Digital Signal Processing for Coherent Communication and Compensation of Photonic Integration Imperfections

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Master dissertation submitted in order to obtain the academic degree of

Erasmus Mundus Master of Science in Photonics

Academic year 2011-2012
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Masterproef ingediend tot het behalen van de academische graad van Erasmus Mundus Master of Science in Photonics
Acknowledgements

I am very fortunate to have Dr. Christopher R. Doerr as my supervisor. Since the time I asked to do my thesis under his supervision in Acacia Communication, his willingness to help me, his immense knowledge, patience and the way he bring light to any of my difficulties have been amazing. I am greatly indebted to him, I simply say: thank you so much.

I would also like to greatly thank my promoter Prof. Geert Morthier, for his concern and giving me his feedbacks on the thesis work.

I owe a great deal of thanks to Prof. Roel Baets for giving me the permission to do my master thesis in Acacia Communication.

My sincere gratitude goes to the European Commission for sponsoring me through the Erasmus Mundus program. It would have been unthinkable to persuade my master study abroad without the scholarship.

Special thanks to Diedrik Vermeulen, who not only has given me a ride in his car, but has been instrumental in my work with his feedback and courage.

My many thanks are also to the staff member of Acacia communication who helped me a lot (especially to Jonas Geyer, Graeme Pendock, Long Chen, Scott Stulz, Mehmet Aydinlik, Saeid Azemati and Seo Yeon Park).

Nebiyu Adello, thank you for encouraging me during my studies and giving me your feedback in my write up.

Finally, my deepest gratitude goes to my family for their never ending support, guidance, love and care.
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Tadesse Mulugeta
17 June 2012
Abstract

Coherent detection in combination with digital signal processing (DSP) provides new capabilities such as enabling the use of highly spectrally efficient modulation formats and compensating a wide variety of transmission impairments. However, coherent transceivers are extremely complex and contain many interconnected optical and electrical components. Photonic integrated circuits (PICs) are a key technology to not only reduce the complexity, cost, and footprint of the transceivers but enable future scaling. PICs do have design trade-offs due to the integration, and these need a deeper understanding.

In this thesis, the most important DSP algorithms used for coherent receiver are studied and simulations are performed. Many PIC imperfections are discussed, and novel DSP mitigation techniques for compensation of some of the imperfections are proposed, simulated, and, in one case, experimentally demonstrated.
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<tr>
<td>ADC</td>
<td>Analog to digital convertor</td>
</tr>
<tr>
<td>ASE</td>
<td>Amplified spontaneous emission</td>
</tr>
<tr>
<td>ASIC</td>
<td>Application specific integration circuit</td>
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<tr>
<td>BPSK</td>
<td>Binary phase shift keying</td>
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<td>CD</td>
<td>Chromatic dispersion</td>
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<tr>
<td>CMOS</td>
<td>Complementary Metal Oxide Semiconductor</td>
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<td>DAC</td>
<td>Digital to analog convertor</td>
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<td>DB</td>
<td>Duobinary</td>
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<td>DC</td>
<td>Direct current</td>
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<td>DSP</td>
<td>Digital signal processing</td>
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<td>EDFA</td>
<td>Er-doped fiber amplifier</td>
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<td>EIC</td>
<td>Electronic integrated circuit</td>
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<td>FEC</td>
<td>Forward-error correction</td>
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<tr>
<td>GSOP</td>
<td>Gram-Schmidt Orthogonalization procedure</td>
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<td>GVD</td>
<td>Group velocity dispersion</td>
</tr>
<tr>
<td>ITLA</td>
<td>Integrable tunable laser assembly</td>
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<tr>
<td>LO</td>
<td>Local oscillator</td>
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<tr>
<td>OOK</td>
<td>On-off keying</td>
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<tr>
<td>PDL</td>
<td>Polarization dependent loss</td>
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<td>PDM</td>
<td>Polarization division multiplexing</td>
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<tr>
<td>PIC</td>
<td>Photonic integrated circuit</td>
</tr>
<tr>
<td>PMD</td>
<td>Polarization mode dispersion</td>
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<tr>
<td>Pol-SK</td>
<td>Polarization shift keying</td>
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<tr>
<td>PRBS</td>
<td>Pseudo-random binary sequences</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>QAM</td>
<td>Quadrature amplitude modulation</td>
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<td>QPSK</td>
<td>Quadrature phase shift keying</td>
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<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>SE</td>
<td>Spectral efficiency</td>
</tr>
<tr>
<td>TIA</td>
<td>Transimpedance amplifier</td>
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<td>WDM</td>
<td>Wavelength division multiplexing</td>
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CHAPTER 1

1. INTRODUCTION

1.1 Introduction

Fiber optic communication is a method of conveying information from one place to another by sending pulses of light through an optical fiber. The light forms an electromagnetic carrier wave that is modulated with the information signal. Due to the high carrier frequency of roughly of 200THz, 200,000 times a typical microwave carrier frequency, much higher data transmission rate can be achieved in fiber-optic system. [1] Within the last 30 years, the transmission capacity of optical fibers, which have a low-loss bandwidth of ~10 THz, has been increased enormously. Fiber optic communication has revolutionized the telecommunication industry and has played a major role in the field of data communication.

Compared to traditional copper wire, optical fiber has many advantages such as: its much lower attenuation, high immunity from electrical interference, light weight and huge data transmission capacity. As a result, it has been a choice for many telecommunication companies especially for long distance and high demand applications. Driven by the exponential growth of data traffic, high-capacity optical transmission has experienced orders of magnitude growth in capacity in the past two decades. The capacity growth has been enabled by key technology breakthroughs such as the erbium-doped fiber amplifier (EDFA), wavelength-division multiplexing (WDM), dispersion compensation and management, and fiber nonlinearity management [4].

Today network traffic continues to grow exponentially driven by bandwidth hungry digital applications, video-on-demand, telepresence, wireless backhaul, cloud computing & services, etc. To keep up with this unprecedented traffic growth, the transmission capacity of fiber optics has to be increased further. However, high bit rate optical transmission is limited by optical impairments such as chromatic dispersion, attenuation, and nonlinearity. These impairments limit the available bandwidth in the fiber as well as the transmission reach. For years these impairments have been dealt with in the optical domain through optical components such as erbium-doped fiber amplifiers (EDFA) to compensate the attenuation introduced by the fiber or dispersion compensating fibers (DCF) to undo chromatic dispersion of the fiber. These methods however do have several drawbacks such as their high cost, large physical size, additional loss, latency, and lack of flexibility.

Recent developments in high speed electronic DSP technology created a new paradigm in optical communication. The realization of high throughput DSP devices has enabled compensation of fiber optic impairment electronically. Thus, one can have more adaptability, reduced cost and simplification in the optical link.

In conjunction with coherent detection, where the field rather than the power is detected, DSP enables the adoption of more spectrally efficient modulation formats and extends the electronic
mitigation possibility. Fiber impairments, such as, the linear effects of chromatic dispersion (CD) and polarization mode dispersion (PMD) can be implemented easily by adaptive electronic filtering at the receiver [2]. Spectrally efficient modulation formats such as quadrature phase shift keying (QPSK); with polarization multiplexing become a reality.

Today, transceivers (i.e. transmitter and receiver) have wavelength division multiplexing capability and can support advanced modulation formats. As the level of sophistication increases, so does the price. And operators are at a constant drive to reduce their price per bit per second while installing more and more expensive equipment. Integrating distinct optical components into a single package is key to reduce cost (or energy) per bit of the optical system. Photonic integrated circuits can allow optical systems to be made more compact, higher performance and lower cost than with discrete optical components.

One of the greatest technological innovations of the twentieth century was the development of the integrated circuit (IC). A PIC is conceptually very similar to an electronic IC. While the latter integrates many transistors, capacitors and resistors, a PIC integrates multiple optical components such as lasers, modulators, detectors, attenuators, multiplexers/de-multiplexers and optical amplifiers on a single chip. And PIC’s functionality is for information signal in optical wavelength, typically in visible or infrared region (850nm-1650nm).[3]

1.2 Motivation
DSP in optical coherent systems can overcome a wide variety of transmission impairments. Transceivers supporting advanced modulation formats and coherent detection are complex as they require many individual optical and optoelectronic components. Monolithic integration of the entire transceiver components into a PIC will reduce its size and packaging cost significantly. However, PICs have design trade-offs because all the components are made at once. DSP capability can even be exploited to compensate some PIC design trade-offs. As the signal processing electronics can be implemented either at the transmitter or receiver, PIC imperfections can be compensated at either end. It is important to know which PIC imperfections can be well compensated by DSP algorithms and which cannot.

In this thesis, DSP algorithms used for coherent communication are studied, with the main goal being to investigate the most relevant PIC imperfections and to evaluate how well DSP algorithms and optical component adjustments can compensate for these imperfections and the remaining penalties. Some novel compensation techniques are presented.

1.3 Report outline
Chapter 1 introduces the material. Chapter 2 gives background information on advanced modulation formats, coherent receivers, photonic integrated circuits (PICs) and their imperfections. In Chapter 3, digital signal processing for coherent communication is reviewed. Simulation work is presented using MATLAB to model the most important digital coherent receiver subsystems. Chapter 4 presents compensation of PIC imperfections, analytical analysis
followed by simulation. Offline DSP is performed on real data captured from a PDM-QPSK coherent system in Chapter 5. A conclusion and recommendations for future work are given in Chapter 6.

1.4 References


Chapter 2

2. BACKGROUND

2.1 Advanced modulation Formats

Increasing the transmission capacity of a single-mode fiber requires increasing the spectral efficiency (SE), (i.e. the ratio of a channel bit rate to WDM channel spacing). A high spectral efficiency (SE) can be achieved by using modulation formats in which more than 1 bit of information is transmitted in a symbol. Up until recently nearly all optical transmission systems employed binary on-off keying (intensity modulation) with direct detection. Higher SE can be achieved by exploiting all three optical field attributes--intensity, phase and polarization--to carry information. In advanced modulation formats two often used terminologies are bit rate and symbol rate. Bit rate is the total delivered information given in bits per unit time. Symbol rate is the delivered symbol per unit time, one symbol able to carry more than one bit.

We classify modulation formats based on the physical quantity used to convey information, and the transmission property of the formats. Systems start to no longer rely on the conventional binary modulation of intensity (OOK), as a result transition to binary or multilevel phase modulation and encoding information on the polarization of light become a necessity. In the optical communications community, all formats that go beyond OOK have earned the qualifier advanced. The location of the data points in the space of the optical signal is called a constellation. This space can have three dimensions (in-phase, quadrature, and polarization), but usually only two dimensions are shown. The constellation diagram in Fig.2.1 shows the most important optical modulation formats. [1] Fig. 2.1(a) represents traditional On-Off (OOK) format displayed with two vectors. Information is coded in 2 symbols, 0 and 1 only in the amplitude of the light. Whereas, in BPSK (fig.2.1 (b)) information is coded in the phase of the light and the number of symbols is still 2. Fig.2.1(c) shows another common optical modulation called duobinary (DB). In DB whenever a bit pattern of 1 0 1 is encountered, it will be encoded to +1 0 -1. DB has 1 bit/symbol like the other two binary encodings. However, it has higher tolerance to CD and narrowband filtering. Fig.2.1 (d)-(e) show true multi-level modulation formats where a symbol represents more than one bit. In QPSK, the four symbol alphabet codes 2 bits in one transmission clock cycle, as a result one vector position in the polar plane codes 2 bits, this increases the SE by a factor of 2. In 16-QAM format (Fig.2.1 (e)), 16 symbols code 4 bits, for the same bit rate the SE will be 4 times higher than the binary counterpart. The SE for OOK and BPSK is ~1 bit/s/Hz. The SE for QPSK is ~2 bit/s/Hz. The SE for 16-QAM is ~4 bit/s/Hz.
Formerly, the polarization is used in research experiments to increase the spectral efficiency, either by transmitting two different signals at the same wavelength but in two orthogonal polarizations [polarization-division multiplexing (PDM)] or by transmitting adjacent WDM channels in alternating polarizations to reduce coherent WDM crosstalk or non-linear interaction between the channels (polarization interleaving).[1] The introduction of high speed ADC/DAC and the advance in DSP (digital signal processing) for coherent communication makes it possible to undo the random polarization changes in optical fiber in the electronic domain. As a result, a polarization diversity receiver in conjunction with dual polarization format like, PDM-OOK, PDM-QPSK and PDM-16QAM become real. [2]-[7]. PDM enables doubling the number of transported bits while keeping the same symbol rate compared to a standard single polarization signal, i.e., it can double the SE.

An intensity detector will not be able to recover an advanced modulation format as it cannot detect the phase information. Phase information can be recovered in two ways: differential detection—to detect differentially modulated transmitted signal (i.e. DPSK, DQPSK) by measuring the phase difference between the consecutive bits to intensity difference—and coherent detection where the incoming data signal interferes with local oscillator (LO) before its conversion to an electrical signal.

### 2.2 Coherent Receiver

Optical coherent systems were the subject of intensive research in the 80s because of its high sensitivity. Once the (EDFA) was invented, work on coherent decreased significantly. This is because a comparable sensitivity was achieved by optical pre-amplified direct detection. It made
Coherent detection has three categories based on the frequency difference between the signal and LO. Fig 2.2 shows these three categories. Homodyne detection (a) needs optical frequency and phase locking with the signal and is very difficult to achieve. Heterodyne detection (c) requires extremely high-frequency photodiodes and receiver electronics as it gives a large offset frequency. Whereas, intradyne detection (b) has relative small offset frequency, with the improvement of CMOS electronics, one can now have enough processing power to do the frequency offset estimation and phase locking in electronics, after the signal has been received, by post-processing in real time. This makes intradyne detection the most common type of coherent detection used in today's telecommunication links. In addition to frequency and phase locking the real-time signal processing can also perform polarization demultiplexing and enables compensation of fiber-optic transmission impairments, such as CD, PMD, and to some extent of fiber non-linearities.

Figure 2.2 Three categories of Coherent detection. (a) homodyne, (b) intradyne, and (c) heterodyne.

Figure 2.3 shows the evolution of coherent receivers. Early coherent receivers detected only a single quadrature and were mainly used for heterodyne detection. Today's coherent receivers detect both polarizations and both quadrature [10].
2.3 Photonic Integrated Circuit

With the introduction of wavelength division multiplexing (WDM) and advanced modulation format, the complexity and cost of today’s optical transceiver (i.e. transmitter and receiver) increased tremendously. In a typical coherent communication link shown in Fig 2.4. [10], the transmitter alone contains 23 optical components with 29 inter-component connections, and the receiver contains 13 optical components with 13 inter-component connections. Optical integration can reduce the cost and size of such a transponder. Optical integration is a way to consolidate multiple optical components into a single photonic integrated circuit (PIC). Similar to electronic ICs, PICs involves building multiple optical components into a common wafer, so all optical couplings occur within a single substrate. Packaging takes the major share the cost of an optical device. Optical integration greatly reduces the number of components and assembly steps in an optical package.
The choice of the substrate material is important to get the most out of the PIC functionality. Optical components are made using many materials, which can have quite different optical characteristics. Most commonly used materials are: Group III-V semiconductor materials i.e. indium phosphide (InP) and gallium arsenide (GaAs), Group IV semiconductor materials i.e. silicon (Si) and germanium (Ge) and lithium niobate (LiNbO$_3$).

Lithium niobate (LiNbO$_3$) is a commonly employed material for optical modulators, however, it is difficult to use for optical integration as it uses a diffused waveguide and there is no known way to directly integrate lasers or detectors.

III-V semiconductor materials allow integration of both active and passive components. Due to their direct band gap property and availability of heterogeneous lattice matched ternary and quaternary materials, all major optoelectronics functionality such as light generation, amplification and detection can be realized.

Group IV semiconductor materials have been a promise for large scale integration of passive optical devices such as arrayed waveguide gratings (AWGs), optical switches and VOAs [14].

However, there are a lot of drives to realize silicon PICs for active optical devices, too. The major driver is cost reduction; silicon is the second most abundant element in the earth’s crust (about 27% by mass). In addition, Si-PICs can be built with already matured CMOS technology. The practical difficulty of Si to detect light at 1550 nm can be solved by using germanium (Ge). Crystalline Ge is only 4% lattice-mismatched to Si, allowing it to be grown on Si using a buffer layer of SiGe without too many dislocations. [10]. Si is a very weak electro-optic material. However, the high refractive index ($n=3.48$) of Si allows to make up a high index contrast waveguide combined with the fact that it can be surrounded on all sides by oxide with an index of 1.45. Thus the cross-section of the waveguide can be very small to allow one to move carriers.
in and out of the light beam passing through the Si in a short enough time to be used for high speed modulation[9].

An example of a Si-PIC integrating the entire dual polarization phase diversity coherent receiver is shown in Fig.2.5. In addition to miniaturizing the opto-electronic front end, the monolithic integration facilitates path-length matching and balancing. The superior performance of balanced detection is obtained. The PIC is made with silicon on insulator waveguides with germanium as photodiodes (PD)[13]. A grating coupler is used to couple the signal and LO to the waveguide and thermo-optic phase shifters to adjust the phases in the 90 hybrids.

![Figure.2.5. Layout of Si dual polarization, dual-quadrature coherent receiver [11]](image)

**2.4 Photonic integrated circuit imperfection.**

Advanced-modulation-format transmitters and coherent receivers are complex and require many elements. This makes them ideal candidates for integration. However, PICs have design trade-offs because of the integration. Some common imperfections in PICs that are applicable to advanced-modulation-format transmitters and coherent receivers are

- Limited bandwidth
- Electrical crosstalk
- Limited polarization extinction ratio
- Limited modulator extinction ratio
- Multi-path interference due to reflections
- Imperfect hybrid phase
- Residual amplitude imbalance in balanced detection
- Residual amplitude modulation in modulators
- Non-linearity in phase modulator
Non-linearity in photodiodes

Most of the imperfections will give distortion to the constellation diagram. In some cases the distortion is constant and in others it is rapidly varying due to a frequency dependence. One can sometimes guess the possible root cause of the distortion visually from the received constellation diagram. Fig. 6 shows QPSK constellations for some of I-Q transmitter and receiver impairment, (a) shows an ideal QPSK, (b) shows the presence of a carrier frequency offset, (c) shows when there is a non-linearity and frequency offset, e.g., clipping in I and Q due to saturation, and (d) shows when there is I-Q power imbalance; this kind of effect may happen when there is unequal splitting between the I and Q MZMs, or when I and Q channels have unbalanced driver gain. Another common imperfection in dual quadrature modulation is an imperfect hybrid phase or an error in the 90 degree phase shifter at the transmitter, leading to non-orthogonality of I and Q and giving a rhombic constellation shape as seen in (e). Limited extinction ratio at the modulators will lead to translation along I and Q axis as shown in (f). Bandwidth limitation in components such as the MZMs, DAC, driver, photodiodes, and TIAs causes a distortion to the constellation diagram as shown in (g). Due to the complexity of the coherent front end, it is difficult to have perfect timing alignment between the I/Q ports when the coherent receiver is made of discrete optics. In the presence of this skew, the QPSK constellation looks like (h). However, for PICs, this skew can be precisely designed to be nearly zero, and thus skew is generally not a concern with PICs.

![Figure 6 QPSK constellation diagram for different types of imperfection. Borrowed from [15]](image-url)
2.5 References

Chapter 3

3. Digital Signal Processing for Coherent Optical Communications

3.1 Introduction
The main reason for a recent revival of coherent communication is advances in DSP. DSP enables one to recover the complex amplitude of the signal in a phase/polarization diversity receiver without the need for dynamic polarization controller and phase locking despite the fluctuation in the carrier phase and the signal state of polarization (SOP). Moreover, a coherent receiver aided with DSP opens up the possibility of compensating fiber-optic transmission impairment. In this chapter, the most important digital coherent receiver subsystems are reviewed and the algorithms are implemented in MATLAB to recover PDM-QPSK and PDM-16QAM signals.

3.2 Phase and polarization diversity receiver
The main purpose of a coherent receiver is detecting the complex electric field of the light signal. In this section, the fundamental concept behind a polarization diversity coherent receiver together with the main subsystem in the DSP is described in more detail.

In traditional on-off (OOK) direct detection, the power of light signal is directly converted to an electrical signal using a photodiode. However, in addition to the intensity, detecting the phase of the light signal is paramount for many advanced modulation formats. A phase-sensitive receiver enables us to employ a variety of spectrally efficient modulation formats such as M-ary phase-shift keying (PSK) and quadrature-amplitude modulation (QAM). In addition, since the phase information is preserved after detection, we can realize electrical post-processing functions such as compensation for CD and PMD in the digital domain. [1]

While encoding information on the phase and intensity of light is widely used in modulation of high-speed communication, the spectral efficiency can be increased further by transmitting the signal in two orthogonal polarizations. Fig.3.1 shows dual-polarization coherent receiver. The incoming signal with arbitrary polarization and a LO signal are first split in two orthogonal polarizations (these orthogonal polarizations not necessarily aligned with the original signal polarizations) using a polarization beam splitter (PBS). The complex electric field in each polarization is detected by letting the incoming data signal to interfere with the local oscillator (LO) in an optical 90° hybrid followed by four pair of balanced detectors.

Let the incoming modulated signal in one polarization be:

\[ E_s = A_s \exp \left[ -j(\omega_0 t + \phi_s) \right] \]  
(3.1)

Where \( A_s \) is the complex amplitude, \( \omega_0 \) is the angular frequency and \( \phi_s \) is the signal phase. The field of the local oscillator is:
\[ E_{\text{LO}} = A_{\text{LO}} \exp[-i(\omega_{\text{LO}} t + \phi_{\text{LO}})] \]  
(3.2)

Where \( A_{\text{LO}} \) is the complex amplitude, \( \omega_{\text{LO}} \) is the angular frequency and \( \phi_{\text{LO}} \) is the phase of the LO. The complex amplitudes \( A_s \) and \( A_{\text{LO}} \) are related to the signal powers \( P_s \) and LO power \( P_{\text{LO}} \) by:

\[ P_s = \frac{|A_s|^2}{2} \quad \text{and} \quad P_{\text{LO}} = \frac{|A_{\text{LO}}|^2}{2}. \]

The 90° optical hybrids shown in Fig 3.2 are used to detect both the in-phase and quadrature components of the signal. A balanced detector is used to maximize the signal photocurrent and to subtract DC components. The output optical fields from the 90° optical hybrids are given by:

\[ E1 = \frac{1}{2}(E_{\text{LO}} + E_s) \]  
(3.3)
\[ E2 = \frac{1}{2}(E_{\text{LO}} - E_s) \]  
(3.4)
\[ E3 = \frac{1}{2}(E_{\text{LO}} + e^{j\pi}E_s) \]  
(3.5)
\[ E4 = \frac{1}{2}(E_{\text{LO}} - e^{j\pi}E_s) \]  
(3.6)

The in-phase (I) and quadrature (Q) photocurrent differences after the balanced detectors are given as:

\[ I_I = R(|E1|^2 - |E2|^2) = R\sqrt{P_s P_{\text{LO}}} \cos(\omega_{\text{IF}} t + \phi_s - \phi_{\text{LO}}) \]  
(3.7)
\[ I_Q = R(|E3|^2 - |E4|^2) = R\sqrt{P_s P_{\text{LO}}} \sin(\omega_{\text{IF}} t + \phi_s - \phi_{\text{LO}}) \]  
(3.8)

Where \( \omega_{\text{IF}} \) is the intermediate frequency (IF) given by \( \omega_{\text{IF}} = |\omega_s - \omega_{\text{LO}}| \). when the value of \( \omega_{\text{IF}} \) is higher than the modulation bandwidth of the optical signal the receiver termed as a heterodyne coherent receiver and when it is exactly zero, the receiver termed as a homodyne coherent receiver. We will focus on an intradyne coherent receiver in this paper. The responsivity, \( R \), of a photodiode given by

\[ R = \frac{e \eta}{h \omega_s} \]  
(3.9)

Where \( e \) refers to electric charge, \( \eta \) to the quantum efficiency of the photodiode, and \( h \) to Planck’s constant. By combining Eq. 3.7 and 3.8 the complex amplitude on \( \exp(\omega_{\text{IF}} t) \) is given by:
\[ E_x = I_t + jQ_t = R \sqrt{P_s P_{LO}} \exp\{j(\phi_s(t) + \phi_n(t))\} \] (3.10)

Where \( \phi_s(t) \) is the phase modulation and \( \phi_n(t) \) is the total phase noise which accounts for the transmitter noise, additive noise and LO phase. Once the received signal is down-converted, Eq. 3.10 shows the complex amplitude of the transmitted signal is fully recovered except for the additional phase noise.

Figure.3.1. Coherent receiver employing phase and polarization diversity. [1]
The outputs from the dual polarization coherent receiver are converted to digital data with an analog digital converter (ADC) and processed by the DSP circuit. The DSP circuit brings the following benefits:

- It is possible to synchronize the frequency and phase of transmitter with the LO light without the need for a bulky and expensive phase locked loop (PLL).
- As a result the LO laser requirement will be relaxed, and a commonly available WDM laser can be used as free-running LO.
- It is possible to track the continuously fluctuating state of polarization (SOP) of the incoming signal.
- Compensation of transmission impairments will be possible by distorting the digital waveform.

Figure 3.3 depicts the main sequence of operations in a typical DSP circuit. The 4-channel ADC samples the in-phase and quadrature component of both polarizations, usually with a rate of 2 samples/symbol. Then the accumulated CD is compensated with a digital filter. After that, the dual polarization signal is demultiplexed and the PMD is compensated using an adaptive filter in a butterfly arrangement. After estimating the offset frequency between the transmitter and the LO, the signal is down-converted. Then, the carrier phase is estimated. Finally, the symbol estimation and decoding is performed. We now describe these blocks in more detail.
3.3 Chromatic Dispersion Compensation

CD is usually the main transmission impairment in a fiber-optic link. It is a phenomenon where different spectral components of a signal propagate at different speeds along the length of the fiber. If left uncompensated, it causes broadening to in signal pulses and thus significantly reduces the data carrying capacity of the fiber. Traditionally, dispersion compensating fiber (DCF) has been used to undo the dispersion in the fiber. However DCF incurs additional loss, latency, and nonlinearity. CD compensation (CDC) using electronic signal processing removes the need for these bulky and expensive optical components and offers additional flexibility needed for future optical networks.

The propagation of a light pulse in optical fiber is described by the non-linear Schrödinger equation (NLSE). [2]

\[
j \frac{\partial A}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - j \frac{\alpha}{2} A - \gamma |A|^2 A
\]  

(3.11)

Where \( A \) stands for the field (could be the vector potential), \( \gamma \) for nonlinear coefficient, \( \alpha \) for attenuation coefficient and \( \beta_2 \) group velocity dispersion (GVD). Neglecting attenuation and nonlinearity in equation 3.11, the effect of CD can be expressed as:

\[
\frac{\partial A(z, t)}{\partial z} = j \frac{D \lambda^2}{4\pi c} \frac{\partial^2 A(z, t)}{\partial t^2}
\]  

(3.12)

Where the dispersion coefficient \( D \) given by:

\[
D = - \frac{2\pi c}{\lambda^2} \beta_2
\]  

(3.13)

Solving (3.12) in the frequency domain gives:[3]

\[
A(z, \omega) = A(0, \omega) \exp \left( -j \frac{D \lambda^2}{4\pi c} \omega^2 z \right)
\]  

(3.14)

Thus, the effect of chromatic dispersion on the transmitted signal is modeled as an all-pass filter with a quadratic phase response. Figure 3.4 shows the parabolic phase response of several amounts of CD. The complex transfer function in frequency domain given by:

\[
H_f(\omega) = \exp \left( -j \frac{D \lambda^2}{4\pi c} \omega^2 \right)
\]  

(3.15)
Figure 3.4. Phase response of dispersive fiber for several fiber length [4].

The effect of dispersion can be reversed or compensated by filtering with a transfer function $H_f(\omega) = H^*_f(\omega)$. Such a filter can be realized by using an FIR filter, as shown in fig 3.5 [5]. With the coefficient given by the discrete inverse Fourier transform of $b_n = \left(\frac{1}{N}\right) \sum_{k=0}^{N-1} H^*_f(\omega_k) \exp(j\omega_k n)$.

Figure 3.5 Schematic of an FIR filter for compensating chromatic dispersion
3.4 Polarization de-multiplexing

Transmitting two different signals at the same wavelength but in two orthogonal polarizations (polarization multiplexing) enables doubling the number of transported bits while keeping the same symbol rate compared to a single polarization transmission.[6]. In the other way, for the same bit rate requirement polarization multiplexing will half the clock speed and the symbol rate. While the symbol rate reduction accounts to increased tolerance to inter symbol interference (ISI), two times lower clock speed translate to a considerable power saving in the DSP core and relaxing the ADC/DAC speed requirements. However the state of polarization (SOP) is not preserved upon propagation on a standard single mode fiber unless polarization maintaining fiber (PMF) is deployed for data transmission. Therefore, the receiver sees a linear combination of vertical and horizontal signal in one polarization channel that lead to substantial crosstalk.

DSP is capable of estimating the jones matrix of the transmission channel and is able to undo the polarization scrambling inside the fiber. There are two way of de-multiplexing polarizations: one is by learning the channel matrix using a training sequence; the other is by using blind channel equalization filter. The later being a more feasible solution as it does not need a prior knowledge about the signal and will be examined in this section.

In addition to de-multiplexing the two polarizations, blind equalization is capable of compensating other impairments like PMD, polarization dependent loss (PDL) and residual CD. Its implementation is by using four FIR filters arranged as a butterfly structure (shown in fig 3.6).

The functionality of this filter structure is to perform inverse jones matrix of the channel, the output is given by. [7]

\[
X(k) = h_{11}x(k) + h_{12}y(k) \quad (3.16)
\]
\[
Y(k) = h_{21}x(k) + h_{22}y(k) \quad (3.17)
\]

Where \( h_{11}, h_{12}, h_{21}, \) and \( h_{22} \) are N-tap FIR filters, with \( x(k) \) and \( y(k) \) representing a sliding block of N input samples to which the filter is applied, such that,

\[ x(k) = [x_{in}(k), x_{in}(k - 1), x_{in}(k - 2), \ldots, x_{in}(k - N),] \]. The coefficient of the filter tap is updated by the well-known constant modulus algorithm (CMA).
For the case of a polarization-division multiplexed quadrature phase shift keying (PDM-QPSK) signal, the channel model and the CMA can be analyzed mathematically as follow. [7] Let us express the transmitted signal in the form

\[
\begin{bmatrix}
  x(t) \\
  y(t)
\end{bmatrix} = \begin{bmatrix}
  \sum_k (a_k + jb_k)p(t - kT) \\
  \sum_k (c_k + jd_k)p(t - kT)
\end{bmatrix}
\]  

(3.18)

Where \(a_k, b_k, c_k, d_k \in \left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\), \(T\) is the symbol period and \(p(t)\) is the pulse shape.

After sampling the symbol synchronously eq. 3.18 gives:

\[
\begin{bmatrix}
  x_{in}(k) \\
  y_{in}(k)
\end{bmatrix} = e^{j\vartheta(k)} \begin{bmatrix}
  1 \\
  e^{j\pi n/2}
\end{bmatrix}  
\]

(3.19)

Where \(\vartheta(k) = \arg(a_k + jb_k)\) and \(n = \frac{2}{\pi}(\arg(a_k + jb_k) - \arg(c_k + jd_k))\).
The main idea behind constant modulus algorithm is that the two polarization constellations should have a constant modulus (as seen in eq. 3.19) because they are QPSK signals (except for the transitions between symbols).

Neglecting polarization dependent loss (PDL), the channel can be modeled by a 2x2 unitary matrix R.

\[
R = \begin{bmatrix}
\cos(\theta) & e^{-j\phi}\sin(\theta) \\
-e^{j\phi}\sin(\theta) & \cos(\theta)
\end{bmatrix}
\] (3.20)

Where \( \theta \) and \( \phi \) corresponds for the azimuth and elevation angle of rotation respectively,

Thus, the received sample symbol is given by:

\[
\begin{bmatrix}
x(k) \\
y(k)
\end{bmatrix} = R \begin{bmatrix}
x_{in}(k) \\
y_{in}(k)
\end{bmatrix} = e^{j\phi(k)} \begin{bmatrix}
\cos(\theta) + e^{j\phi/2}e^{-j\phi}\sin(\theta) \\
-e^{j\phi}\sin(\theta) + e^{j\phi/2}\cos(\theta)
\end{bmatrix}
\] (3.21)

Calculating the modulus of the signal, eq.3.21 gives:

\[
\begin{bmatrix}
|x(k)|^2 \\
|y(k)|^2
\end{bmatrix} = \begin{bmatrix}
1 + \sin(2\theta)\cos(\phi - \frac{n\pi}{2}) \\
1 - \sin(2\theta)\cos(\phi - \frac{n\pi}{2})
\end{bmatrix}
\] (3.22)

Note that the total power in the two polarizations is constant by unitary rotation, with only the ratio of power between the two polarizations changing. The output, x-polarization (X) and y-polarization (Y) from the CMA is given by:

\[
\begin{align*}
X(k+1) &= x(k) + h_{11}(k) * x(k) + h_{12}(k) * y(k) \\
Y(k+1) &= x(k) + h_{21}(k) * x(k) + h_{22}(k) * y(k)
\end{align*}
\] (3.23, 3.24)

Where \( x(k) \) and \( y(k) \) is the input sample to the CMA and \( h_{11}, h_{12}, h_{21}, h_{22} \) are the coefficients of the four FIR filters and with an update algorithm given by:

\[
\begin{align*}
h_{11}(k+1) &= h_{11}(k) + \mu \varepsilon_x x(k)x^*(k+1) \\
h_{12}(k+1) &= h_{12}(k) + \mu \varepsilon_x y(k)x^*(k+1) \\
h_{21}(k+1) &= h_{21}(k) + \mu \varepsilon_y x(k)y^*(k+1) \\
h_{22}(k+1) &= h_{22}(k) + \mu \varepsilon_y y(k)y^*(k+1)
\end{align*}
\] (3.25, 3.26, 3.27, 3.28)

Where \( \mu \) is the step size, \( \varepsilon_x \) and \( \varepsilon_y \) is the cost functions given by \( \varepsilon_x = 1 - |X|^2 \), \( \varepsilon_y = 1 - |Y|^2 \).
3.5 Frequency offset estimation

Ideally an intradyne coherent receiver requires lasers with narrow linewidth often on the order of 100 kHz, and very limited offset frequency between the transmitter laser and the local oscillator (LO). However, it is desirable to use the commonly available WDM lasers to be able to reduce the deployment cost.

Although the carrier phase estimation (to be discussed in the next section) will tolerate a laser line width often of the order of 100 MHz, the presence of an offset frequency will degrade the performance of this algorithm. Commercially available tunable lasers have a frequency accuracy of ±2.5 GHz over the lifetime. [8]. If left uncompensated the frequency offset causes a rotation in constellation of a symbol, thus, the decoder will not be able to identify the symbol truthfully.

As a result, incorporating a separate frequency offset estimator circuit to the DSP core is important. Depending on the type of modulation format used, many ways of estimation can be proposed. Straight-forward way estimation for QPSK format can be discussed. If the input signal takes the form:

\[ X_{in}(k) \sim \exp\{j(\vartheta(k) + 2\pi f kT)\} \quad (3.29) \]

Where \( \vartheta(k) \) is the signal phase and \( T \) is the symbol period. A QPSK signal has four possible phases \( \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{-3\pi}{4}, \frac{-\pi}{4} \right\} \), such that the power of four operations can eliminate this signal phase. Operating on consecutive samples gives:

\[ [X_{in}(k)X^*_{in}(k-1)]^4 \sim \exp\{j(4\Delta\vartheta(k))\} \quad (3.30) \]

In [7], it was shown that, in the absence of additive noise, \( 4\Delta\vartheta \) has a circular Gaussian distribution, with mean \( 8\pi f\Delta T \) due to the laser phase noise. And the probability density function (pdf) for \( 4\Delta\vartheta(k) \) is:

\[ f(4\Delta\vartheta) = \frac{\exp\{kc\cos(4\Delta\vartheta - 8\pi f\Delta T)\}}{2\pi I_0(k)} \quad (3.31) \]

Where, \( k \) corresponds to the linewidth of the laser. From the pdf, the offset frequency can be estimated using maximum likelihood technique as: [7]

\[ \Delta f_{est} = \frac{1}{8\pi T} \text{arg} \left( \sum_{k=1}^{N} [X_{in}(k)X^*_{in}(k-1)]^4 \right) \quad (3.32) \]

Where \( N \) is the number of samples used for the estimation.
3.6 Carrier phase estimation

Carrier synchronization in optical coherent systems can be achieved using digital phase estimation techniques, allowing for a free running local oscillator without an optical phase-locked loop [9].

Like the frequency estimation, carrier phase can be estimated using power of four operations for QPSK modulation format. Taking the laser phase into account, the input signal can be expressed as:

\[ X_{in}(k) \sim \exp\{j(\theta_s(k) + \theta_c(k))\} \quad (3.33) \]

Where \( \theta_s(k) \) the data phase, takes on four values \( \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, -\frac{3\pi}{4}, -\frac{\pi}{4} \right\} \) and \( \theta_c(k) \) is transmitter laser phase in reference to the LO. When the sample is raised to the fourth power, we get:

\[ X_{in}(k)^4 \sim -\exp\{j(4\theta_c(k))\} \quad (3.34) \]

The fourth power operation strips off the data phase ( \( \exp\{j(4\theta_s(k))\} = -1 \)). The carrier phase can then be estimated and subtracted from the phase the received signal to recover the data phase as shown in fig 3.8.[5]

\[
X_{in}(k) \sim \exp\{j(\theta_s(k) + \theta_c(k))\}
\]
\[
\theta_s(k) = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, -\frac{3\pi}{4}, -\frac{\pi}{4} \right\}
\]

\[
X_{in}(k)^4 \sim -\exp\{j(4\theta_c(k))\}
\]

Figure 3.7 Feed-forward phase estimation algorithm for QPSK modulation.

The feed-forward algorithm is used only for ideal situations where there is no additive noise present. In reality, the received signal will contain noise dominated by either amplified spontaneous emission (ASE)-LO beat noise or shot noise of the LO. [5] The phase estimation have to be modified to manage these noises, given as:[7].

\[ \theta_c(k) = \text{arg} \left\{ 1 \sum_{n=-N}^{N} w[n] X_{in}(k + n)^4 \right\} \quad (3.35) \]

Where \( w[n] \) is a weighting function, which depends on the ratio of additive white Gaussian noise to the laser phase noise. For a constant tap weighting function, assuming the carrier phase is constant over the sequence of sample, the variance of the phase estimation error due to additive noise will be reduced by a factor \( 2N+1 \). However, the size of the weighting function has to be limited as the filtering itself may introduce error since the carrier phase can’t be constant over a large sequence of samples.
Since the phase estimate is forced in value in range \(0 \leq \theta_c(k) \leq \frac{\pi}{2}\), there is a fourfold phase ambiguity. It is important to differentially pre-code data to avoid cycle slips [5]. At the receiver, differential decoding may also be implemented to reduce the impact of cyclic slips which can create a catastrophic effect otherwise.

### 3.8 Simulation result

In addition to the theoretical analysis, the algorithms (i.e. CMA, offset frequency estimation, carrier phase estimation) are implemented in MATLAB to recover both QPSK and 16-QAM modulation formats for the line rate of 120Gb/s. The electrical data signals each consist of delayed copy of \(2^{15}\) pseudo-random binary sequences (PRBS). The noise of the optical preamplifier was modeled as additive white Gaussian noise. The signal and local oscillator were mixed by a 90° optical hybrid and the in-phase and quadrature channel of the signal is extracted by balanced detectors.

Figure 3.9 shows a 30Gbaud PDM-QPSK signal recovery by using: CMA, frequency offset estimation and carrier phase estimation algorithms. The received data constellation diagram is shown in (a) for X-polarization. After running the CMA algorithm on the input samples, the polarization separation is seen in the donut shape constellation in (b). After applying the phase and frequency compensation algorithms the QPSK signal is recovered, Fig(c) shows the recovered constellation for X-polarization.

Figure 3.8. PDM-QPSK Digital signal processing, (a) received symbol for X-pol (b) X-pol after CMA polarization separation, with 100MHz offset frequency (c) X-pol after frequency and phase compensation.(note: black star represent the ideal constellation points)

Similarly, it is also possible to recover 16-QAM with CMA algorithm. Fig.3.10 shows 15Gbaud 16-QAM signal recovery for two OSNR values: 12dB and 20dB (0.1nm-resolution bandwidth), in the presence of 100MHz offset frequency. After the frequency and phase estimation the recovered symbols have BER of \(2 \times 10^{-2}\) for 12dB OSNR, and BER of \(3 \times 10^{-5}\) for 20dB OSNR.
Figure 3.9. PDM-16QAM Digital signal processing, (i) for 12dB OSNR (0.1nm resolution bandwidth) (ii) for 20dB OSNR (0.1nm bandwidth), (a) received symbol for X-pol (b) X-pol after CMA polarization separation (c) X-pol after frequency and phase compensation. (note: red star represent the ideal constellation points)
3.8 References


Chapter 4

4. COMPENSATION OF PHOTONIC INTEGRATED CIRCUIT IMPERFECTION

4.1 Introduction

Most PIC imperfections can be compensated with DSP and/or optical element adjustments. The imperfections can be in the transmitter or receiver, and likewise the DSP compensation algorithms can be in the transmitter or receiver. In this chapter, some of the PIC imperfections are analyzed and possible compensation mechanisms are proposed.

The DSP algorithms must be computationally efficient, because they occur in real time. The number of two-variable multiplies must be minimized. They ideally should be able to be pipelined.

First, limited extinction ratio could happen in the MZMs due to e.g., imperfect splitting of the 3-dB couplers or unbalanced waveguide loss in the two arms of the MZM. In Sec. 4.2, a novel compensation mechanism is proposed to improve the penalty due to finite extinction ratio.

Second, the effect of limited bandwidth and non-linearity is a pervasive imperfection in high-speed transceivers and is discussed in Sec. 4.3. A high speed silicon modulator based on free carrier plasma dispersion effect suffers from capacitance variation; in Sec. 4.4 a novel pre-compensation scheme is described.

Finally, post compensation methods to correct I-Q phase error in the receiver or transmitter are discussed in Secs. 4.5 and 4.6 respectively.

4.2 Compensation limited extinction ratio at MZM

One common non-ideality at the modulator is imperfect power splitting ratio in a balanced MZM, leading to finite extinction ratios. When data transmission is considered, the finite extinction ratio manifests itself in BER degradation. VOAs could be used to control the power splitting in the MZMs, as perfect splitting is difficult to be achieved in realistic situations. However, adding VOAs adds cost and complexity. In this section, novel electronic tuning and pre-compensation mechanisms are proposed to compensate MZM finite extinction ratio. A simulation for 16-QAM is performed.

a) Compensation by pre-distorting driver voltage

For a dual-parallel MZM with push-pull configuration, let the ratio of the amplitudes of the electric fields from the two arms in the upper MZM is $1:\gamma_2$, in the lower MZM is $1:\gamma_3$ and the outer MZM has splitting ratio of $1:\gamma_1$, as shown in Fig 4.1. In the ideal case, $\gamma_1 = \gamma_2 = \gamma_3 = 1$.
Figure 4.1. Dual Parallel MZM with unequal splitting ratio

If all the other parameters are perfect, and $\gamma_1 = 1$, the output field can be expressed as

$$E_{out} = \frac{E_{in}}{2} \left\{ \frac{1}{2} \left( e^{j \frac{\pi}{V_n} (V_{driver1} + V_{bias1})} + \gamma_2 e^{-j \frac{\pi}{V_n} (V_{driver1} + V_{bias1})} \right) + \gamma_1 \frac{1}{2} j \left( e^{j \frac{\pi}{V_n} (V_{driver2} + V_{bias2})} + \gamma_3 e^{-j \frac{\pi}{V_n} (V_{driver2} + V_{bias2})} \right) \right\} \tag{4.1}$$

Assuming the DC bias voltages for both inner Mach-Zehnder are $-V_n/2$, the modulated optical field is then expressed as:

$$E_{out} = \frac{E_{in}}{2} \left\{ \frac{1}{2} \left( (1 + \gamma_2) \sin(\phi_1) - j(1 - \gamma_2) \cos(\phi_1) \right) + \gamma_1 \frac{1}{2} j \left( (1 + \gamma_3) \sin(\phi_2) - j(1 - \gamma_3) \cos(\phi_2) \right) \right\} \tag{4.2}$$

Where, $\phi_1 = \frac{\pi}{V_n} (V_{driver1})$, $\phi_2 = \frac{\pi}{V_n} (V_{driver2})$ and $\frac{\pi}{V_n} (V_{bias1}) = \frac{\pi}{V_n} (V_{bias2}) = -\frac{\pi}{2}$

For a small drive voltage, after small angle approximation Eq.4.2 is reduced to:

$$E_{out} \approx \frac{E_{in}}{2} \left\{ \left( \frac{(1+\gamma_2)}{2} \right) (\phi_1) + \left( \frac{(1-\gamma_3)}{2} \right) \right\} + j \left( \frac{(1+\gamma_3)}{2} \phi_2 - \frac{(1-\gamma_2)}{2} \phi_1 \right) \tag{4.3}$$

The output signal encounters scaling and translation term in each quadrature due to the imperfect splitting. However, if $\gamma_2$ and $\gamma_3$ are close to 1, the effect of scaling can be neglected (i.e. $\frac{(1+\gamma_2)}{2} \approx 1$, $\frac{(1+\gamma_3)}{2} \approx 1$) and Eq.4.3 is finally approximated as:
Driven with a small signal, the output optical field gives a linear combination of two electrical drive voltages with an additional translation due to imperfect splitting. Imperfection at the in-phase MZM causes back-ward translation to the quadrature channel while the imperfection at the quadrature MZM causes a forward translation to the in-phase channel. 

If we shift $\phi_1$ backward with $\frac{(1-\gamma_2)}{2}$ and $\phi_2$ forward with $\frac{(1-\gamma_2)}{2}$, by pre-distorting the electrical driver signal, we will be able to get the desired optical signal from the imperfect modulator.

For a dual parallel-MZM, we have two drive voltages. In order to generate a desired optical signal with specified amplitude and phase at the output of modulator, first, two required electrical drive voltages are back-calculated based on an inverse modulator transfer function. Since $\phi_1 = \frac{\pi}{V_{\pi}} (V_{\text{driver1}})$ and $\phi_2 = \frac{\pi}{V_{\pi}} (V_{\text{driver2}})$, the the pre-distorted drive voltages will be:

$$\begin{aligned}
V_{\text{driver1}} &= V_{\text{back1}} - \frac{V_{\pi}}{\frac{(1-\gamma_2)}{2}} \\
V_{\text{driver2}} &= V_{\text{back2}} + \frac{V_{\pi}}{\frac{(1-\gamma_2)}{2}}
\end{aligned}$$

Where, $V_{\text{back1}}$ and $V_{\text{back1}}$, the back calculated voltages for the ideal modulator 

Note that, since the driver voltages are linearly proportional to data signals, a DC shift in the RF voltage can give a similar compensation.

b) Compensation by scaling bias voltages

While changing the DC level of the drive voltage in accordance with splitting ratio works well to compensate the imperfection, a more elegant way of compensation is proposed by adjusting the bias voltages in the two MZMs.

To see the effect of scaling the bias voltages, let us modify the output of dual parallel-MZM by adding scaling coefficients to bias voltages:

$$E_{\text{out}} = \frac{E_{\text{in}}}{2} \left\{ \cos \left( \phi_1 + \frac{\pi}{V_{\pi}} K_1 * V_{\text{bias1}} \right) + j \cos \left( \phi_2 + \frac{\pi}{V_{\pi}} K_2 * V_{\text{bias2}} \right) \right\}$$

Where, $K_1$ and $K_2$ is the scaling coefficient for the bias voltages 

For a similar bias point (i.e. $\frac{\pi}{V_{\pi}} (V_{\text{bias1}}) = \frac{\pi}{V_{\pi}} (V_{\text{bias2}}) = -\frac{\pi}{2}$), after small angle approximation, Eq.4.6 can be expressed as:
Scaling bias voltage in the in-phase (upper) MZM gives a linear translation in the in-phase channel and scaling bias voltage in the quadrature (lower) MZM gives a linear translation in the quadrature channel. Comparing Eqs. 4.4 and 4.7, it is possible to compensate the amplitude imbalance by scaling the bias voltages as:

\[
\begin{align*}
(1 - K_1)\pi &\approx -(1 - \gamma_3) \\
(1 - K_2)\pi &\approx (1 - \gamma_2)
\end{align*}
\]  

(4.8)

The scaling in the upper MZM corresponds for the imbalance in the lower MZM and vice versa.

**c) Simulation of a finite extinction ratio compensation**

In addition to the theoretical analysis, the proposed method of pre-compensation was numerically investigated using MATLAB. A 16QAM modulation format with 15Gbaud symbol rate is used. The four multiplexed data signals each consist of 15-Gb/s \(2^{15}\) PRBS. At the receiver, the noise of the optical preamplifier was modeled as additive white Gaussian noise. The signal and local oscillator were mixed by a 90° optical hybrid and the in-phase and quadrature channel of the signal is extracted by balanced detectors. Assuming \(V_p = 3v\) and \(V_{pp} = 0.76v\), Fig.4.2 (a) and (b) shows the constellation diagram with and without compensation for the case when the splitting ratio in both MZM is 1 to 0.8 (~19dB extinction ratio), and Fig (c) and (d) for 1 to 0.65 splitting ratio in both MZM (~13.4dB extinction ratio).

Both Fig (a) and (c) show that limited extinction ratio causes linear translation along the real and imaginary axes of the constellation diagram, and if left unattended will lead to substantial BER degradation. It appears that the bias voltage scaling mechanism fully recovers the constellation diagram for the case of ~19dB extinction ratio (Fig 4.2 (b)). However, because of the approximation involved in the analytical analysis of the previous section and the assumption of small signal modulation, the proposed method of compensation performs suboptimally for large signal modulation or very low extinction ratio (i.e. less than 10dB) case. The compensation is not perfect for the case of ~13.4dB (fig.4.2(d)) as compared to the case in (b).
In this section, a simple but novel method of compensating finite extinction ratio of a MZM is presented. The method with bias voltage scaling will not add any computational burden to the DSP circuit, yet it can reduce the number of VOA’s required in the modulator which will in turn simplify the packaging and cost of photonic integrated circuit (PIC). The bias voltage defined in Eq. 4.1 can be a very sensitive parameter to play with especially in silicon modulator where a bias point can solely decide the speed of the modulator, however, in Si modulator there is a separate thermo-optic phase shifter in each arm of the MZM, a similar bias voltage scaling of the phase shifters can do the job.[13]

4.3 Compensation of limited bandwidth and non-linearity
A high-speed optical transmitter comprises of a digital-to-analog convertor, a DSP, an electrical driver and the actual phase modulators. Fig 4.3 shows the block diagram for a single polarization transmitter. Encoding, waveform generation, DSP and DAC is executed in an ASIC (application-specific integrated circuit). Common sources of limitations in such kinds of
transmitter are: non-linearity and non-flat frequency response of the DACs, drivers and optical modulator and non-linearity in the phase modulator, driver and DACs. Non-linearity in the phase modulator is unavoidable, however the presence of non-linearity in the driver and DAC will lead to additional signal distortion if not compensated with the DSP. The DSP circuit is used to implement a linear pre-emphasis filter and a non-linear equalizer to compensate bandwidth limitation and nonlinearity of those devices. Figure 4.4 shows the sequential operation needed to be performed in the ASIC.

Figure 4.3 Transmitter block-diagram

If the nonlinearity in the driver is neglected, the pre-emphasis filter can be computed by merging the frequency responses of the DACs, Drivers and DP-MZM. And the back calculated driver voltages are modified as:

\[ V_{\text{driv1}}(t) = F^{-1}(F(V_{\text{back1}}(t)) \ast (H_{\text{mod}} \ast H_{\text{driver}} \ast H_{\text{dac}})^{-1}) \]  \hspace{1cm} (4.9)
\[ V_{\text{driv2}}(t) = F^{-1}(F(V_{\text{back2}}(t)) \ast (H_{\text{mod}} \ast H_{\text{driver}} \ast H_{\text{dac}})^{-1}) \]  \hspace{1cm} (4.10)

Where \( V_{\text{back1}}(t) \) and \( V_{\text{back2}}(t) \) are the back calculated driver voltage based on the desired complex symbols. And \( F() \) and \( F^{-1}() \) denote the Fourier transform and the inverse Fourier transform, respectively. However, in the presence of driver nonlinearity, the compensation should follow the sequence in Fig 4.4.
At a given bit rate, PDM-16QAM signal has half the bandwidth requirement of PDM-QPSK. As a result, 16-QAM has a more relaxed device requirement than the QPSK format in terms of bandwidth for a given bit rate. A simulation is performed for 120-Gb/s polarization-division multiplexed (PDM) quadrature phase-shift-keyed (QPSK) signal with I and Q components consisting of delayed copies of 30-Gb/s $2^{15}$ PRBS. Each quadrature data signal experiences bandwidth limitation in the DAC, driver and modulator (e.g. 20GHz bandwidth is assumed for DAC and driver, and 15GHz for the modulator). Figure 4.5 shows the simulation results performed for a 15-dB OSNR 120-Gb/s signal. The bandwidth limitations severely degrade the performance, unless it is compensated with a digital filter. A comparison between the phase diagrams indicates that the pre-equalizing filter has eliminated the effect of limited bandwidth and allows detection of the four phases of QPSK.
Figure 4.5 Phase diagrams and constellation diagrams without and with compensation of bandwidth limitation in the transmitter for a 120-Gb/s PDM-QPSK signal.

4.4 Compensation of varying capacitance in silicon-modulator
(a) Si-modulator

Silicon-based devices like waveguides, modulators and detectors will enable high-speed communication at a low cost, low power consumption and small footprint due to its compatibility with mature silicon technology and the potential of combining electronics and optics technology.

Silicon, unlike the traditional materials used for high-speed modulators, exhibits no linear electro-optic effect and only very weak Kerr and Franz-Keldysh effects, and optical modulation through the Pockels effects is not possible in silicon. [3]

High-speed modulation in silicon can instead be achieved through the plasma dispersion effect, where a change in carrier concentration causes a change in absorption, and through the Kramer–Kronig relations, a change in the refractive index. A carrier density variation can be reached by carrier injection in a forward biased p-i-n diode, by accumulation in a metal–oxide–semiconductor capacitor, or by depletion in a reverse biased p-n diode. The three types of Si modulators are shown in Fig.4.6. (a) Shows a carrier-injection modulator. While this effect has better modulation efficiency ($V_\pi L$ of 2.5 V-mm is possible), however due to its high injection
capacitance and slow free-carrier recombination achieving high speed modulation will be difficult, it has typical speed below 5 GHz. Speeds up to 10 Gb/s have been achieved [2], but require electronic pre-emphasis. (b) Shows a carrier depletion modulator. The effect is much weaker (VₐL of 25 V-mm is typical) but speeds in excess of 40 GHz are possible. 50-Gb/s modulation has been reported [4]. Figure 4.6 (c) shows a metal-oxide-semiconductor (MOS) modulator. The thin layer of oxide allows a larger carrier density change, thus, it has better modulation efficiency than carrier-depletion in a reverse-biased pn junction (VₐL of 2.5 V-mm is possible). However, the speed of MOS-type modulator is limited by the high capacitance from the thin oxide layer. [1, 2]  

![Figure 4.6 various type of Si modulator (a) carrier injection (b) carrier depletion (c) MOS](image)

Carrier depletion in reverse biased p-n diodes have an advantage on the operational speed since it relies on the electric-field induced majority carrier dynamics. It is a widely used mechanism to realize high-speed modulation in silicon. However, carrier depletion has low modulation efficiency due to a relatively small overlap between the carrier depletion region and the optical mode [3]. As a result there is a tradeoff between speed, insertion loss and drive voltage of the modulator.

The real and imaginary parts of the refractive index change of Si with free carrier distribution at 1.55μm wavelength are given by Soref’s relation [5].

\[
\Delta n_r = -8.8 \times 10^{-22} N_e - 8.5 \times 10^{-18} N_h^{0.8} \quad 4.11
\]

\[
\Delta n_i = 2.5 \times 10^{-26} N_e^{1.2} + 2.5 \times 10^{-24} N_h^{1.08} \quad 4.12
\]

Where \(N_e\) and \(N_h\) are the electron and hole densities, respectively. The hole density has a stronger effect on the real part of the index change than the electron density yet the absorption is approximately the same for hole and electron concentrations. For this reason, Si modulators are typically designed to have a larger optical mode overlap with the p-doped region.

A typical plot of the phase response and normalized capacitance for varying driver voltage is shown in Fig. 4.7. For a reverse bias voltage, the depletion width increases with the square root of the applied voltage, thus the capacitance reduces as the voltage increases.
As part of reducing the power consumption in an optical transponder, one of the main performance requirements in the modulator is low RF power. Although, working at high modulation efficiency region using a pre-emphasis driving signal may achieve this goal, there is no fixed frequency response as the capacitance varies with the driving voltage.

(b) Pre-compensation

Device bandwidth limitation can be overcome by using a digital pre-compensation filter when the frequency roll-off of the device is not too steep. Multilevel modulation in Si involves a relatively large RF voltage swing. This results in a significant modulator capacitance change during modulation. Thus there is not just one frequency response to use for the pre-compensating filter. In other words, the impulse response of the modulator changes with drive voltage. In this section a novel pre-compensation technique for a silicon depletion dual parallel MZM is proposed.

Let the bias voltage at both arms of a MZM be zero, and assume there is an additional 180 degree phase shifter in the lower arm. If there is no capacitance variation in the modulator then the output field from the dual parallel MZM can be expressed as:

\[
E_{\text{out}} = \frac{E_{\text{in}}}{2} \left\{ \frac{1}{2} \left( e^{i \pi \frac{V_{\text{driver1}}}{V_{\text{drive}}}} - e^{-i \pi \frac{V_{\text{driver1}}}{V_{\text{drive}}}} \right) + \frac{1}{2} j \left( e^{i \pi \frac{V_{\text{driver2}}}{V_{\text{drive}}}} - e^{-i \pi \frac{V_{\text{driver2}}}{V_{\text{drive}}}} \right) \right\}
\]  

(4.13)
This is the model commonly used for an MZM. However, this model is significantly inaccurate, because the two arms of an MZM have different impulse responses. For example, at a given moment of driver voltage, the positive voltage side arm experiences high capacitance while the negative side experiences low capacitance.

Considering the four driver voltages applied to the four arms of the dual parallel MZM separately (i.e. V1=Vdriver1, V2=-Vdriver1, V3=Vdriver2, V4=-Vdriver2)

For \( V_{pp} = 0.768 \text{V}, \ V_\pi = 3 \text{V}, \) assuming RC time constant varies from 12-30ps. The output voltages with R-C model are calculated as follows:

\[
\begin{align*}
R &= 60; & \text{% serious resistor in ohm} \\
Q1(1) &= 0; & \text{% initial charges} \\
Q2(1) &= 0; \\
Q3(1) &= 0; \\
Q4(1) &= 0; \\
\text{for } i = 1: \text{length}(V\text{driver}1) & \text{% for } (V_{pp}) = 0.768 \text{V} \\
C1 &= 1e-13*(3.75*V1(i)+3.5); \\
C2 &= 1e-13*(3.75*V2(i)+3.5); \\
C3 &= 1e-13*(3.75*V3(i)+3.5); \\
C4 &= 1e-13*(3.75*V4(i)+3.5); \\
V01(i) &= Q1(i)/C1; & \text{% the output voltage from the RC circuit} \\
V02(i) &= Q2(i)/C2; \\
V03(i) &= Q3(i)/C3; \\
V04(i) &= Q4(i)/C4; \\
Q1(i+1) &= Q1(i)+Dt*(C1*V1(i)-Q1(i))/(R*C1); \\
Q2(i+1) &= Q2(i)+Dt*(C2*V2(i)-Q2(i))/(R*C2); \\
Q3(i+1) &= Q3(i)+Dt*(C3*V3(i)-Q3(i))/(R*C3); \\
Q4(i+1) &= Q4(i)+Dt*(C4*V4(i)-Q4(i))/(R*C4); \\
\end{align*}
\]

\( R \) is the series resistance of the modulator, \( C \) is the voltage-dependent capacitance, \( Q \) is the charge in the capacitor, and \( Dt \) is the sampling period.

If we a use piece-wise approximation for the capacitance curve, we can model the capacitance effect by adding different impulse response in both arms. For two-level driver voltage the output field is modified as:

\[
E_{\text{out}} = \frac{E_{\text{in}}}{2} \left\{ \frac{1}{2} \left( e^{\frac{i\pi}{\sqrt{2}}(V_{\text{driver}1}*h_{1,2})} - e^{-\frac{i\pi}{\sqrt{2}}(V_{\text{driver}1}*h_{2,1})} \right) + \frac{1}{2} \left( e^{\frac{i\pi}{\sqrt{2}}(V_{\text{driver}2}*h_{1,2})} - e^{-\frac{i\pi}{\sqrt{2}}(V_{\text{driver}2}*h_{2,1})} \right) \right\} \quad (4.14)
\]
where "*" denotes convolution, and h is the local impulse response. When the first arm impulse response is $h_1$, the second arm will be $h_2$ and vice versa.

For relatively small phase shift in the modulator, the above non-linear equation can be linearized and approximated to get a resultant linear response.

After first order approximation and simplification, Eq. 4.14 gives:

$$E_{out} = \frac{E_{in}}{2} \left\{ \left[ \frac{\pi}{V_{\pi}} V_{\text{driver}1} \ast \frac{(h_{1,2} + h_{2,1})}{2} \right] - \left[ \frac{\pi}{V_{\pi}} V_{\text{driver}2} \ast \frac{(h_{1,2} + h_{2,1})}{2} \right] \right\}$$  \hspace{1cm} (4.15)

Figure 4.8. (a) shows a push-pull dual parallel MZM, and the corresponding linear filter equivalence of a single MZM is shown in (b).

![Figure 4.8](image-url)

When there is no capacitance variation, the expected small-signal modulated output field from the DP-MZM can be expressed as:

$$E_{out} = \frac{E_{in}}{2} \left\{ \left[ \frac{\pi}{V_{\pi}} V_{\text{driver}1} - \frac{\pi}{V_{\pi}} V_{\text{driver}2} \right] \right\}$$  \hspace{1cm} (4.16)

Using the approximated linear response model, it is possible to pre-equalize the RF driver voltage to overcome distortion induced by the non-linear bandwidth limitation of Si modulator. For example, the two required pre-compensated drive voltages become as follows:

$$V_{\text{driver}1}^{pre}(t) = F^{-1} \left\{ F \left[ V_{\text{driver}1}(t) \ast \left( \frac{H_1(\omega) + H_2(\omega)}{2} \right)^{-1} \right] \right\}$$  \hspace{1cm} (4.17)

$$V_{\text{driver}2}^{pre}(t) = F^{-1} \left\{ F \left[ V_{\text{driver}2}(t) \ast \left( \frac{H_1(\omega) + H_2(\omega)}{2} \right)^{-1} \right] \right\}$$  \hspace{1cm} (4.18)

Where $H_1(\omega)$ and $H_2(\omega)$ corresponds to the frequency responses of the modulator at the positive and negative driver voltage respectively. In other words, we have discovered that we
can take the average of the impulse responses for the two diode conditions as the impulse response to invert and use in the compensator.

Another possible way to track the modulator capacitance variation is to employ two drivers for a single MZM, so that the two RF voltages can be distorted separately in the DSP circuit. In this case two adaptive filters are used for a single quadrature. The filter coefficients are chosen depending on the value of the driver voltage. For example, in 16-QAM modulation each driver can have four different voltage levels, which means the equalizing filter for a single RF voltage can have four different filter characteristics and chosen adaptively.

A simulation is performed for 15-Gbaud 16-QAM modulation, both static equalization (i.e based on Eqs. 4.17 and 4.18) for a single-drive MZM and adaptive equalization for double drive MZM is considered. Figure 4.9 (a) shows the piecewise approximation of the capacitance curve employed in the compensation, (b) the pre-emphasis filter used for both type of equalization, the black curve corresponds to the static equalizing filter, the other four curves for the adaptive equalizing filter when two driver voltages used. For the simulation, the peak-to-peak driver voltage ($V_{pp}$) is assumed to be 0.76V and the RC time constant varies from 18ps to 30ps linearly with voltage.

The distorted constellation and closed eye diagram in Fig.4.10 (a) are due to the RC time constant variation of the modulator. A single equalizing filter however gives a much better performance, shown in (b). For the case of dual drive voltage the result of piecewise adaptive equalizer is shown in (c).

![Figure 4.9](image_url)

**Figure 4.9** (a) capacitance curve piecewise approximation (b) equalizing filters amplitude response (black curve being for static equalizer)
Figure 4.10 constellation and eye diagram for 16QAM Si-modulator, (a) without pre-compensation, (b) single driver MZM compensated with static average filter (b) double driver MZM compensated with adaptive filter by piecewise capacitance approximation.

One can see that the piecewise filter adapting equalization does not give a better performance despite its computational and hardware burden, whereas a single filter approach gives actually slightly better performance with much more simple computation.

4.5 Compensation of imperfect hybrid phase and photodiodes responsivity mismatch

Imperfection of 90° hybrid and photodiode responsivity mismatch are important imperfection at the phase diversity receiver. If left uncompensated, these imperfections will introduce gain and phase imbalance between the in-phase and quadrature channels, commonly known as quadrature imbalance.

Ideally, the I and Q channels of a phase-diversity receiver are orthogonal. However, the presence of quadrature imbalance destroys the orthogonally between the two received channels and these degrade the performance of the system. Several digital signal processing (DSP) algorithms have been proposed so far to compensate quadrature imbalance for quadrature phase shift keying (QPSK). [7-10]. Most of the quadrature imbalance compensation techniques have been based on the Gram-Schmidt Orthogonalization procedure (GSOP). However, the impact of quadrature imbalance compensation for M-QAM format and the effect of common mode rejection ratio (CMMR) have not been analyzed yet. In [11], decision directed least-mean–square (DD-LMS) adaptive filter method is proposed for 16-QAM case. In this section, a method of compensating both amplitude and quadrature imbalance is proposed. A simulation for both QPSK and 16QAM modulation format is performed to verify the proposed algorithm.
Referring to Fig. 3.1 of polarization diversity receiver, there are 8 photodiodes and 2 90° hybrids for both polarizations.

Let the local oscillator split equally to both polarization and responsivity of the four photodiodes for x-polarization be: r1, r2, r3 and r4. And let δ be the phase error in the 90° hybrid.

The output current from the four photodiodes will be:

\[ A1 = r1 \frac{1}{2} |E_{LO} + E_s|^2 \]  
\[ A2 = r2 \frac{1}{2} |E_{LO} - E_s|^2 \]  
\[ A3 = r3 \frac{1}{2} |E_{LO} + e^{j(\frac{\pi}{2} - \delta)}E_s|^2 \]  
\[ A4 = r4 \frac{1}{2} |E_{LO} - e^{j(\frac{\pi}{2} - \delta)}E_s|^2 \]  

Where the field from the LO, \( E_{LO} = |E_{LO}|e^{-j\Delta \omega t} \), with \( \Delta \omega \) being the offset frequency between the laser and LO and signal field \( E_s = |E_s|e^{j\theta_s(t)} \).

After the balanced detector, the two output channels for x-polarization give,

\[ I = A1 - A2 = (r1 + r2) |E_{LO}| |E_s| \cos(\Delta \omega t + \theta_s(t)) + (r1 - r2) \frac{(|E_{LO}|^2 + |E_s|^2)}{2} \]  
\[ Q = A3 - A4 = (r3 + r4) |E_{LO}| |E_s| \sin(\Delta \omega t + \theta_s(t) + \delta) + (r3 - r4) \frac{(|E_{LO}|^2 + |E_s|^2)}{2} \]

The 90 degree hybrid phase error gives rise to phase error to the quadrature term, whereas, unequal responsivity gives dc offset in other word reduces the common mode rejection ratio (CMRR). For a constant-level modulation format like QPSK, the DC offset may be corrected by subtracting the average levels from each channel. However, for a multi-level modulation format, the DC offset depends on the signal, and hence the constellation diagram will be deformed.

(a) Quadrature Imbalance compensation for QPSK modulation

After subtracting the DC term from each channel, I and Q can be approximated as,

\[ I = \frac{r1+r2}{r3+r4} (r3 + r4) |E_{LO}| |E_s| \cos(\Delta \omega t + \theta_s(t)) \]  
\[ Q = (r3 + r4) |E_{LO}| |E_s| \sin(\Delta \omega t + \theta_s(t) + \delta) \]
Expressing in matrix form

\[
\begin{bmatrix}
I
Q
\end{bmatrix}
= |M| \begin{bmatrix}
I_o
Q_o
\end{bmatrix}
= \begin{bmatrix}
\frac{r_1+r_2}{r_3+r_4} & 0 \\
\frac{r_3+r_4}{\sin(\delta)} & \cos(\delta)
\end{bmatrix}
\begin{bmatrix}
I_o
Q_o
\end{bmatrix}
\] (4.27)

Where,

\[I_o = (r_3 + r_4) |E_{LO}| |E_s| \cos(\Delta \omega t + \theta_s(t))\] and \[Q_o = Q = (r_3 + r_4) |E_{LO}| |E_s| \sin(\Delta \omega t + \theta_s(t))\] are the expected output without phase error and responsivity imbalance.

Calculating inverse of the error matrix gives:

\[
|M|^{-1} = \begin{bmatrix}
\frac{r_3+r_4}{r_1+r_2} & 0 \\
\frac{r_3+r_4}{r_1+r_2} \frac{\sin(\delta)}{\cos(\delta)} & 1
\end{bmatrix}
\approx
\begin{bmatrix}
\frac{r_3+r_4}{r_1+r_2} & 0 \\
\frac{r_3+r_4}{r_1+r_2} \frac{\sin(\delta)}{\cos(\delta)} & 1
\end{bmatrix}
\] (4.28)

As the phase error is expected to be small, the scaling term \(\cos(\delta)\) is neglected in the inverse matrix. After compensation, I and Q will be nearly equal and orthogonal.

\[
\begin{bmatrix}
I_f
Q_f
\end{bmatrix}
= (r_3 + r_4) |E_{LO}| |E_s| \begin{bmatrix}
\cos(\Delta \omega t + \theta_s(t)) \\
\cos(\delta) \sin(\Delta \omega t + \theta_s(t))
\end{bmatrix}
= \begin{bmatrix}
I_o \\
Q_o
\end{bmatrix}
\] (4.29)

Next, the matrix coefficient is derived from the received sample signal,

Considering Eqs 4.25 and 4.26, multiplying I and Q gives:

\[
I \ast Q = \frac{(r_1+r_2)}{2(r_3+r_4)} ((r_3 + r_4) |E_{LO}| |E_s|)^2 \{ \sin(2\Delta \omega t + 2\theta_s(t) + \delta) + \sin(\delta) \}
\] (4.30)

After averaging over sufficient samples:

\[
\langle I \ast Q \rangle = \frac{(r_1+r_2)}{2(r_3+r_4)} ((r_3 + r_4) |E_{LO}| |E_s|)^2 \sin(\delta)
\] (4.31)

The average power for in-phase and quadrature components is given by: \(\langle P_I \rangle = \frac{1}{2} \frac{r_1+r_2}{r_3+r_4} (r_3 + r_4)|E_{LO}| |E_s|^2\) and \(\langle P_Q \rangle = \frac{1}{2} (r_3 + r_4)|E_{LO}| |E_s|^2\).

Thus, the inverse matrix in equation 10 calculated as:

\[
|M|^{-1} = \begin{bmatrix}
\frac{r_3+r_4}{r_1+r_2} & 0 \\
\frac{r_3+r_4}{r_1+r_2} \frac{\sin(\delta)}{\cos(\delta)} & 1
\end{bmatrix}
= \begin{bmatrix}
\sqrt{\frac{\langle P_Q \rangle}{\langle P_I \rangle}} & 0 \\
\frac{\langle I \ast Q \rangle}{\langle P_I \rangle} & 1
\end{bmatrix}
\] (4.32)
Given I and Q are the received samples, the resulting pair of equal and orthogonal signal denoted by $I_f$ and $Q_f$ computed as follows:

$$\begin{align*}
    I_f &= I \cdot \sqrt{\frac{(P_Q)}{(P_I)}} \\
    Q_f &= Q - \frac{(I+Q)}{(P_I)}
\end{align*}$$

(4.33)

A simulation was performed for a QPSK signal with a 15-degree hybrid phase error and equal photodiode responsivity. The received samples give a rhombic constellation diagram. The signal is recovered into four orthogonal phase states using the method of Eq. 4.33. The result is shown in fig.4.11.

![Figure 4.11. Constellations diagram (a) with QI and frequency offset of 10MHz. (b) QI compensated. (c) QI and frequency offset compensated. (Black dot represent the ideal constellation symbol) While the presence of offset frequency between the laser and LO causes rotation of the constellation, the QI makes the shape elliptical (Fig.4.11a). The correlation method (Eq. 4.33) correct the IQ mismatch and transforms the ellipse in to a circle (shown in fig.4.2b). After offset frequency compensation, it is possible to identify and detect the signal as a QPSK signal as shown in (c).](image)

(b) Quadrature imbalance compensation for 16-QAM modulation

The effect of amplitude imbalance (e.g. due to responsivitiy inequality or due to imperfection in the optical 90-degree hybrid) for 16-QAM modulation format is much more complex than QPSK. Simple DC subtraction will not solve the amplitude imbalance problem, this is due to the fact that some symbols in 16-QAM have different power levels (shown in Fig. 4.12)
Figure 4.12. 16-QAM constellation points and QPSK partitioning

The compensation for the hybrid phase error will be the same as with the case of QPSK.

If \( r_1 \approx r_2, r_3 \approx r_4 \), Eq. 4.31 gives

\[
\langle I \times Q \rangle = \frac{(r_1 + r_2)}{2(r_3 + r_4)} ((r_3 + r_4) |E_{LO}| |E_s|)^2 \sin(\delta)
\]

Where,

\[
|E_s| = \sqrt{\frac{R_a^2 + 2R_b^2 + R_c^2}{4}}
\]

(4.34)

However, in 16-QAM a small amplitude imbalance will give a distorted constellation diagram. Considering Eqs 4.23 and 4.24, the translation term will result in non-uniform shift due to the multi-level signal \( E_s \).

Fig. 4.13 is a simulation result for 16QAM in the presence of 15-degree hybrid phase error and responsivity mismatch.

Figure 4.13. Constellation diagrams for 16-QAM signal with 15 degree hybrid phase error and unequal photodiodes responsivity (a) without compensation (b) phase error compensated (c) phase error compensated and channel DC value subtracted (* represent the ideal constellation symbol)
In addition to non-orthogonality, the amplitude imbalance creates a non-even shift of the symbol constellation point and non-uniform narrowing and spreading of the Gaussian noise (Fig.4.13a). After compensating the non-orthogonality (b) using correlation method as Eq. 4.15, the problem of amplitude imbalance can be clearly seen. (c) Shows the constellation diagram after subtracting the average values from in-phase and Quadrature channels. It shows that a simple DC subtraction will not fully remove the effect of amplitude imbalance. One way of reducing the effect of amplitude imbalance for a higher modulation format is to increase power of the LO. This can be seen in Eqs.4.23 and 4.24. When power of the LO is higher, the dominant DC term will be from the LO and not the varying signal power.

4.6 Compensation of 90 degree phase error at MZM

The dual parallel MZM of Fig. 4.1 may experience error in the π/2 phase shifter. If it is not corrected from the transmitter side, it will also contribute to quadrature imbalance in the received signal and can be compensated in a similar manner with the previous section.

Let β be the error in π/2 phase shifter, the output field from the transmitter is then expressed as

\[ E_{\text{out}} = \frac{E_{\text{in}}}{2} \left[ \sin(\phi_1) + e^{j\left(\frac{\pi}{2} - \beta\right)} \sin(\phi_2) \right] \]  \hspace{1cm} (4.35)

Where \( \phi_1 = \frac{\pi}{\sqrt{\pi}}(V_{\text{driver1}}) \), and \( \phi_2 = \frac{\pi}{\sqrt{\pi}}(V_{\text{driver2}}) \)

Expressing in terms of signal amplitude and phase (i.e

\[ |E_s| \cos(\theta_s(t)) = \frac{E_{\text{in}}}{2} \sin(\phi_1) \] \hspace{1cm} \text{and} \hspace{1cm} \[ |E_s| \sin(\theta_s(t)) = \frac{E_{\text{in}}}{2} \sin(\phi_2) \]

\[ E_{\text{out}} = |E_s| \left[ \cos(\theta_s(t)) + \sin(\beta) \sin(\theta_s(t)) + j\cos(\beta) \sin(\theta_s(t)) \right] \] \hspace{1cm} (4.36)

Considering the offset frequency between the transmitter and LO, the balanced detector output can be approximated as:

\[ I \approx 2|E_{\text{LO}}||E_s| \cos(\Delta \omega t + \theta_s(t) - \beta) \] \hspace{1cm} (4.37)

\[ Q \approx 2|E_{\text{LO}}||E_s| \cos(\beta) \sin(\Delta \omega t + \theta_s(t)) \] \hspace{1cm} (4.38)

Correlation between the two channels gives:

\[ \langle I \ast Q \rangle = 2\cos(\beta) (|E_{\text{LO}}||E_s|)^2 \sin(\beta) \] \hspace{1cm} (4.39)

Like the previous section, non-orthogonal I and Q the received samples can be transformed to pair of orthogonal samples \( I_f \) and \( Q_f \) as follows:
\[
\begin{align*}
I_f &= \sqrt{\frac{\langle P_I \rangle}{\langle P_Q \rangle}} \left( I - Q \frac{\langle I+Q \rangle}{\langle P_Q \rangle} \right) \approx I - Q \frac{\langle I+Q \rangle}{\langle P_Q \rangle} \\
Q_f &= \sqrt{\frac{\langle P_I \rangle}{\langle P_Q \rangle}} Q \approx Q
\end{align*}
\] (4.40)

Where, $\langle P_I \rangle$ and $\langle P_Q \rangle$ is the average power of in-phase and quadrature channel respectively.

Phase error at both transmitter and receiver in the presence of offset frequency gives an elliptical and distorted constellation (Fig. 4.14 (a)). If the frequency offset is compensated but if there is phase error at both the transmitter and receiver the constellation diagram looks rhombic as shown in (b), whereas the constellation diagram takes on a non-rhombic shape if the phase error is only at the transmitter or receiver as shown in (c). Like the hybrid phase error compensation, the phase error of the transmitter is nicely compensated using Eq. 4.40. The recovered constellation is shown in (d).

Eq.4.33 and 4.40 is valid to compensate a single quadrature phase error. However, phase error could happen at both the transmitter and receiver or it could happen at both quadratures of either end. A more generic solution is still to be devised to compensate all at once.
Figure 4.14 Constellation diagram illustrating the effect of 90 degree phase error at MZM and hybrid detector (a) with 100MHz offset frequency, $12^\circ$ phase error at transmitter and receiver (b) with offset frequency compensated (c) with offset frequency compensated and in the presence of $12^\circ$ phase error at transmitter (d) both offset frequency and phase error compensated. (note: a closer look at the shapes of the constellation, (b) looks like rhombic whereas (c) looks like parallelogram).

An efficient method for compensation of 90 degree phase shifter error at the MZM for the case of 16-QAM modulation format is proposed. The effectiveness of the algorithm is also verified with numerical simulations.
4.7 References


Chapter 5

5. EXPERIMENTAL RESULT AND ANALYSIS

5.1 Introduction
To verify the validity of the algorithms discussed in Chap. 3 for a coherent receiver, offline DSP post processing of real transmission data is performed. Using a DSP algorithm, a PDM-QPSK signal is recovered from the data captured from a 100-Gb/s coherent system. The symbol rate is 30Gbaud, the system employing 20% overhead for forward-error correction (FEC). In addition, BER degradation caused by nonlinearity in the detector response due to the saturation of the transimpedance amplifier (TIA) is measured. A nonlinear equalization algorithm is applied to compensate for this imperfection and hence improve the BER.

5.2 Experimental set up
The experimental setup for back-to-back measurement is shown in Fig. 5.1. An amplified spontaneous emission noise source is used to obtain different OSNR levels. The transmitter generates two orthogonal QPSK signals, by using an electrical data pattern of 2\(^{15}\)-1 PRBS. Each electrical channel has a data rate of 30Gb/s yielding a single-channel line rate of 120 Gb/s. At the receiver, the signal is combined with a free-running LO in a polarization diversity 90-degree hybrid followed by four balanced detectors. The four signal components (Ix, Qx, Iy, Qy) are sampled with a real-time scope. Off-line data processing was subsequently performed using algorithms implemented in MATLAB. The sampling rate of the scope is 50Gsamp/s. The samples are up-sampled to two samples per symbol resulting in 60Gsamp/s.

4 x 12-tap adaptive butterfly-structured FIR filters are applied on the up-sampled data to realize polarization separation and equalization of the signal. These filters are adapted with a CMA algorithm. Once the CMA converges, the received signals are separated into two orthogonal polarizations. Later on, the frequency offset between the transmitter laser and LO laser is estimated with a 100-sample window. Eventually, the carrier phase is estimated with an 8-tap feed-forward algorithm.

To avoid the problem of cycle slip (discussed in Sec. 3.6), differential encoding is employed at the transmitter. Likewise, after thresholding the recovered constellation, the decision is differentially decoded. Finally the bit errors are counted to evaluate the performance of the post-processing algorithms. No FEC operations were employed, i.e., the BER reported here is raw.
5.3 Result and analysis

Figure 5.2 shows a QPSK signal recovery through a sequence of DSP algorithms. The received data constellation diagram without DSP is shown in (a) for X-polarization. After running the CMA algorithm on the input samples with convergence parameter $\mu$ chosen to be $10^{-3}$, the polarization separation is seen in (b). At this stage, it is noticed that the shape of the constellation looks like a donut as the carrier phase and frequency offset is yet to be compensated. Then, the offset frequency is estimated and removed using the method mentioned in Eq. 3.32. Finally, a symbol by symbol laser phase tracking (Eq. 3.35) with constant weighting function is applied. Figures (c) and (d) show the final recovered QPSK signal for X polarization and Y polarization, respectively, after the phase and frequency compensation.
Figure 5.2 Constellation diagrams illustrating sequence of DSP processing on the real PDM-QPSK signal. 120Gb/s, 20dB OSNR, (a) Input X-pol (b) X-Pol after CMA (c) X-pol after phase and offset frequency compensation (d) Y-pol after phase and offset frequency compensation.

It is demonstrated that the PDM-QPSK signal is recovered for different OSNR values (0.1-nm reference bandwidth). Figure 5.3 shows the back-to-back bit error ratio (BER) performance for both polarizations.
Figure 5.3 BER performance at 30Gbaud: dual-polarization 120Gb/s QPSK

Figure 5.4. BER versus LO power for 0dBm signal power. The green square after nonlinear equalization is for 15.5 dBm LO power.

We furthermore studied data corruption due to saturation in the TIA response. A BER curve for one of the polarizations for different LO powers is shown in Figure 5.4. Significant degradation is seen once the TIA reaches saturation. (e.g. 15.5 dBm LO power results in BER~10^{-1}). After the application of some nonlinear equalization to the received symbols, with cubic function to the four channels given as:
\begin{align*}
  lxe &= lx + k \cdot lx^3 \\
  Qxe &= Qx + k \cdot Qx^3 \\
  lye &= ly + k \cdot ly^3 \\
  Qye &= Qy + k \cdot Qy^3
\end{align*} \\
(5.1)

Where, \( lx, Qx, ly, Qy \) are the input samples, \( k \) is optimization coefficient and \( lxe, Qxe, lye, Qxe \) are the equalized samples.

The BER is improved by about two orders of magnitude (from \( 10^{-1} \) to \( \sim 10^{-3} \)).
Chapter 6

6. CONCLUSION AND RECOMMENDATIONS

6.1 conclusions

In this thesis, DSP algorithms used in current optical coherent communication links are studied. Some of the most common PIC imperfections are analyzed and some mitigation techniques are proposed.

The proposed compensation mechanisms for PIC imperfections include:

- Tuning the bias or phase shifter voltage of the modulator to compensate the effect of limited modulator extinction ratio
- Electronic pre-distortion of the modulator driving voltage to compensate imperfections associated with bandwidth limits and nonlinearities of the devices at the transmitter
- Electronic pre-distortion of the modulator driving voltage to compensate the effect of capacitance variation in high-speed silicon modulators
- Post compensation at the receiver to correct I-Q phase error in the receiver or transmitter
- Post compensation at the receiver to correct for nonlinearity in the receiver

DSP algorithms used for a coherent receiver are verified by applying a post-processing algorithm on real data experimentally captured from a 100-Gb/s coherent optical transmission setup. Successful recovery of a PDM-QPSK signal is demonstrated, and the BER is measured. In addition, it is shown that nonlinear equalization algorithms can be employed to improve the BER induced by nonlinear behaviors in devices either at transmitter or receiver side. This is verified by applying a nonlinear equalization to the received symbols in order to compensate for imperfections caused by saturation in the TIA circuit at the detector. A two-order-of-magnitude improvement in the BER is achieved with this technique. Optimized algorithms can lead to further BER improvements.

Despite the fact that DSP promises a high potential for mitigating PIC imperfections, future progress must be concurrently made to make the algorithms efficient both in energy and in computation. In addition, more robust and informative performance measures for an optical data link will be required to make the DSP techniques more effective. For instance, BER measurements or constellation diagrams can be ineffective to isolate the root causes as an imperfection can practically involve complex origins.
6.2. Future work

Future work would involve the study of other important PIC imperfections such as those associated with nonlinearities in photodiodes, limited polarization extinction ratios, and electrical crosstalk. Moreover, the computational complexity of all the proposed algorithms needs to be evaluated and optimizations have to be done to make them suitable for real time operations.

While it has been numerically demonstrated that DSP can be used to compensate PIC imperfection, it is important to see how the DSP algorithms perform experimentally on the real PIC.

Finally, the proposed algorithms need to be employed in an actual ASIC and be measured in real time.