table>

Table 5.1: Conductivity versus skin depth with $H_{SR}$=0.025 m.
5.1. SURFACE ROUGHNESS AS A HALF DISK

mentation of the material boundaries. However the frequency shouldn’t be too high in order to limit the simulation time. The last parameter in the setup is the conductivity. As said earlier we want to study the behavior of the surface roughness in copper, but the same effects will happen when we are dealing with a medium of lower conductivity than that of copper. Hence, we can choose other values of conductivity than that of copper.

We are particularly interested in the following aspects of the current density:

- How does the current density behave in the x-direction?
- When we increase the skin depth, will the current density in the x-direction then take the upper hand of the current density in the y-direction (in the surface roughness)?
- How large is the current density in the y-direction in the surface roughness? As this stands in relationship with the attenuation caused by the surface roughness element.

To obtain a good interpretation of the behavior of the current density we will plot the current density in two separate graphs, one graph for the magnitude of the current density $|J|$ and one graph that shows the direction of the current density $\angle J$, where the angle is measured w.r.t. the x-axis and is plotted in absolute value, as then the results are more easily to interpreted on the color bar.

We have simulated the current density with the parameters from table 5.1.

We have calculated the values of the current density in a surface roughness with a geometry of a half disk and in a surface roughness which has a geometry of a trapezoid.

5.1 Surface Roughness as a Half Disk

One of the things in which we are interested is what happens with $|J_x|$ on the top of the surface roughness, and when the current density starts to deviate from flowing in the x-direction and starts to have a significant y-component. Or when does the
5.1. **SURFACE ROUGHNESS AS A HALF DISK**

In this section we simulate the surface roughness as a half disk with a radius of 2.5 cm, the observation box (see figure 5.1) in which we calculate the fields has a lower left corner of (-10.1 m, -0.1 m) and a upper right corner of (-9.9 m, 0.1 m) and has a resolution of 200 points in both x-direction and y-direction. The results are shown in figures 5.3 to 5.10.

For the simulation with a conductivity of 60 S/m, the observation box which is then used to calculate the fields has a lower corner of (-10.03 m, -0.03 m) and a upper right corner of (-9.97 m, 0.03 m) and has a resolution of 60 points in both x-direction and y-direction. The result is shown in Fig. 5.11 and Fig. 5.12.

From the figures 5.11 and 5.12 one can see clearly that when we decrease the skin depth, the current density is higher in the surface roughness on the border and that it is more inclined to follow the border.

When we impose a conductivity of 0.5 S/m, we clearly see that the skin depth is larger than the height of the surface roughness (see figures 5.4 and 5.3). But still, we can not neglect the surface roughness because their is still a significant part of the current which is flowing in the surface roughness compared with the other current in the rest of the conductive medium. This is because the skin depth is still not much larger than the height of the surface roughness. Here we can also see the need for the research in this master’s thesis, because it is not easy to say from which point one has to stop using the traditional calculations.

When one is dealing with surface roughness, then it is more important to know when the surface roughness is taking the upper hand than to know when the current will follow exactly the border of the surface roughness. We can see clearly the transient...
effect in figures 5.5 to 5.10. We are more interested in the transient effect instead of the extreme case where the current follows exactly the border, because this effect will only happen with extremely high frequencies. We are speaking then of several terahertz. Like we can see in figure 5.2, we are then in the beginning of the area of the infrared light. And on this frequencies we will no longer use copper traces for connecting our devices on the PCB.

The general conclusions of the simulations with the half disk as a surface roughness are:

- The current density follows the shape of the surface roughness when the skin depth is only about 10 to 20 \% of the height of the surface roughness.

- When the skin depth is larger than the height of the surface roughness we see that the current density becomes more homogeneous in and around the surface roughness element. This can be seen most clearly in figures 5.3 and 5.4.

- In the cases where the skin depth is everything in between the previous two cases, the current density shows a nice transient process. We see in our simulations that the current density is somewhat higher on the left side of the surface roughness, this is due to the fact that the wave impinges on the left side of the surface roughness element.
5.1. SURFACE ROUGHNESS AS A HALF DISK

Figure 5.3: $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 0.5 S/m.

Figure 5.4: $\angle J$ in and around the surface roughness at a conductivity ($\sigma$) of 0.5 S/m.
5.1. SURFACE ROUGHNESS AS A HALF DISK

Figure 5.5: $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 1 S/m.

Figure 5.6: $\angle J$ in and around the surface roughness at a conductivity ($\sigma$) of 1 S/m.
Figure 5.7: $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m.

Figure 5.8: $\angle J$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m.
5.1. SURFACE ROUGHNESS AS A HALF DISK

Figure 5.9: $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 4 S/m.

Figure 5.10: $|\angle J|$ in and around the surface roughness at a conductivity ($\sigma$) of 4 S/m.
5.1. SURFACE ROUGHNESS AS A HALF DISK

Figure 5.11: $|J|$ in the surface roughness at a conductivity ($\sigma$) of 60 S/m.

Figure 5.12: $|\angle J|$ in the surface roughness at a conductivity ($\sigma$) of 60 S/m.
5.2 Surface Roughness as a Trapezoid

In this section we will investigate trapezoidal surface roughness instead of a half disk, as some surface practical roughness elements more closely resemble a trapezoid than a half disk (e.g., Gaussian shape surface roughness). A second reason to discuss a trapezoid surface roughness is because we think that in this geometry the segmentation of the surface roughness will be more uniform since the boundary is composed entirely of straight lines, avoiding the need for a fine segmentation to approximate the curved arc shape of the previous section.

Figure 5.13: Observation window to calculate the fields inside this window.

The height of the trapezoid is 2.5 cm (just like the half disk) and the angle between the oblique flank and the x-axis is 65°. The observation box has again a lower left corner of (-10.1 m, -0.1 m) and a upper right corner of (-9.9 m, 0.1 m) and has a resolution of 200 points in both x-direction and y-direction, (-9.97 m, 0.03 m) and has a resolution of 60 points in both x-direction and y-direction (see figures from 5.14 to 5.21).

For the simulation with a conductivity of 60 S/m, the observation box which is then used to calculate the fields has a lower corner of (-10.03 m, -0.03 m) and a upper right corner of (-9.97 m, 0.03 m) and has a resolution of 60 points in both x-direction and y-direction, see figures 5.22 and 5.23.

The condition number of the system matrix for the waveguide with the trapezoid is 5 to 6 times smaller than in the case with the half disk. When we compare the results of the current densities with a half disk or with a trapezoid we see more or less the same. So, the general conclusions we made in the case of a half disk are similar to the case of a trapezoid.
5.2. SURFACE ROUGHNESS AS A TRAPEZOID

Figure 5.14: $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 0.5 S/m.

Figure 5.15: $|\angle J|$ in and around the surface roughness at a conductivity ($\sigma$) of 0.5 S/m.
5.2. **SURFACE ROUGHNESS AS A TRAPEZOID**

Figure 5.16: $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 1 S/m.

Figure 5.17: $|\angle J|$ in and around the surface roughness at a conductivity ($\sigma$) of 1 S/m.
5.2. SURFACE ROUGHNESS AS A TRAPEZOID

Figure 5.18: $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m.

Figure 5.19: $|\angle J|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m.
Figure 5.20: $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 4 S/m.

Figure 5.21: $|\angle J|$ in and around the surface roughness at a conductivity ($\sigma$) of 4 S/m.
5.2. SURFACE ROUGHNESS AS A TRAPEZOID

Figure 5.22: $|J|$ in the surface roughness at a conductivity ($\sigma$) of 60 S/m.

Figure 5.23: $|\angle J|$ in the surface roughness at a conductivity ($\sigma$) of 60 S/m.
5.3. Relation between the Fields and Current Density Inside the Waveguide

If we now compare the results of the fields inside the waveguide (from chapter 3) to what we see in the simulations of the current density, then we can clearly see that these results are consistent with each other. Because in chapter 3 we saw that we had attenuation due to the surface roughness and that this was expressed on the left-side of the surface roughness. And now we see clearly a larger current density on the left-side of the surface roughness, which implies Joule losses on that side of the fields in the waveguide.

Although the simulations results of chapter 3 did not give much inside in how the current density is distributed in and around the surface roughness, we now have a much better understanding of the current density behavior, thanks to the results of chapter 5.

5.4 Scaling the Dimensions

All the previous simulations where done at a frequency of 300 MHz, with a smaller conductivity than that of copper and a surface roughness height of 2.5 cm. When one wants to convert the simulation results to a realistic situation where the dimensions are much smaller and the frequency and conductivity are much larger then one can scale the dimensions in Maxwell’s equations.

Let’s take the situation where we want to scale the dimension $d$ to a dimension $d' = \alpha d$ with a factor $\alpha$ (with $\alpha < 1$). So then $dx = \frac{1}{\alpha} dx'$. If we apply this to the first Maxwell law we obtain:

$$\nabla \times E(r) = -j\omega \mu H(r), \quad (5.1)$$

$$\nabla = \partial_x u_x + \partial_y u_y + \partial_z u_z$$

$$= \alpha \nabla' = \alpha \left( \partial'_x u'_x + \partial'_y u'_y + \partial'_z u'_z \right). \quad (5.2)$$

If we now say that the fields in both cases are the same, i.e. $E(r) = E'(r)$ and $H(r) = H'(r)$ we get:

$$\alpha \nabla' \times E'(r) = -j\omega \mu H'(r), \quad (5.3)$$
5.5 Influence of Surface Roughness on a PCB Transmission Line Model

As we now have gained more insight into the behavior of the current density, we can make an attempt to look at the influence of the surface roughness on a PCB transmission line model. We use the RLGC model for the PCB transmission line model (see Fig. 5.24).

![RLGC model diagram](image)

Figure 5.24: The RLGC model.

The admittance per-unit of-length (p.u.l) (G) and capacitance (p.u.l) (C) of the RLGC model are not influenced by the surface roughness, as they mainly depend on the dielectric properties (dielectric losses, permittivity, substrate thickness) of the PCB. We can also neglect the change in thickness of the dielectric due to the surface roughness as the thickness of the dielectric of the PCB is around 1 mm, and the
surface roughness height is only of the order $\mu$m’s. Hence only the resistance (p.u.l) (R) and the inductance (p.u.l) (L) will vary in value due to the surface roughness.

One can calculate the resistance (in $\Omega/m$) as follows:

$$R_{ac} = \frac{2P_d}{|I|^2} = \frac{\int_S dS \mathbf{E} \cdot \mathbf{J}^*}{\int_S dS |\mathbf{J}|^2},$$

(5.7)

$$R_{ac} = \frac{\int_S dS |\mathbf{J}|^2}{\int_S dS |\mathbf{J}|^2},$$

(5.8)

where $P_d$ is the dissipated power in the conductive volume and I is the current. If we apply this on a waveguide without surface roughness then we get the result which can be seen in Fig. 5.25. Here the simulated curve is obtained by means of the above formula and the calculated curve is obtained with the formula (in $\Omega/m$):

$$R_{ac} = \frac{l}{\sigma \delta},$$

(5.9)

where $l$ is the length of the calculated conductive medium.

Figure 5.25: AC resistance in a conductive medium without surface roughness.

However, the calculation of the resistance in surface roughness is much more difficult to calculate because of the rapid changes in directions of the current density in the surface roughness. Therefore, we will only discuss intuitively the influence
5.5. INFLUENCE OF SURFACE ROUGHNESS ON A PCB TRANSMISSION LINE MODEL

of the surface roughness on the resistance of the RLGC model: one can easily see that the resistance will increase as the skin depth becomes smaller because of the narrower passages of the current density and the extra path length which the surface roughness introduces. Once the skin depth is that small compared to the height of the surface roughness, so that the current density follows nicely the shape of the surface roughness, then the surface roughness introduces only an extra path length in the path of the current density.

If we now try to model the surface roughness effect on the resistance by an enhancement factor (\(K_{SR}\)) multiplied by the traditional resistance (resistance in absence of surface roughness)

\[
R' = RK_{SR}, \tag{5.10}
\]

then the enhancement factor behaves as in Fig. 5.26.

Figure 5.26: Enhancement factor (\(K_{SR}\)) in function of the skin depth (\(\delta\)).

where the enhancement factor increases as the current density moves further into the surface roughness, and where the enhancement factor saturates as the skin depth
becomes that small, that the current density follows nicely the shape of the surface roughness and the surface roughness only introduces an extra path length in the path of the current density. These results are comparable with the results of Tsang et al. [6], who plotted the enhancement factor of Gaussian surface roughness (the Gaussian surface roughness can be approached as a trapezoid surface roughness), as shown in Fig. 5.27.

Figure 5.27: Gaussian rough surface with \( h=0.2, 0.8, 1.5 \) \( \mu m \), and width of the surface roughness is \( 1.5h \) [6].

Because inductance is closely related to the delay of a transmission line, and a proper relationship between the resistance and inductance is necessary to preserve causality for wide band transient simulations of high-speed digital circuits. We calculate the inductance as in paper [1]

\[
L_{\text{tot}}(\omega) = L_{\text{external}} + L_{\text{internal}}(\omega), \quad (5.11)
\]

\[
L_{\text{tot}}(\omega) = L_{\text{external}} + \frac{R(\omega)}{\omega}. \quad (5.12)
\]

With use of the enhancement factor we obtain:

\[
L_{\text{tot}}(\omega) = L_{\text{external}} + \frac{R(\omega)K_{SR}}{\omega}. \quad (5.13)
\]
From this, we can conclude that the inductance will have a somewhat higher value due to the surface roughness, this is consistent with the result of [2].
So, now we have an idea how the surface roughness influences the parameters of the RLG C model.
Analysis of the Fields on the Border

"The most exciting phrase to hear in science, the one that heralds new discoveries, is not 'Eureka!' but 'That’s funny...’”

– Isaac Asimov

Although we can clearly see the behavior of the current density in and around the surface roughness, we have still some small 'hills' below the conductive medium as can be seen in the figures where we plot $|\angle J|$. Therefore we will take a look at the fields on the boundary of the conductive medium, to see what can be cause of that.

6.1 The Anomalous Effect

The anomalous effect of which we are talking about are the small 'hills' below the conductive medium. To make this effect more visible we plotted the current density in its x-component and y-component for a segmentation length of $\frac{\lambda}{10}$ (see Fig. 6.1 and Fig. 6.2) and a segmentation length of $\frac{\lambda}{20}$ (see Fig. 6.3 and Fig. 6.4).

As one would expect at the bottom, we would only see a current density in the x-direction and no current density in the y-direction. But when we look at the graph of the $|J_y|$ current density we clearly see some hills in the current density, which would imply that there is current density running in the y-direction at those particular places, Which cannot be true. That’s why we will try to find a explanation for this effect in this chapter by means of analyzing the fields (solutions) on the boundary material from which then the fields (and the associate current density) in the conductive medium are calculated. We see in our calculations that when we apply a maximum segmentation length of $\frac{\lambda}{10}$, the period of the hills is also $\frac{\lambda}{10}$. And
6.1. THE ANOMALOUS EFFECT

Figure 6.1: $|J_x|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m at a segmentation length of $\frac{\lambda}{10}$.

Figure 6.2: $|J_y|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m at a segmentation length of $\frac{\lambda}{10}$.

Figure 6.3: $|J_x|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m at a segmentation of $\frac{\lambda}{20}$.

Figure 6.4: $|J_y|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m at a segmentation of $\frac{\lambda}{20}$.

when we apply a maximum segmentation length of $\frac{\lambda}{20}$, the period of the hills is also $\frac{\lambda}{20}$. 
6.2 Fields on the Border with a Terminated Parallel-Plate Waveguide

Since we were looking at the surface roughness at the lower plate of the parallel-plate waveguide in the previous chapter to study the current density, we will also look at the fields of the border of the lower part of the waveguide as can be seen in figure 6.5.

![Figure 6.5: The solutions on the border which will be studied are marked in red.](image)

To analyze the anomalous effect of the current density we take a look at what happens if we change our segmentation length. We will study the results by a maximum segmentation length of $\frac{\lambda}{10}$ and $\frac{\lambda}{20}$. The current density in the previous chapter was calculated by taking the curl of $H_z(\rho)$, from which the $H_z$ component is calculated in the Nero2d program by the formula:

$$H_{z,i}(\rho) = \oint_{C_i} \left( H_z(\rho') \frac{\partial G_i(\rho|\rho')}{\partial n_i} + j\omega \epsilon_i E_i(\rho') G_i(\rho|\rho') \right) dC_i, \quad (6.1)$$

such that we need both the $H_z$ and $E_t$ solutions on the border. So, we will take a look at both these field components on the border.

6.2.1 Segmentation with $\frac{\lambda}{10}$

When we apply a segmentation of $\frac{\lambda}{10}$ then we get the following results.

- On the border on top of the conducting plate we see what one could expect (see Fig. 6.8 and Fig. 6.10). In the tail of the the plot we see some small oscillations, these small oscillations are due to wave phenomenon which may be caused by the reflection at the conducting wall.
6.2. **FIELDS ON THE BORDER WITH A TERMINATED PARALLEL-PLATE WAVEGUIDE**

- On the border of the lower part of the conducting plate we see highly oscillations, with a period of half a wavelength (see Fig. 6.7 and Fig. 6.9). In Fig. 6.9 we see a peak on the edges of the waveguide. This can be explained with help of [13] as follows. In [13] the field behavior at a conducting wedge is discussed with help of Fig. 6.6.

![Figure 6.6: Conducting wedge](image)

In [13] it is demonstrated that the charge density on the conductor, is

\[ \rho_s \propto r^{\pi^{-1}}, \]  

(6.2)

where \( \rho_s \) is the charge density on the surface of the wedge and \( r \) the distance to the tip of the wedge. Thus, at a sharp edge the charge density becomes infinite at the tip but without charge accumulation because, when we take the integral of the charge density over the contour, we obtain

\[ j \propto \int_c r^{\pi^{-1}}dr = r^{\pi} + C, \]  

(6.3)

where \( j \) is the magnitude of the current density. From (6.3) we see that although there is an infinite charge density, there is no charge accumulation because the current density stays bounded (for a PEC) at the tip of the wedge. This implies that \( |H_z| \) is bounded (for a PEC) at the edges of a wedge, but can have a sharp peak due to the \( r^{\pi} \) (i.e. \( C=0 \) for \( x=0 \) in this case) behavior.

And indeed, there is a peak in the plot of \( |H_z| \) on the sharp edges of the waveguide who tends to go to zero.
6.2. FIELDS ON THE BORDER WITH A TERMINATED PARALLEL-PLATE WAVEGUIDE

Figure 6.7: $|E_t|$ field on the lower border of the lower conducting strip.

Figure 6.8: $|E_t|$ field on the upper border of the lower conducting strip.
6.2. **FIELDS ON THE BORDER WITH A TERMINATED PARALLEL-PLATE WAVEGUIDE**

Figure 6.9: $|H_z|$ field on the lower border of the lower conducting strip.

Figure 6.10: $|H_z|$ field on the upper border of the lower conducting strip.
6.2. FIELDS ON THE BORDER WITH A TERMINATED PARALLEL-PLATE WAVEGUIDE

6.2.2 Segmentation with $\frac{\lambda}{20}$

When we apply a segmentation of $\frac{\lambda}{20}$ then we get the following results.

- On the border on top of the conducting plate we see again what one could expect (see Fig. 6.12 and Fig. 6.14). In the tail of the plot we see some small oscillations, these small oscillations are due to the wavephenomenon.

- On the border of the lower part of the conducting plate we see again the same plots as in previous section (see Fig. 6.11 and Fig. 6.13). The peak on the edge of the waveguide is due to the same reason as explained in the previous section. Again, we see highly oscillations of the solutions of the fields on the bottom of the conductor with a period of half a wavelength. We can conclude that the oscillations are a standing wave pattern, which may be the cause of reflections on the edges of the parallel-plate waveguide.

![Figure 6.11: $|E_t|$ field on the lower border of the lower conducting strip.](image)

Although, we now have studied the solutions on the border of the parallel-plate waveguide, we still have no explanation why there is a current density in the y-
Figure 6.12: $|E_t|$ field on the upper border of the lower conducting strip.

direction on the bottom of the lower conducting plate. So, further research is needed.
6.2. **FIELDS ON THE BORDER WITH A TERMINATED PARALLEL-PLATE WAVEGUIDE**

Figure 6.13: $|H_z|$ field on the lower border of the lower conducting strip.

Figure 6.14: $|H_z|$ field on the upper border of the lower conducting strip.
Conclusions

"A conclusion is the place where you got tired of thinking."

– Arthur Bloch

In this master’s thesis we studied the effect of surface roughness, to extend our knowledge and to improve our intuition on the effects of surface roughness in copper. For this we solve a 2D electromagnetic problem using a boundary element method, together with the method of moments. We made use of a parallel-plate waveguide to observe the behavior of the fields inside the waveguide, as well as the current distribution in and around the surface roughness of the conductive plates of the waveguide, by means of simulations. To calculate the fields inside a conductive medium, we extended the Nero2d program with a function to calculate the fields inside a conductive medium.

In our observations of the fields inside the waveguide, we see that the surface roughness has indeed an attenuation effect. From the pattern of the envelope of the fields we clearly observe two different regions before and after the surface roughness. The pattern between the surface roughness and the termination has the typical attenuation pattern for a waveguide with losses. However, in the region between the surface roughness element and the open end of the waveguide, we clearly see the influence of the reflection (and attenuation) of the surface roughness element.

In our observations of the current density in and around the surface roughness element, we came up with the following general conclusions:
When the skin depth is less than 20 % of the surface height we see that the current density follows the border of the surface roughness, very well. In case the skin depth is larger than the height of the surface roughness we see that the current density becomes more homogeneous in and around the surface roughness element. In the
cases where the skin depth is everything in between the last two cases, the current density shows a nice transient process. Although the simulations where done for rather small values of the conductivity and rather large dimensions of the surface roughness, we can scale the dimensions of the surface roughness (and keeping the values of the fields the same), which will automatically increase the conductivity and the frequency. From the current density simulations, one can now better understand which impact a rough surface has on the PCB transmission line model (RLGC model).

One aspect which has not been investigated in this thesis is the case with multiple surface roughness elements. In this thesis we did not discuss this case, because when we insert more than one surface roughness element in the waveguide the segmentation becomes so nonuniform causing the the inaccuracy to increase and we are not able anymore to calculate accurate results. So, more research is still needed for the case where we have multiple surface roughness elements.
Appendix A

A.1 Derivation of Wavenumber and Wavelength in a Good Conductor

In a conductive medium the permittivity is given by \( \epsilon = \epsilon_0 - j \frac{\sigma}{\omega} \) and the permeability is \( \mu_0 \), with \( \epsilon_0 \) the permittivity of vacuum. The wavenumber becomes:

\[
\gamma = \omega \sqrt{\mu_0 \epsilon_0 \left( 1 + \frac{\sigma}{j\omega\epsilon_0} \right)}.
\]

(A.1)

In a good conductor \( \sigma \gg \omega\epsilon_0 \), leading to

\[
\gamma \approx \omega \sqrt{\mu_0 \epsilon_0 \left( \frac{\sigma}{j\omega\epsilon_0} \right)}.
\]

(A.2)

With \( \frac{1}{\sqrt{j}} = \frac{\sqrt{2}}{1+j} \) it becomes:

\[
\gamma \approx \sqrt{\frac{2\sigma \mu_0 \omega}{1+j}} \cdot \frac{1-j}{1-j}
\]

(A.3)

\[\Rightarrow \gamma \approx \sqrt{\frac{\sigma \mu_0 \omega}{2}} (1-j)
\]

(A.4)

With skin depth \( \delta \) equals to \( \sqrt{\frac{2}{\omega\mu_0\sigma}} \) we get:

\[
\gamma \approx \frac{1-j}{\delta}
\]

(A.5)

The real part \( \beta = \frac{2\pi}{\lambda} \) of the wavenumber is given by

\[
\frac{2\pi}{\lambda} = \frac{1}{\delta}.
\]

(A.6)

Which then leads to the following expression for the wavelength \( \lambda \) inside the conductor:

...
A.2 Derivation of the $r_{\text{cut}}$ Distance

In [14] the derivation and definitions of the cut-off distance $r_{\text{cut}}$ are given. This derivation goes as follows. For large $|\gamma r|$ the Hankel function of the second kind and order $\nu$ behaves as:

$$H^{(2)}_{\nu}(\gamma r) \approx \left(\frac{2}{\pi \gamma r}\right)^{1/2} e^{-j\gamma r + j\frac{\pi}{4}(2\nu+1)}$$  \hspace{1cm} (A.8)

In our case the Green’s function is the zeroth order Hankel function of the second kind.

$$G(\gamma r) = \frac{j}{4} H^{(2)}_{0}(\gamma r) \approx \left(\frac{2}{\pi \gamma r}\right)^{1/2} e^{-j\gamma r + j\frac{\pi}{4}} \cdot \frac{j}{4}$$  \hspace{1cm} (A.9)

At a sufficiently large $r_{\text{cut}}$ (small $\Delta_{\text{cut}}$), the asymptotic expression (latter equation) holds and imposing:

$$|G(\gamma r_{\text{cut}})| = \left(\frac{-1}{8\pi |\gamma| r_{\text{cut}}}\right)^{1/2} e^{3\gamma r_{\text{cut}}} = \Delta_{\text{cut}}$$  \hspace{1cm} (A.10)

Taken the square of this:

$$8\pi |\gamma| \Delta_{\text{cut}}^2 = \frac{1}{r_{\text{cut}}} e^{23\gamma r_{\text{cut}}}$$  \hspace{1cm} (A.11)

Multiplying both sides with $-23\gamma$ and putting it in the right form for applying the LambertW function gives:

$$\frac{-23\gamma}{4\pi |\gamma| \Delta_{\text{cut}}^2} = -2r_{\text{cut}}3\gamma e^{-23\gamma r_{\text{cut}}}$$  \hspace{1cm} (A.12)

Making use of the LambertW function gives:

$$W\left(\frac{-23\gamma}{4\pi |\gamma| \Delta_{\text{cut}}^2}\right) = -2r_{\text{cut}}3\gamma$$  \hspace{1cm} (A.13)

Which lead to a value of $r_{\text{cut}}$ of:
A.2. DERIVATION OF THE $R_{\text{cut}}$ DISTANCE

\[ r_{\text{cut}} = \frac{-1}{2\Im\gamma} W \left( \frac{-\Im\gamma}{4\pi|\gamma|\Delta_{\text{cut}}^2} \right) \]  
(A.14)

When we now include the fact that we are in a good conductive medium ($\gamma = \frac{1-j}{\delta^2}$), we become:

\[ r_{\text{cut}} = \frac{\delta}{2} W \left( \frac{1}{4\pi\sqrt{2}\Delta_{\text{cut}}^2} \right) \]  
(A.15)


[14] D. Dobbelaere, H. Rogier, and D. De Zutter. Accurate 2.5-D boundary element method for conductive media. *Accepted for publication in Radio Science*.

[15] The electromagnetic spectrum. "[http://hyperphysics.phy-astr.gsu.edu/hbase/ems1.html](http://hyperphysics.phy-astr.gsu.edu/hbase/ems1.html)"


[17] Integral equations in electromagnetics, Massachusetts Institute of Technology 6.635 lecture notes.
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Surface profile measurement of a rough copper foil</td>
<td>2</td>
</tr>
<tr>
<td>1.2</td>
<td>Surface profile measurement of rough copper foil</td>
<td>3</td>
</tr>
<tr>
<td>2.1</td>
<td>Geometry and incoming field</td>
<td>9</td>
</tr>
<tr>
<td>2.2</td>
<td>Hat function as basis and test function</td>
<td>13</td>
</tr>
<tr>
<td>2.3</td>
<td>Pulse function as basis and test function</td>
<td>13</td>
</tr>
<tr>
<td>3.1</td>
<td>The open waveguide with length $L$</td>
<td>15</td>
</tr>
<tr>
<td>3.2</td>
<td>The terminated waveguide</td>
<td>16</td>
</tr>
<tr>
<td>3.3</td>
<td>Electric field inside the shorted waveguide with PEC</td>
<td>18</td>
</tr>
<tr>
<td>3.4</td>
<td>Electric field inside the shorted waveguide with losses</td>
<td>19</td>
</tr>
<tr>
<td>3.5</td>
<td>Electric field inside the shorted waveguide with losses (detail)</td>
<td>20</td>
</tr>
<tr>
<td>3.6</td>
<td>Fitting the data of the lower envelope to the theoretical function</td>
<td>20</td>
</tr>
<tr>
<td>3.7</td>
<td>The terminated waveguide with length of 20 m and with a surface roughness</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>element in the middle of the length (10 m).</td>
<td></td>
</tr>
<tr>
<td>3.8</td>
<td>Detail of the lower part of the envelope for the terminated waveguide</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>with length of 20 m and with a surface roughness element in the middle of the</td>
<td></td>
</tr>
<tr>
<td></td>
<td>length (10 m) with $\sigma = 5.8 \cdot 10^6$ S/m</td>
<td></td>
</tr>
<tr>
<td>3.9</td>
<td>Detail of the lower part of the envelope for the terminated waveguide</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>with length of 20 m and with a surface roughness element in the middle</td>
<td></td>
</tr>
<tr>
<td></td>
<td>of the length (10 m) with $\sigma = 5.8 \cdot 10^7$ S/m</td>
<td></td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.10</td>
<td>Plot of the field behavior between the surface roughness and the open end of the waveguide by means of calculation.</td>
<td>24</td>
</tr>
<tr>
<td>3.11</td>
<td>Field behavior between the surface roughness and the open end of the waveguide when $</td>
<td>E_1</td>
</tr>
<tr>
<td>3.12</td>
<td>Field behavior between the surface roughness and the open end of the waveguide when $</td>
<td>E_2</td>
</tr>
<tr>
<td>4.1</td>
<td>Visual image of the solution to limit the integration area</td>
<td>28</td>
</tr>
<tr>
<td>4.2</td>
<td>Visual image of the algorithm</td>
<td>28</td>
</tr>
<tr>
<td>4.3</td>
<td>Visual image of the solution</td>
<td>30</td>
</tr>
<tr>
<td>4.4</td>
<td>Situation when the remained interval gives only rise to a hat integration.</td>
<td>33</td>
</tr>
<tr>
<td>4.5</td>
<td>Situation when the remained interval gives rise to pulse and hat integration.</td>
<td>33</td>
</tr>
<tr>
<td>4.6</td>
<td>Plane wave impinging on a copper rectangular region</td>
<td>33</td>
</tr>
<tr>
<td>4.7</td>
<td>Current density $</td>
<td>J_y</td>
</tr>
<tr>
<td>4.8</td>
<td>Current density $</td>
<td>J</td>
</tr>
<tr>
<td>5.1</td>
<td>Observation window to calculate the fields inside this window.</td>
<td>38</td>
</tr>
<tr>
<td>5.2</td>
<td>Frequency spectrum [15].</td>
<td>39</td>
</tr>
<tr>
<td>5.3</td>
<td>$</td>
<td>J</td>
</tr>
<tr>
<td>5.4</td>
<td>$</td>
<td>\angle J</td>
</tr>
<tr>
<td>5.5</td>
<td>$</td>
<td>J</td>
</tr>
<tr>
<td>5.6</td>
<td>$</td>
<td>\angle J</td>
</tr>
<tr>
<td>5.7</td>
<td>$</td>
<td>J</td>
</tr>
<tr>
<td>5.8</td>
<td>$</td>
<td>\angle J</td>
</tr>
<tr>
<td>5.9</td>
<td>$</td>
<td>J</td>
</tr>
<tr>
<td>5.10</td>
<td>$</td>
<td>\angle J</td>
</tr>
<tr>
<td>5.11</td>
<td>$</td>
<td>J</td>
</tr>
<tr>
<td>5.12</td>
<td>$</td>
<td>\angle J</td>
</tr>
</tbody>
</table>
5.13 Observation window to calculate the fields inside this window. 45
5.14 $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 0.5 S/m. 46
5.15 $\angle J$ in and around the surface roughness at a conductivity ($\sigma$) of 0.5 S/m. 46
5.16 $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 1 S/m. 47
5.17 $\angle J$ in and around the surface roughness at a conductivity ($\sigma$) of 1 S/m. 47
5.18 $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m. 48
5.19 $\angle J$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m. 48
5.20 $|J|$ in and around the surface roughness at a conductivity ($\sigma$) of 4 S/m. 49
5.21 $\angle J$ in and around the surface roughness at a conductivity ($\sigma$) of 4 S/m. 49
5.22 $|J|$ in the surface roughness at a conductivity ($\sigma$) of 60 S/m. 50
5.23 $\angle J$ in the surface roughness at a conductivity ($\sigma$) of 60 S/m. 50
5.24 The RLGC model. 52
5.25 AC resistance in a conductive medium without surface roughness. 53
5.26 Enhancement factor ($K_{SR}$) in function of the skin depth ($\delta$). 54
5.27 Gaussian rough surface with $h=0.2, 0.8, 1.5 \mu m$, and width of the surface roughness is $1.5h$. 55
6.1 $|J_x|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m at a segmentation length of $\frac{\lambda}{10}$. 58
6.2 $|J_y|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m at a segmentation length of $\frac{\lambda}{10}$. 58
6.3 $|J_x|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m at a segmentation of $\frac{\lambda}{20}$. 58
6.4 $|J_y|$ in and around the surface roughness at a conductivity ($\sigma$) of 3 S/m at a segmentation of $\frac{\lambda}{20}$. 58
6.5 The solutions on the border which will be studied are marked in red. 59
6.6 Conducting wedge [13]. 60
6.7 $|E_z|$ field on the lower border of the lower conducting strip. 61
6.8 $|E_z|$ field on the upper border of the lower conducting strip. 61
6.9 $|H_z|$ field on the lower border of the lower conducting strip. 62
6.10 \( |H_x| \) field on the upper border of the lower conducting strip. . . . . . . 62
6.11 \( |E_t| \) field on the lower border of the lower conducting strip. . . . . . . 63
6.12 \( |E_t| \) field on the upper border of the lower conducting strip. . . . . . . 64
6.13 \( |H_x| \) field on the lower border of the lower conducting strip. . . . . . . 65
6.14 \( |H_z| \) field on the upper border of the lower conducting strip. . . . . . . 65
List of Tables

5.1 Conductivity versus skin depth with $H_{SR}=0.025$ m. . . . . . . . . . . 36