LIQUID-CRYSTALLINE GRADIENT-INDEX LENSES FOR 3D-DISPLAYS

by

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Master’s Thesis in fulfillment of the requirements for the Degree of Master of Engineering Physics

Academic Year 2006-2007
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Gent, 30 mei 2007

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Summary

Current 2D/3D auto-stereoscopic displays consist of a cylindrical lenticular filled with liquid crystal (LC). These lenticulars are expensive to produce. A possible alternative is a liquid crystalline gradient index (GRIN) lens. These lenses refract light due to a gradient in refractive index. The gradient is created by aligning the LC molecules with an appropriate electric field. A basic design of the GRIN lens, in order to have an appropriate electric field in the LC, was available. The electric field was first calculated without taking LC interaction into account. Based on this design GRIN lens samples were created. In an experiment the refraction angle of the lens was measured. Two GRIN lens samples had a large useful area i.e. the part of the GRIN lens which refracts the light as desired. GRIN lens sample G22 had a useful area of 63% and G35 one of 75%. A useful area of 90% is desired. With this experiment the refraction angle of the GRIN lens could not be measured above the electrodes. There also was no technique to measure the thickness of the LC layer accurately in the GRIN lens.

Therefore a model of the GRIN lens with LC interaction, including the area above the electrodes, is made. This is based on an LC simulation program. The influence of different design-parameters of the GRIN lens is investigated in order to improve the useful area. To check the model, the refractive index of the GRIN lens is measured experimentally. In addition the problem of the inaccurate determination of the LC thickness is solved.

Keywords: Liquid crystal, gradient index lenses, birefringence, 3D-display
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ABBREVIATIONS

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<tr>
<td>GRIN</td>
<td>Gradient Index</td>
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<tr>
<td>LC</td>
<td>Liquid Crystal</td>
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<td>LCD</td>
<td>Liquid Crystal Device</td>
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<td>ITO</td>
<td>Indium Tin Oxide</td>
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<tr>
<td>AC</td>
<td>Alternating current</td>
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<td>DC</td>
<td>Direct current</td>
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SYMBOLS

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<thead>
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<tr>
<td>$n$</td>
<td>index of refraction</td>
</tr>
<tr>
<td>$n_o$</td>
<td>ordinary refractive index</td>
</tr>
<tr>
<td>$n_e$</td>
<td>extra ordinary refractive index</td>
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<tr>
<td>$\epsilon$</td>
<td>relative electric permittivity</td>
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<tr>
<td>$c$</td>
<td>speed of light in vacuum</td>
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Chapter 1

Introduction to switchable 2D/3D displays

Looking at a 3D display, the viewer has the perception of depth: he sees objects coming out of the screen, or objects moving behind the screen. In paragraph 1.1 it is explained how somebody perceives depth. The technique to create the perception of depth with an autostereoscopic displays is described, together with its desired properties, like multi-viewing and a good resolution. A switchable 2D/3D display is a 3D display with an extra advantage: it can operate in a 2D mode and a 3D mode. An alternative technique for this switchable 2D/3D display is the use of a gradient index (GRIN) lens based on liquid crystals.

1.1 Depth perception and 3D displays

1.1.1 Depth perception

A person is able to see depth because he looks with his two eyes at an object. His two eyes see the same object at a slightly different angle, resulting in two slightly different perceived images, see figure 1.1. From these images the brain determines the position of the objects and the person perceives depth.
Chapter 1. Introduction to switchable 2D/3D displays

1.1.2 Auto-stereoscopic 3D-displays

How can a display give the perception of depth? An old technique is the use of polarized glasses. The polarized glasses separate the images for the left and right eyes, based on polarization of the light. To this end the glasses as well as the offered images for the left and right eye are perpendicularly polarized. If the images of the left and right eye contain the correct information, the viewer perceives depth.

Another technique is the auto-stereoscopic display, which provides a depth perception without the need of glasses. The technique is illustrated in the next figure.

Figure 1.1: Two slightly different images are seen by the left eye the right eye.

Figure 1.2: An array of cylindrical lenses enables auto-stereoscopic vision
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The image meant for the left eyes is placed on a part of the pixels of the LCD and the image meant for the right eye is placed on another part of the pixels. An array of cylindrical lenses is placed upon the LCD, causing the light from the pixels to be focused into certain directions. Because the left and right eye have different positions in relation to the display, the lens focuses the light coming from the pixels of the left-eye image, on the left eye and the right-eye image on the right eye. The two slightly different images perceived by the left and right eye, creates the illusion of depth.

1.1.3 Multi-view

If a 3D display presents only two views for the viewer (one for the left eye and one for the right eye), the viewer has to be in a specific region to perceive the depth effect. The region where the viewer perceives depth, is called the viewing area, illustrated in the next figure.

![Figure 1.3: Auto-stereoscopic display with two views (a) and six views (b). For the two-view display, the central viewing area is indicated with ‘L’ (position of the left eye) and ‘R’ (position of the right eye) in the two black diamonds. The viewer perceives depth the best when looking from this position at the display. For the six-view display the viewing area is the big black diamond. The viewer perceives depth the best when looking at the display from anywhere within the black diamond.](image)

The vertical black dashed line depicts the ideal viewing distance. At this distance, the depth is perceived the best because the views do not overlap. For example the left view does not overlap with the right view (1.3 a) or view 1 does not overlap with view 2 (1.3 b). Along the ideal viewing distance the viewing area is repeated.

The two-view display has a central viewing area, indicated with black diamonds. The viewing area for the left and the right eye, indicated with blue diamonds, keeps repeating itself along the black dashed line. If the viewer moves from the central viewing area to the left or right, the image for the left eye is delivered to the right eye and vice versa. This unwanted effect of
inverting the experienced depth limits the freedom of position of the viewer. A solution for this problem is a multi-view display, figure 1.3 b.

More than two images are supplied by the auto-stereoscopic multi-view display, increasing substantially the viewing area (the black diamond in figure 1.3 b). The observer can move anywhere within the diamond. Also this viewing area is repeated along the ideal viewing distance. The next figure shows a person looking at a 7-view display.

![Figure 1.4: Observer at the ideal viewing distance watching a 7-view 3D-display.](image)

### 1.1.4 Slanting of lenticulars

A multi-view autostereoscopic display loses resolution because of the different views. When the lenticular is placed vertical on the LCD, so without slanting, the resolution is reduced with a factor 1/N in the horizontal direction. Where N is the number of views. In the vertical direction, the array of cylindrical lenses placed on the LCD, leaves the resolution undisturbed, as shown in the next figure.
Chapter 1. Introduction to switchable 2D/3D displays

Figure 1.5: The left picture shows an LCD on which a lenticular is positioned without slanting. For a N-view display, each cylinder of the lens covers N columns of pixels. The pixels seen by one eye (only one of the views) are displayed in the right image. In the horizontal direction the resolution is reduced while in the vertical direction the resolution is untouched.

To solve this difference in resolution between the vertical and horizontal direction, slanting is used. Slanting is the placing of the lenticular at an angle on the display as shown in the next figure.

Figure 1.6: The left picture shows an LCD on which a lenticular is positioned with slanting. Now each cylinder of the lens covers a lot of different columns. The right picture shows the pixels of one view.

For an N-view display, the lenticular is placed at an angle $\beta = \frac{1}{N}$. At this angle, the resolution in both the horizontal and the vertical direction is reduced with a factor $\frac{1}{\sqrt{N}}$. 
1.2 Switchable 2D/3D displays

A multi-view auto stereoscopic display with a slanted lenticular loses resolution as described in the previous paragraph. This resolution loss is not desired. A solution is to make the 3D-display switchable between a 2D mode, with full resolution but no depth perception, and a 3D-mode with depth perception. Such a switchable 2D/3D display uses a lenticular which can be turned on (2D mode) and off (3D mode) with a switch. This technology uses liquid crystal (LC), as shown in the next figure.

![Switchable lens of a 2D/3D-display.](image-url)

**Figure 1.7:** Switchable lens of a 2D/3D-display. The replica lens plate is filled with LC and placed in front of the LCD. Two conducting layers of Indium-Tin-Oxide (ITO) control the voltage applied on the lens. The orientation of the LC is indicated when the voltage is on (2D mode) and when the voltage is off (3D mode). The blue arrows depict the path of the light.

In the 2D mode, a voltage is applied on the ITO layers and the LC is aligned with the electric field. More information about the properties of a liquid crystal is given in section 2.1.1. In this LC configuration, the refractive index of the LC matches with the refractive index of the replica. The replica is a plastic lens plate made by the use of a mold. The light of the LCD travels without being refracted, so the display is in 2D mode, without loss of resolution.

In the 3D mode, there is no electric field in the LC. The LC is now aligned parallel to the rubbed polymers present on the replica and on the glass plate above the LCD. Now the refractive index of the LC is larger than that of the replica. This results in a positive lens effect and light coming from the display will be refracted.

1.3 An alternative, gradient index lens based on liquid crystal

The 3D-display consists of a cylindrical lenticular. There is no production process for this cylindrical lenticular in conventional LCD factories, which makes it rather expensive. There-
fore a cheaper alternative is proposed: a GRIN lens based on LC. This is a lens made of LC, without the need of a cylindrical lenticular.

Conventional lenses, like glass lenses, refract light at the boundary between two materials. A GRIN lens changes the direction of light as a result of a gradient in the refractive index in the material. The material used here is LC, and the gradient in refractive index is created with an electric field. Liquid crystals are birefringent and have the property to align with an electrical field. Birefringence means that the refractive index will depend on the orientation of the liquid crystal. Creating an appropriate electrical field in the LC, results in an refractive index modulation in the LC, creating a lens. The basics of GRIN lenses and liquid crystal are explained in next chapter. An illustration of a GRIN lens is given in the next figure.

Figure 1.8: GRIN lens for a switchable 2D/3D-display. The orientation of the Liquid crystal is indicated when the voltage is off (2D mode) and when the voltage is on (3D mode). The blue arrows depict the path of the light.
Chapter 2

Basics of GRIN lenses and previous work

The GRIN lenses are based on LC, therefore a short introduction into the theory of liquid crystals is given. The design of the GRIN lens is described as well as the theory necessary to derive the refractive index and refraction angle of the GRIN lens. The ideal refractive index profile and the ideal angular profile of the GRIN lens are given. The chapter ends with resuming the most important results of previous investigations done on GRIN lenses.

2.1 Theory of liquid crystals

2.1.1 Definition of Liquid Crystal

Liquid crystals are a class of organic materials that behave like an intermediate phase between solids and liquids. The next figure illustrates the liquid crystal phase.

![Liquid Crystal Phase](image)

**Figure 2.1:** The solid, liquid crystal and liquid phase

In the liquid crystal phase the constituent molecules have some freedom to move around in the material (translation freedom) just like in a liquid. But at the same time, the molecules
are subjected to some long range ordering in their orientation, just as in a solid, crystalline material (orientational ordering).

There are many different types of LC phases such as smectic C, smectic A and nematic. The most important for display and other opto-electronic components are the nematic liquid crystals.

The nematic molecules have mirror plane symmetry and are therefore achiral. The permanent dipole moment $\vec{p}$ of the molecule is laying in the plane of symmetry. The molecules are free to rotate around their long axis, so the phase has full rotational symmetry. This symmetry is compatible with uniaxiality, so all macroscopic properties will have the uniaxial symmetry [1]. Because the nematic molecules are able to rotate freely and switch head and tail, they are often represented by cylinders or prolate ellipsoids. The average direction of the long axis is represented by a vector, the director $\vec{L}$. These could be just as well be represented by the opposite vector $-\vec{L}$. The director $\vec{L}$ is illustrated in the next figure.

![The director of liquid crystals](image)

**Figure 2.2:** The director of liquid crystals

### 2.1.2 Birefringence

A uniaxial material is birefringent. Birefringence, or double refraction, is the decomposition of a ray of light into two rays (the ordinary ray and the extra-ordinary ray) depending on the polarization of the light. Birefringence can be formalised by assigning two different refractive indices to the material for different polarizations. The birefringence magnitude is then defined by:

$$\Delta n = n_e - n_o$$

where $n_o$ and $n_e$ are the refractive indices for polarizations perpendicular (ordinary) and parallel (extraordinary) to the axis of anisotropy respectively. The birefringence of the nematic liquid crystal is depicted in the next figure.
In the left of figure 2.3, the polarization is perpendicular on the long axis of the molecule. The refractive index for the light is the ordinary refractive index $n_o$.

In the right of figure 2.3, the polarization is parallel with the long axis of the molecule. The refractive index for the light is the extra-ordinary refractive index $n_e$.

In the middle figure 2.3, the direction of propagation makes an angle $\theta$ (inclination angle) with the long axis of the molecule. The refractive index for the light is the effective refractive index $n_{eff}$,

$$n_{eff} = \frac{1}{\sqrt{\cos^2 \theta \frac{n_0}{n_0^2} + \sin^2 \theta \frac{n_e}{n_e^2}}}$$  \hspace{1cm} (2.2)

2.1.3 Influence of the electric field

If an electric field $\vec{E}$ is applied to the liquid crystal, the LC molecule experiences an electric torque,

$$\vec{T}_e = \vec{p} \times \vec{E}$$  \hspace{1cm} (2.3)

For the nematic crystals with a permanent dipole moment $\vec{p}$ parallel with the long axis of the molecule, the director will align preferably with the electric field.
2.1.4 Interface interaction

The director orientation of the LC molecules is also influenced by the surfaces containing the liquid crystal. One way is by using polymer layers at the surface. Rubbing or buffing the surface with a soft cloth with appropriate pressure lead to a preferential azimuthal alignment, usually in combination with a small deviation from the horizontal orientation (pre-tilt). The preferential orientation can be due to structural or chemical modification of the surface. The rubbing of surfaces is illustrated in the next figure [1].

![Rubbing direction](image)

**Figure 2.4:** Alignment of the LC with the rubbing direction of the polymer layer. The azimuthal alignment along the rubbing direction is illustrated in the left picture. The pre-tilt with the plane of the substrate is depicted in the right figure.

2.1.5 Elastic deformation in nematic liquid crystals

The configuration of the directors is determined by minimalizing the total free energy. The two most important terms in the total free energy are the electric and elastic energy. It is assumed that the interface interaction can be neglected in the GRIN lens configuration. This is because the interaction between the surface and the LC is so strong that the boundaries are fixed (strong anchoring). The elastic and electric energy are discussed in the next two paragraphs.

**Elastic energy**

The elastic energy is zero in case the director distribution is homogeneous. When there are variations the elastic energy is given by the Oseen-Franck expression [1],

$$f_{elastic} = \frac{1}{2} \left[ K_1 (\nabla \vec{L})^2 + K_2 \left( \vec{L} \cdot (\nabla \times \vec{L})^2 \right) + K_3 \left( \vec{L} \times (\nabla \times \vec{L})^2 \right) \right]$$  \hspace{1cm} (2.4)

with $\vec{L}$ the director. The constants $K_1$, $K_2$ and $K_3$ are independent constants related to the energy required for splay, twist and bend deformations. For most nematic crystals $K_3 > K_1 > K_2$. The next figure illustrates the twist, bend and splay [2].
Chapter 2. Basics of GRIN lenses and previous work

Energy related with the electric field

The electrical energy density is given by,

\[ f_{\text{electric}} = -\frac{1}{2}\epsilon_{\perp}E^2 - \frac{1}{2}\Delta\epsilon(\vec{E} \cdot \vec{L})^2 \] (2.5)

For nematic liquid crystals with \( \Delta\epsilon > 0 \), the energy density \( f_{\text{electric}} \) is smaller when \( (\vec{E} \cdot \vec{L})^2 \) is larger. This is when the angle between \( \vec{E} \) and \( \vec{L} \) is smaller, so when the directors are aligned parallel with the electrical field.

2.2 GRIN lenses for switchable 2D/3D displays

2.2.1 Design of a GRIN lens

The design of the GRIN lens was investigated profoundly in the master thesis of L. Kusters [3]. Conventional lenses refract light at the boundary between two materials. A gradient refractive index (GRIN) lens changes the direction of light as a result of a gradient in the refraction index in the material, as shown in the next figure.
Figure 2.6: Gradient refractive index lens (blue). The red arrows indicate the path of the light. The refractive index modulation $n$ is given as function of the position of incidence $x$.

If LC is used as material, this gradient can be created by an appropriate electric field in the LC. As explained in the previous section, the LC is birefringent and the refractive index will depend on the LC orientation. The next figure illustrates the basic design of a GRIN lens and shows the electric field and the LC alignment in the GRIN lens.

Figure 2.7: The electric field (red) in the GRIN lens (left) and the alignment of the LC in the GRIN lens (right).

The left figure shows a cross section of the GRIN lens. The structure repeats in horizontal
direction, and is invariant in the direction perpendicular to the paper. The LC layer is placed in a part of the electric field distribution. Below the LC-layer there is a dielectric layer. In this layer the electric field is mostly oriented horizontally between the electrodes. Therefore the LC is placed further away from the electrodes, where the shape of the electric field is more bent more gradually. In the dielectric layer the (line) electrodes are positioned on fixed distances, corresponding with the distance of several pixels (depending on the desired number of views). The voltages on the electrodes have an equal amplitude but an opposing polarity. The polarity is changed at a frequency of approximately 100 Hz. This frequency prevents a charging effect of the LC. Because the directors align with the direction of the electric field and not with its polarity, using the AC voltage does not influence the LC configuration.

This configuration is optimized with an extra electrode-plate connected to the ground, placed above the LC-layer. This improves the shape of the electric field in the LC layer, see next figure. More information can be found in the master thesis of L. Kusters [3].

![Figure 2.8](image)

**Figure 2.8:** The field lines (red) in the GRIN lens without the upper electrode (left) and with the upper electrode (right). The black dashed lines depict the path of the light traveling through the GRIN lens.

The model of figure 2.8 (right) with the dielectric layer and the extra electrode connected to the ground above the LC layer, is used for the GRIN lens. The next figure gives a schematic representation of an array of GRIN lenses.
Chapter 2. Basics of GRIN lenses and previous work

Figure 2.9: An array of GRIN lenses. Because of the gradient refractive index, the perpendicular incident polarized light is refracted at an angle $\alpha$ and focused. The light rays are depicted in blue and the polarization in red.

To understand how a GRIN lens based on LC refracts light, it is necessary to know the relation between the LC configuration and the refractive index modulation, and the relation between the refractive index modulation and the refraction angle. The refraction angle $\alpha(x)$, indicated in figure 2.9, is the angle at which the light is refracted when leaving the GRIN lens. These relations are derived in the next two paragraphs.

2.2.2 Relation between the LC configuration and the refractive index profile

The incident light of figure 2.9 is polarized perpendicular to the direction of light and parallel to the paper. So all light travels with the effective extraordinary index of refraction. The effective refractive index $n_{eff}(x, z)$ is a smoothly varying refractive index in the x-and z-direction, with x the horizontal direction in the LC layer and z the vertical direction. If $\theta(x, z)$ for each director positioned at (x, z) in the LC is known, $n_{eff}(x, z)$ can be calculated with formula 2.2.

$$n_{eff}(x, z) = \frac{1}{\sqrt{\frac{\cos \theta(x, z)^2}{n_0^2} + \frac{\sin \theta(x, z)^2}{n_e^2}}}$$

(2.6)

The direction of each LC molecule can be found using liquid crystal simulation programs. This is done in chapter 3. From the effective refractive index $n_{eff}(x, z)$, the average refractive index $n(x)$ of the LC layer is derived. For the derivation it is assumed that all light rays impinge
perpendicularly on the LC layer with polarization in the x-direction, that diffraction effects may be neglected and that the changes in the director profile are smooth. When assuming that the refractive index varies smoothly, the average refractive index \( n(x) \) can be calculated by averaging \( n_{eff}(x, z) \) in the z-direction,

\[
n(x) = \langle n_{eff}(x, z) \rangle .
\]  

In this model the light propagates remains propagating in the z-direction in the LC layer and the refraction occurs when the light leaves the LC layer.

### 2.2.3 Relation between the refractive index profile and the angular profile

The relation between the refractive index profile \( n(x) \) and the angular profile \( \alpha(x) \) is derived using the Huygens sphere method, illustrated in the next figure.

\[ \text{Figure 2.10: Refraction at a surface with a gradual refractive index modulation} \]

\( n_g \) is the refractive index of glass. \( x_1 \) and \( x_2 \) are two points at the surface of the LC. To determine the wave vector \( \vec{k} \) from a light ray after passing through the LC layer, the Huygens spheres emanating from these points are considered. The wave front is tangent to the Huygens spheres is and \( \vec{k} \) is perpendicular to the wave front. The Huygens spheres will develop differently in point \( x_1 \) and \( x_2 \) because of the varying indices of refraction. The average effective index of refraction at point \( x_1 \) is \( n(x_1) = n_1 \) and at point \( x_2 \) is \( n(x_2) = n_2 \). The evolution of the two Huygens spheres are determined as follows. Without loss of generality it is assumed that \( n_1 > n_2 \). In order to calculate the time \( \tau \) the light needs to pass through
the LC layer, it is assumed that the light rays remain propagating along the z-direction in
the LC layer and are only refracted when leaving the LC. The time $\tau_i$ the light needs to pass
through the LC layer with thickness $h$ and refractive index $n_i$ (i=1,2) is

$$\tau_i = \frac{h n_i}{c}. \quad (2.8)$$

Now the radii of the Huygens spheres in the glass can be derived. From $n_1 > n_2$ follows that
$\tau_1 > \tau_2$ and at a time $t > \tau_1$, the radii $r_i$ (i=1,2) are

$$r_i = (t - \tau_i) \frac{c}{n_i}. \quad (2.9)$$

Substituting 2.8 into 2.9 gives,

$$r_i = r_0 - h \frac{n_i}{n_g}, \quad (2.10)$$

with $r_0 = \frac{tc}{n_g}$.

Now the angle of the wave vector in glass is determined from figure 2.10.

$$\sin \alpha_g = \frac{r_2 - r_1}{x_2 - x_1} \quad (2.11)$$

Substituting expression 2.9 for the radii into formula 2.11 gives,

$$\sin \alpha_g = \frac{h n_1 - n_2}{n_g x_2 - x_1} \quad (2.12)$$

Using snell’s law for the glass-air interface

$$\sin(\alpha) = n_g \sin(\alpha_g) \quad (2.13)$$

and substituting expression 2.12 into expression 2.13 gives the refraction angle $\alpha$ in air in
terms of the gradient of the effective of refractive index, averaged over the LC layer.

$$\alpha = -\arcsin \left[ h \frac{\partial n(x)}{\partial x} \right] \quad (2.14)$$

With this formula, the angular profile $\alpha(x)$ can be determined by calculating the gradient in
the x-direction of $n(x)$ and substituting it in formula 2.14.

Now that the relations between the LC configuration and the refractive index profile $n(x)$,
and between $n(x)$ and the angular profile $\alpha(x)$ are known, the ideal profile of the GRIN lens
can be derived.

### 2.2.4 Ideal angular profile and refractive index profile

The light coming from the pixels has to travel in certain directions. This is more clear
when the reverse path of the light is followed, which is possible because of the time-reverse
independence of Maxwell’s equations. The view of the observer looking perpendicular at the
3D-display, has to be focused on the different pixels, if not a blurry image is perceived. The focus point \( f \) on the optical axis (\( x=0 \) in figure 2.9) is defined as

\[
f = \frac{x}{\tan \alpha(x)}
\]  

(2.15)

In the middle of the lens (\( x=0 \) in figure 2.9), the light propagates unperturbed. The further away from the middle, the larger the refraction angle will be. All light will be focused in the same point (pixel), if \( f \) is a constant in formula 2.15. This means that for small angles, \( \alpha \) has to be linear in \( x \).

If \( \alpha \) is linear for small angles, it follows from formula 2.14 that \( n(x) \) is parabolic. The desired profiles are resumed in the next figure.

![Figure 2.11: Desired profiles of the GRIN lens: a parabolic refractive index \( n(x) \) and a linear refraction angle \( \alpha(x) \)](image)

2.3 Previous work

2.3.1 Samples G22 and G35

In the master thesis of L. Kusters [3] is described how different GRIN lens samples are made. Two good configurations are GRIN lens samples G22 and G35. A schematic representation of the two lenses together with important parameters are given in the next figure.
The model shows only a unit cell of GRIN lens. It is invariant in the direction perpendicular on the paper, and repeats itself in the horizontal direction. The material constants and the size of the different layers are indicated. The material used for the dielectric layer in sample G22, figure 2.12(a), is Su8. This is a photoresist polymer. The glass used in sample G35 is Eagle 2000. This is a substrate commonly used in flat panel displays. The two models have, apart from the two line electrodes under the LC layer, an extra plate electrode above the the LC layer. This improves the shape of the electric field in the LC layer.

The lenses were characterized by a theoretical model and with an experiment.

2.3.2 Theoretic characterization of GRIN lens G22

For GRIN lens G22, the electric potential in the LC layer was calculated based on a model with boundary conditions. This is done in the master thesis of L. Kusters [3]. The next figure shows the electrical potential in the LC layer of GRIN lens G22.
Electrical Potential

Theoretical electrical potential is calculated for GRIN lens G22 without taking LC interaction into account, assuming the LC is a homogeneous material. Instead of electrodes with finite dimensions, singular point sources were used. The electrical field is perpendicular to the equipotential lines. It is assumed that all LC molecules align with the electric field. From the LC configuration, it is possible to calculate the refractive index modulation $n(x)$. This was explained in paragraph 2.2.2. The calculated $n(x)$ is plotted in the next figure.

Figure 2.13: Equipotential lines of GRIN lens G22 as function of $z[\mu m]$ and $x[\mu m]$
Chapter 2. Basics of GRIN lenses and previous work

Refractive index

![Graph of refractive index vs x(µm) for GRIN lens G22. The 'e' indicates the x-position of the center of the electrodes.](image)

**Figure 2.14:** $n$ vs $x[µm]$ for GRIN lens G22. The 'e' indicates the x-position of the center of the electrodes.

The refractive index modulation of GRIN G22 is parabolic between the electrodes, with a maximum equal to $n_e$ and a minimum equal to $n_o$. Closer to the electrodes, the refractive index profile decreases more slowly and starts deviating from a parabolic profile. In the same way as described in paragraph 2.2.3, it is possible to derive the angular profile $\alpha(x)$ from the refractive index profile $n(x)$. The next graph shows the analytical result, see the master thesis of L. Kusters [3].

Angular profile

![Graph of angular profile vs x(µm) for GRIN lens G22.](image)

**Figure 2.15:** $\alpha[deg]$ vs $x[µm]$ for GRIN lens G22
The refraction angle has the desired linear profile in the interval \([115\mu m; 220\mu m]\). This is 63% of the whole lens area (166 \(\mu m\)). Closer to the electrodes, the absolute value of the refraction angle starts decreasing slowly. The refraction angle of the ideal GRIN increases linearly in the whole interval between the electrode.

### 2.3.3 Experimental angular profile of G35

In the work of L. Kusters [3], an experiment to measure the refraction angle is described. When the light of a laser is incident at a certain position \(x\) on the GRIN lens, the light will be refracted when leaving the GRIN lens. When measuring the position of the light after the refraction, the refraction angle of the light at every position \(x\) of the GRIN lens, can be determined. The next plot shows the measured refraction angle as function of the position \(x\) of GRIN lens G35 at 100 V.

![Figure 2.16: \(\alpha[deg] vs x[\mu m]\) for GRIN lens G35](image)

The refraction angle is linear in the middle of the GRIN lens, but close to the electrodes, it deviates. To measure the refraction angle, the light path of a laser beam incident on the GRIN lens was measured. Close to the electrodes the refractive index changes very fast and because the laser beam has a certain width, the light incident on this region will be scattered in different directions. Therefore the measurements above the electrode are inaccurate and it was not possible to characterize the GRIN lens in the region above the electrodes.
Chapter 3

Analysis of the GRIN lens based on simulations

3.1 Introduction

The electrical field and the director configuration of GRIN lens G22 and G35, see section 2.3.1, are determined with the liquid crystal simulation program ‘LCD Master, Shintech’ [5]. From the director configuration the refractive index modulation and the angular profile are derived. Then, the influence of free parameters such as the voltage, the thicknesses of the LC layer and dielectric layer, the dielectric constants of the dielectric layers and the pre-tilt are investigated. Finally the simulations are compared with previous work.

In the previous work, see section 2.3, a theoretical study of the GRIN lens was done without taking LC interaction into account. Also experiments on the GRIN lens were done, but they could not characterize the GRIN lens close to the electrodes. Therefore the LC interaction is studied and an attempt is made to characterize the GRIN lens close to the electrodes. The free parameters are investigated in order to increase the performance of the GRIN lens. Also simulations with the simulation program ‘2dimMOS’ citedimmos were done, but they led to inaccurate results. In the last section of this chapter, the simulation done with the program ‘2dimMOS’ are discussed.

3.2 Simulation model Shintech

3.2.1 Input

The input model is two dimensional and all material parameters and boundaries have to be given. An input model is shown in the next figure.
Figure 3.1 shows the different dielectric layers, the LC and the electrodes. Because it is possible to define periodic boundary conditions, only one unit cell of the GRIN lens is modeled. This unit cell repeats itself in the x-direction.

The simulations are done by defining a DC voltage on the two electrodes. In reality there is an AC voltage, but because the directors tend to orient parallel with the electrical field, positive or negative, simulating with DC voltage will not change the result.

When defining the input model, each dielectric layer and the LC layer are divided into different layers. Also the x-direction is divided into different segments in order to define a rectangular mesh for the numerical calculations, using a finite-difference method.

### 3.2.2 Calculations

The program 'Shintech, LCD Master' calculates the azimuthal $\phi$ and polar angle $\theta$ of the director dependent on the boundary conditions (voltage, pre-tilt, twist angle, anchoring) for every element of the nematic liquid crystal [5].

This is done by evaluating numerically the equation of motion of the director vector based on the Ericksen-Leslie theory. This theory determines the director orientation by minimizing the total free energy as explained in section 2.1.5.

### 3.2.3 Output

There are two output files. One with the direction $(n_x, n_y, n_z)$ of the unit orientation vector for the director at each position $(x, z)$, and another with the value of the electric potential at
Chapter 3. Analysis of the GRIN lens based on simulations

Each position of the GRIN lens. The relation between \((n_x, n_y, n_z)\) and the azimuthal angle \(\phi\) and the polar angle \(\theta\) is shown in the next figure.

\[
\begin{align*}
  n_x &= \cos(\theta) \cos(\phi) \\
  n_y &= \cos(\theta) \sin(\phi) \\
  n_z &= \sin(\theta)
\end{align*}
\]

3.3 Director configuration and electrical field

3.3.1 Electrical potential

On the two electrodes of the GRIN lens G22 and G35 (see figure 2.12 for the design), an opposite voltage is applied and the upper electrode is connected to the ground. This causes an electrical field varying in both x- and z-direction with X and Z axis as indicated in figure 3.1. In the y-direction, perpendicular on the paper and along the line electrode, the electrical potential is constant.

GRIN lens G22

The next figure shows the equipotential lines of sample G22 when 100 V is applied.
The center of the electrode with the positive voltage is positioned at \((x=83 \, \mu m, \, z=15 \, \mu m)\) and the center of the electrode with the negative voltage is positioned at \((x=249 \, \mu m, \, z=15 \, \mu m)\). The electrodes have a width and of 10 \(\mu m\). For convenience the height is also taken equal to 10 \(\mu m\), although the real electrode is much thinner (\(\approx 100 \, nm\)). At \(z=134 \, \mu m\), there is an electrode connected to the ground. In the dielectric layer (\(0 \, \mu m < z < 79 \, \mu m\)) the equipotential lines have an elliptical form, but in the LC layer (\(790 \, \mu m < z < 103 \, \mu m\)) the shape is deformed as a consequence of the internal field and the anisotropy of the LC-layer. The variation of the electrical field in the LC-layer is smaller in the z-direction, because of the high dielectric constant \(\epsilon_\parallel = 10\), which is higher than \(\epsilon = 5.7\) in the Su8-dielectric layer. When applying \(+100 \, V\) and \(-100 \, V\) at the electrodes of the GRIN lens, there is in the LC-layer of G22 a variation from 13 \(V\) to \(-13 \, V\) in the x-direction between the electrodes and a variation from 13 \(V\) to 7 \(V\) in the z direction above the positive electrode. This is only a small part, less than 15\%, of the applied voltage on the GRIN lens.

The electrical potential in the LC layer of sample G22 is shown in more detail in the next figure.
Chapter 3. Analysis of the GRIN lens based on simulations

Figure 3.4: $U[V]$ vs $z[\mu m]$ and $x[\mu m]$ in the LC layer of G22 when 100 V is applied

The position of the center of the positive electrode is indicated at 83 $\mu m$ and that of the negative at 249 $\mu m$. Right above the electrodes the equipotential lines are deformed due to interaction with the internal field of the liquid crystal molecules. The farther from the electrode in the z-direction, the more the equipotential line is deformed. This is because the electrical field decreases and the LC interaction becomes more important. The LC interaction will be investigated further in section 3.3.2.

GRIN lens G35

The electrical potential in the LC layer of sample G35 when 100 V is applied, is shown in the next figure.
The electrodes are located at the same position as for G22 in figure 3.4. For GRIN lens G35, the dielectric layer below the LC and the LC layer itself is thinner. When applying +100 V and −100 V on the electrodes of the GRIN lens, there is in the LC-layer of G35 a variation from 15 V to −15 V in the x-direction between the electrodes and a variation from 15 V to 11 V in the z-direction above the positive electrode. A larger part, 15%, of the applied voltage influences the LC layer than in sample G22 because the dielectric layer in G35 has a smaller value, 55 μm, than in sample G22, 59 μm. Because the LC layer is thinner, there is also a smaller difference between minimum and maximum voltage over the LC layer.

3.3.2 Directors and electrical field

GRIN lens G22

From the electrical potential, the electrical field in the LC layer is derived. In the next figures the directors are plotted together with the electrical field when applying 50 V, 60 V and 100 V on G22.
Figure 3.6: The directors (red) and the electrical field (blue) as function of $x[\mu m]$ and $z[\mu m]$ for GRIN lens G22 with different applied voltages.
In figures 3.6 the directors are depicted in red without an arrow and the electric field vector corresponding with every director is depicted in blue with an arrow indicating the direction of the electrical field. The liquid crystal is located between \( z = 0 \mu m \) and \( z = 24 \mu m \). At \( z = 0 \mu m \) and \( z = 24 \mu m \) the directors are anchored with a pre-tilt of 2°. They make an inclination angle with the electrical field vector but they do not twist. The configuration is totally two-dimensional. The negative electrode is located at \( x = 83 \mu m \) and the positive electrode is located at \( x = 249 \mu m \). Because LC molecules tend to orient parallel with the electric field, positive or negative, changing the sign of both the electrodes will not influence the director alignment and the shape of the electrical field. The directors and electrical field between the electrodes are depicted. This is a unit cell for the director alignment and the shape of the electrical field, which repeats itself in the x-direction. For the figures 3.6 the shape of the electrical field is parabola-like between the electrodes.

In figure 3.6(a) the directors are aligned with the electrical field in the middle between the electrodes. Closer to the electrodes in the x-direction, the directors are less well aligned. The configuration of the directors is determined by minimizing the total free energy, which is the sum of the electric and elastic energy, explained in section 2.1.5. Figure 3.6(a) shows that between the electrodes the field is almost parallel with the directors, but closer to the electrodes the angle between \( \vec{E} \) and \( \vec{L} \) is larger. When the directors tend to align, \( f_{elastic} \) increases and \( f_{electric} \) decreases. The minimum in total free energy is obtained for a certain LC configuration, which at 50 V is shown in figure 3.6(a).

Looking at the variation in the z-direction above the electrodes, the directors closer to the electrodes are more aligned than the directors farther away from the electrodes. This is because the electrical field vector \( \vec{E} \) is larger close to the electrodes, depicted by a bigger blue arrow in figure 3.6(a). So the contribution of \( f_{electric} \) to the total free energy is larger. When minimizing the total free energy, this results in a smaller angle between \( \vec{E} \) and \( \vec{L} \) close to the electrodes.

The electrical field in figure 3.6(b) is stronger than in the electrical field in figure 3.6(a). This results in a better alignment of the directors with the electrical field, especially in the region above the electrodes.

Increasing the voltage up to 100 V, results in the configuration of figure 3.6(c). The field is stronger and the directors are more aligned. As shown in figure 3.4 for the potential of G22 at 100 V, there are more equipotential lines closer to the electrodes. This means the field is stronger closer to the electrodes. The smaller field further away from the electrodes results in less alignment in the region further away from the electrodes.

Because the director configuration changes most above the electrodes, the region above the
The next plot shows the directors and the electrical field for GRIN lens G22 at 50 V in a small region above the electrode. For clarity, approximately the same length for the director is used in x- and z-directions.

Figure 3.7: The directors (red) and the electrical field (blue) as function of $x[\mu m]$ and $z[\mu m]$ above the electrode for GRIN lens G22 with 50 V applied

The x-position of the negative electrode is depicted in orange. It is positioned at $z = -59 \mu m$ below the LC layer and at $x = 83 \mu m$. It has a width of 10 $\mu m$, so the edges of the electrodes are at $x = 78 \mu m$ and $x = 88 \mu m$. The directors only have an inclination ($\theta$) and no twist ($\phi = 0$). For G22 with 50 V applied it is clear that in the region above the electrode the directors are not aligned with the electrical field. This is because the pre-tilt has an important influence, when the electric field is not strong enough. The directors have a horizontal pre-tilt
at the boundaries of the LC layer and they will stay horizontal in the middle of the LC layer, because the field is not strong enough to make them rotate.

The next figure shows the directors and the electrical field, when a higher voltage, 100 V, is applied.
Figure 3.8: The directors (red) and the electrical field (blue) as function of $x[\mu m]$ and $z[\mu m]$ above the electrode for GRIN lens G22 with 100 V applied.

The electrode’s position is again indicated in orange. The green dashed line indicates roughly...
the border of good alignment with the electrical field. Between and above the two dashed green lines the directors are not well aligned along the field lines. In this region the elastic energy is very high because the director rotates between $x \approx 78 \, \mu m$ and $x \approx 88 \, \mu m$ over an angle of almost 180, increasing the splay ($K_1$) and bend ($K_3$). So at the left of the electrode, the director is oriented almost vertically and at the right of the electrode, the director is also oriented almost vertically, but above the electrode, the director is oriented horizontally. The reason for this change in orientation of director, is the influence of the pre-tilt, which keeps dominating in the region between the dashed green line.

**GRIN lens G35**

In the same way as for GRIN lens G22, the electrical field in the LC layer is derived from the electrical potential. In the next figures the directors are plotted together with the electrical field when applying 100 V and 300 V on GRIN lens G35.
Chapter 3. Analysis of the GRIN lens based on simulations

Figure 3.9: The directors (red) and the electrical field (blue) as function of $x[\mu m]$ and $z[\mu m]$ for GRIN lens G35 with different applied voltages

The liquid crystal is now located between $z = 0 \ \mu m$ and $z = 12 \ \mu m$. Again the negative electrode is located at $x = 83 \ \mu m$ and the positive electrode is located at $x = 249 \ \mu m$. The overall shape of the electric field and the LC configuration are the same as for GRIN lens G22.

In figure 3.9(a), the applied voltage is 100 V. The electrical field of G35 is higher than for GRIN G22 at 100V. This is seen by comparing the divergence of the electrical potential of the two GRIN lenses. The contribution of the electrical field in GRIN G35 (figure 3.9(a)) will be larger than in GRIN G22 (figure 3.6(c)), resulting in a better alignment. This can be seen by comparing the configuration of the directors between figure 3.6(c) and figure 3.9(a), where the difference is the clearest in the region above the electrodes.

In figure 3.9(b), the LC configuration and electrical field is determined when 300 V is applied. This is a very high voltage so almost all directors are aligned, especially in the region above
the electrodes. Because the director configuration changes the most above the electrodes, the next plots show the directors and the electrical field in a small region above the electrode positioned at \( x = 83 \, \mu m \).
Chapter 3. Analysis of the GRIN lens based on simulations

Figure 3.10: The directors (red) and the electrical field (blue) as function of $x[\mu m]$ and $z[\mu m]$ above the electrode for GRIN lens G35 with 100 V applied

The electrode is again indicated in orange, and the green dashed line indicates roughly the border of good alignment. Comparing figure 3.10 with figure 3.8 shows that for G35 there is a better alignment. This is because the electrical field in the LC layer is stronger.
The next figure shows the directors and the electrical field at 300 V.

\[ x[\mu m] \quad z[\mu m] \]

Now there is almost complete alignment. The green dashed circles indicate where there is a sudden change in director orientation. In contrast to figures 3.10, 3.7 and 3.8, the director is not oriented horizontally above the electrode and does not rotate over an angle of \( \approx 180^\circ \). At this high voltage, the director is vertically aligned with the electrical field above the electrode. In a small region, indicated by the green circles, there is a sudden change in director orientation and there the elastic energy is very high. The appearance of the disclination changes the director configuration a lot. It is possible to simulate such a disclination, but the voltage by which it appears will depend on the used grid of the simulations. The voltage at which
it appears in reality can be much lower, because disclinations appear easily at irregularities. Figure 3.11 depicts clearly the director configuration if there is a disclination.

3.4 Refractive index profile

In section 2.2.2 it is explained how the refractive index variation \( n(x) \) is calculated from the LC configuration, the inclination angle \( \theta(x, z) \) of each director. A discussion of the refractive index variation for G22 and G35 are given in the next two sections.

3.4.1 GRIN lens G22

A plot of \( n(x) \) for sample G22 derived from the LC configuration of figure 3.6(c) is shown in the next figure.

![Graph n vs x[\mu m] for G22 with 100 V applied](image)

**Figure 3.12:** \( n \) vs \( x[\mu m] \) for G22 with 100 V applied

The plot shows the refractive index variation \( n(x) \) in the interval \([0; 332\mu m]\), this is repeated in the \( x \)-direction. The electrodes are positioned at 83 \( \mu m \) and 249 \( \mu m \). In good approximation, \( n(x) \) has a parabolic shape between the two electrodes, but above the electrodes, \( n(x) \) increases and decreases very fast. This is, as shown in figure 3.8, because above the electrodes the directors are not well aligned.
3.4.2 GRIN lens G35

A plot of $n(x)$ for sample G35 derived from the LC configuration of figure 3.9(a) is shown in the next figure.

![Graph showing $n(x)$ for G35 with 100 V applied.](image)

Figure 3.13: $n$ vs $x[\mu m]$ for G35 with 100 V applied

Again $n(x)$ is close to parabolic between the electrodes, but above the electrode it sweeps up and down again because the directors are not well aligned, as shown in figure 3.10. It is interesting to investigate $n(x)$ when there is full alignment. The next figure shows $n(x)$ when an extremely high voltage, 500 V, is applied at the electrodes of G35.
Between the electrodes, $n(x)$ still has a parabola-like shape but above the electrodes, the refractive index profile does not sweep up and down. Also above the electrodes, $n(x)$ reaches a minimum of $n = 1.545$ which is smaller than the minimum $n = 1.622$ $n(x)$ reaches when 100 V is applied (figure 3.13). This is because the minimal value that $n(x)$ can reach is the ordinary index of refraction $n_o = 1.5271$. This happens when all directors are oriented vertically, which is not possible because of the fixed pre-tilt of 2° at the boundaries of the LC. At 500 V a larger part of the directors are oriented vertically above the electrodes than at 100 V, which results in a smaller minimum in the refractive index. The reason why the minimum of G22 at 100 V, $n = 1.59$ from figure 3.12, is smaller than the minimum of G35, $n = 1.622$, is because the thickness of the LC layer of G35 is smaller and the horizontal pre-tilt has relatively more influence than in the twice as thick LC layer of G22.

3.5 Angular profile

How the angular profile can be calculated from the refractive index was explained in section 2.2.3. The effective refractive index $n(x)$ was determined in paragraphs 3.4.1 and 3.4.2 for respectively GRIN lens G22 and G35. The angular profile of G22 and G35 is discussed in the
next two paragraphs.

3.5.1 GRIN lens G22

A plot of the refraction angle $\alpha(x)$ of sample G22 at 100 V derived from figure 3.12 is shown in the next figure.

![Graph of refraction angle vs x for G22 at 100 V](image)

**Figure 3.15:** $\alpha[^{\circ}]$ vs $x[\mu m]$ of G22 when 100 V is applied. The center of the electrode is indicated with the letter ‘e’ and the orange rectangles represent each half an electrode.

Between the two electrodes, the angular profile is linear in the interval [113; 220]. The useful area is defined as the quotient of the length of the interval where the angular profile is linear (107 $\mu m$) and the length of the unit cell of the GRIN lens (166 $\mu m$). For GRIN lens G22 at 100 V this is about 65%.

Closer to the electrodes, outside the useful area, the absolute value of the refraction angle becomes too small and stays approximately constant. This is because the refractive index modulation, depicted in figure 3.12, tends more to a linear profile than a parabolic profile. Looking at figure 3.6c, for the director configuration of G22 at 100V, it is clear that the directors are not well aligned with the electrical field in this region. Because not all of them rotate vertically, the effective refractive index decreases slower in the x-direction.

Above the electrode, the directors are aligned horizontally again, and the angular profile shows a large deviation.
3.5.2 GRIN lens G35

A plot of the refraction angle $\alpha(x)$ of sample G22 at 100 V derived from figure 3.13 is shown in the next figure.

![Graph showing the refraction angle $\alpha(x)$ vs $x[\mu m]$ of G35 with 100 V applied. The center of the electrode is indicated with the letter 'e' and the orange rectangles represent each half an electrode.]

**Figure 3.16:** $\alpha[deg]$ vs $x[\mu m]$ of G35 when 100 V is applied. The center of the electrode is indicated with the letter 'e' and the orange rectangles represent each half an electrode.

Between the two electrodes, the angular profile is linear in the interval $[103; 230]$. The useful area is 77% which is larger than the useful area of G22. Close to the electrodes there is again a large deviation, the refraction angle increases upto 15°. This is smaller than the 39° for G22, as shown in figure 3.15.

The angular profile of G35 for complete alignment is shown in the next figure.
Chapter 3. Analysis of the GRIN lens based on simulations

Figure 3.17: $\alpha[\text{deg}]$ vs $x[\mu\text{m}]$ of G35 when 500 V is applied. The center of the electrode is indicated with the letter ‘e’ and the orange rectangles represent each half an electrode.

Between the two electrodes, the angular profile is linear in the interval [109; 224]. The useful area is 69% which is a little smaller than the useful area of G35 at 100 V. So for a large useful area it is not necessary to apply a high voltage. Now there is no deviation above the electrodes.

3.6 Influence of free parameters on the refractive index and angular profile

To gain more insight in the model of the GRIN lens, different free parameters of the GRIN lens are varied. The next figure shows the different free parameters of GRIN G22.
Chapter 3. Analysis of the GRIN lens based on simulations

Figure 3.18: Free parameters of GRIN G22

The varied parameters are the applied voltage on the electrodes, the thickness of the LC-layer, the thickness of the dielectric layer below the LC, the dielectric constant $\epsilon_1$ of the dielectric layer below the LC and the dielectric constant $\epsilon_2$ of the dielectric layer above the LC. Also the influence of the upper electrode plate and the pre-tilt of the liquid crystal are discussed. The same free parameters will be investigated for GRIN lens G35.

When one of these parameters is varied, the refractive index modulation and angular profile will be different. An attempt is made to relate each parameter with the part of influence on the refractive index profile and angular profile. From the angular profile, the useful area of the GRIN lens can be derived and an optimum for each parameter is suggested in this section.

Some parameters of the GRIN lens could not be varied. For example, the distance between the electrodes is not a free parameter. Because of the fixed sizes of the pixels, on top of which the lens is placed, the distance between the electrodes is fixed. Also the LC parameters are not varied. The LC that is used has already a very large $\Delta n = n_e - n_o$, which is preferable. If $\Delta n$ would be smaller, a thicker layer of LC has to be used in order to have the same focal strength. The larger the LC layer, the higher the applied voltage has to be to align the LC, which is not desired.

The plots of the refractive index and the refraction angle for each free parameter is shown in
the next sections. In each plot, the center of the electrodes are located at 83 µm and 149 µm and the width of an electrode is 10 µm.

3.6.1 Upper electrode

The GRIN lens is designed with an electrode plate connected to the ground, placed above the LC layer. This improves the shape of the electric field, see figure 2.8. This upper electrode plate changes the electric field and especially the directors in the top of the LC layer, closer to the electrode plate are influenced. In the LC-layer, they will align more vertically above the electrodes, because the electric field is oriented more vertically. The influence on the refraction angle of GRIN lens G35 is shown in the next figure.

![Figure 3.19: α(°) vs x[µm] for G35 at 100 V. The blue line is the angular profile for GRIN lens G35 when the upper electrode is present and the green line when the upper electrode is not present.](image)

When this upper electrode is not present, the angular profile is not linear anymore. So the optimal shape of the electric field is with an electrode plate connected to the ground place above the LC.
3.6.2 Voltage

In this section the influence of the voltage applied on the electrodes below the LC layer is investigated for G22 and G35.

GRIN lens G22

The refractive modulation when different voltages are applied, is shown in the next figure.

When increasing the voltage, the directors align more with the electric field. This is best seen at the minimum of the refractive index modulation, located at the edge of the electrode, which becomes smaller at higher voltages. This is because when the electric field is higher, more directors will orient vertically close to the electrode and the refractive index will have a smaller value, closer to the ordinary refractive index $n_o = 1.5271$. If all directors were oriented vertically, the refractive index would be equal to the ordinary refractive index. So when the voltage is higher, but still smaller than 100 V, the directors are more aligned and the refractive index tends more to a parabola-like curve.

The next figure shows the refraction angle at different voltages.
Increasing the voltage changes the slope of the linear part of the angular profile and enlarges the useful area. The higher the applied voltage the larger the slope of the linear part of the angular profile. This means the focal point of the lens is closer to the GRIN lens, which is preferred. By increasing the voltage, the useful area of the GRIN lens becomes larger. So the GRIN lens has the best performance at 100 V.

**GRIN lens G35**

Because GRIN lens G35 behaves in the same way as GRIN lens G22 at voltages lower than 100 V, the next figure shows the angular profile for GRIN lens G35 at different voltages higher than 100 V.
Increasing the voltage changes the slope of the linear part of the angular profile, but this time does not enlarge the useful area. Also, there is (almost) no deviation above the electrodes when a voltage higher than 300 V applied.

This is explained by the sudden change in LC configuration when the voltage is higher than \(\approx 300\) V, which is shown in figure 3.9(b). At this high voltage, almost all directors are aligned with the electrical field and there is no more influence of the pre-tilt, so there is no more undesired deviation above the electrodes. At voltages lower than \(\approx 300\) V, the pre-tilt influences the LC configuration in a region of \(\approx 20\ \mu m\) in the x-direction centered above the electrode. This means that still a part of the LC is oriented horizontally in this region and right above the electrode, all the LC molecules are oriented horizontally.

So although the undesired deviations above the electrodes disappear at voltages higher than \(\approx 300\) V, the useful area of G35 is not larger when applying voltages higher than 100 V. Also GRIN lens G35, has the best performance at 100 V.

**Figure 3.22:** \(\alpha\) vs \(x[\mu m]\) for G35 at 100 V (blue), 300 V (green) and 500 V (magenta).
3.6.3 Thickness of the LC layer

GRIN lens G22

A plot of the angular profile of sample G22 at 100 V and with a variable thickness of the LC layer is shown in the next figure.

![Figure 3.23: α[deg] vs x[µm] for G22 at 100 V. Each curve corresponds with a different thickness of the LC layer.](image)

Increasing the thickness of the LC layer, increases the slope of the linear part. Also the undesired deviation above the electrodes becomes larger and the linear part between the electrodes becomes smaller. When decreasing the thickness of the LC layer, the slope would become too small and the focal point of the lens would not be close enough. So for GRIN lens G22, the thickness of 24 µm is optimal.

GRIN lens G35

A plot of the angular profile of sample G35 at 100 V and a variable thickness of the LC layer is shown in the next figure.
Increasing the thickness of the LC layer, results in a bigger slope of the linear part. Also for GRIN lens G35, a bigger LC layer results in a smaller useful area between the electrodes and a larger deviation above the electrodes. The original thickness of 12 $\mu$m is optimal. A thinner thickness is not possible because this would make the focal strength of the lens too small.

3.6.4 Thickness of the dielectric layer

GRIN lens G22

The refractive index for sample G22 at 100 V with the thickness of the dielectric layer below the LC varying between 5 $\mu$m and 59 $\mu$m is shown in the next figure.
Figure 3.25: $n$ vs $x[\mu m]$ for G22 at 100 V. The letter 'e' indicates the position of the electrode. The green curve corresponding with a thickness of 59 $\mu m$ is the angular profile of the original GRIN lens G22. Each curve corresponds with a different thickness of the dielectric layer below the LC layer.

Figure 3.25 shows that for thinner dielectric layers, 5 $\mu m$ and 15 $\mu m$, the deviations are smaller above the electrodes: the refractive index increases less and for a smaller region around the electrode, indicated by the letter 'e'. This is because the electric field in the LC will be stronger when the dielectric layer is thinner, the LC molecules are more aligned and the deviations are smaller.

If the dielectric layer is too thin, less than 30 $\mu m$, the refractive index profile is not parabolic anymore between the electrodes. This is caused by the shape of the electric field. When the dielectric layer is too small, the electric field is oriented mostly horizontally in the LC layer. The directors will only rotate close to the electrodes. This results in an undesired refractive index profile.

The next figures show the corresponding angular profiles for different thicknesses of the dielectric layer of sample G22 at 100 V.
Figure 3.26: $\alpha[\text{deg}]$ vs $x[\text{\mu m}]$ for G22 at 100 V. Each curve corresponds with a different thickness of the dielectric layer below the LC layer.

For thicknesses smaller than 15 $\mu m$, the linear part of the angular profile becomes too small. In the next figure, a close up of the region between the electrodes is shown.
Figure 3.27: $\alpha[\text{deg}]$ vs $x[\mu\text{m}]$ for G22 at 100 V. Only the region between the electrodes is depicted. Each curve corresponds with an other thickness of the dielectric layer below the LC layer. The black line is a straight line, placed in order to estimate the linearity of the different angular profiles.

The angular profiles corresponding with smaller thicknesses than $\approx 40 \ \mu\text{m}$ deviate more from the desired linear profile, indicated in black. So the largest useful area for G22 is reached when the dielectric layer has a thickness between $59 \ \mu\text{m}$ and $40 \ \mu\text{m}$.

**GRIN lens G35**

The next figure shows the angular profile of the original G35 and G35 with a smaller dielectric layer.
Figure 3.28: $\alpha[\text{deg}]$ vs $x[\mu\text{m}]$ for G35 at 100 V. Each curve corresponds with a different thickness of the dielectric layer below the LC layer. The blue curve corresponds with the original thickness of sample G35.

When decreasing the original thickness of 55 $\mu$m to 40 $\mu$m, the angular profile changes in the same way when the thickness of the dielectric layer of GRIN G22 decreases too much. Close to the electrodes, the refraction angle becomes too large. This is again because the electric field is more horizontally in the region between the electrodes, and more vertically close to the electrodes. More directors will rotate close to the electrodes, resulting in an undesired angular profile. Decreasing the thickness of the dielectric layer of GRIN lens G35 is not desired.

3.6.5 Dielectric constant of the dielectric layer below the LC

GRIN lens G22

A plot of the refractive index of sample G22 at 100 V and a variable dielectric constant of the dielectric layer below the LC layer is shown in the following figure.
Figure 3.29: $n$ vs $x [\mu m]$ for G22 at 50 V. Each curve corresponds with a different thickness value of the dielectric constant in the dielectric layer below the LC-layer. The red curve corresponding with $\epsilon = 3.4$ is the original dielectric constant of G22.

The lower the dielectrical constant, the more the electric field is reduced in the LC layer. If $\epsilon = 1$, the electrical field is too low to align the LC with the electrical field. When the dielectrical constant is higher than $\epsilon = 5$, the refractive index profile is not parabola-like anymore. The refractive index decreases too fast from the middle of the GRIN lens towards the electrodes, this means too much LC molecules are oriented vertically in the region between the electrodes.

A plot of the angular profile of sample G22 at 100 V and a variable dielectric constant of the dielectric layer below the LC layer is shown in the following figure.
Figure 3.30: $\alpha[\text{deg}]$ vs $x[\mu m]$ for G22 at 50 V for the region between the electrodes. Each curve corresponds with a different thickness value of the dielectric constant in the dielectric layer below the LC-layer. The blue curve corresponding with $\epsilon = 3.4$ is the original dielectric constant of G22.

If the dielectric constant is larger than $\epsilon = 5$, the angular profile starts deviating from a linear profile. If the dielectric constant is smaller than $\epsilon = 5$, the useful area of the GRIN lens starts decreasing. GRIN lens G22 works optimal for a dielectric constant of the layer below the LC equal to $\epsilon = 5$.

3.6.6 Dielectric constant of the dielectric layer below and above the LC

GRIN lens G35

For the sample G35 at 100 V a plot of the refractive index and a variable dielectric constant of the dielectric layer below and above the LC is shown in the next figure.
Figure 3.31: $n$ vs $x$ [µm] for G35 at 100 V. Each curve corresponds with a different value of the dielectric constant in the dielectric layer above ($\epsilon_2$) and below ($\epsilon_1$) the LC-layer. The red curve corresponding with $\epsilon_1 = \epsilon_2 = 3.4$ is the original refractive index of G35.

The red curve corresponds with the original model of GRIN G35. When $\epsilon_1 = 3.4$ is replaced by a dielectric constant twice as high, $\epsilon_1 = 6.8$, the blue curved is achieved. When the dielectric constant is higher, the electric field is less reduced. The effect is best seen close to the electrodes, where the minimum value of the refractive index is lower. This is because more LC is oriented vertical when the field is stronger.

If also the dielectric constant in the layer above the LC is twice as high, $\epsilon_2 = 6.8$, the green curve is obtained. When $\epsilon_2$ is higher, the upper electrode will have more influence. This causes more directors to orient vertical over the whole GRIN lens, resulting in a refractive index profile which decreases faster from $n_e$ to $n_o$ and deviates more from a parabola-like profile. The influence on the refraction angle is depicted in the next figure.
Figure 3.32: $\alpha[\text{deg}]$ vs $x[\mu\text{m}]$ for G35 at 100 V. Each curve corresponds with a different value of the dielectric constant in the dielectric layer above ($\epsilon_2$) and below ($\epsilon_1$) the LC-layer. The red curve corresponding with $\epsilon_1 = \epsilon_2 = 3.4$ is the original angular profile of G35.

If $\epsilon_1$ is twice at high, the angular profile starts deviating from the desired linear profile. Taking $\epsilon_2$ twice at high (the green curve), decreases the useful area substantially. Changing the dielectric constant of the layer above or below the LC-layer is not desired.

3.6.7 pre-tilt

A pre-tilt of $2^\circ$ was used in the previous plots. Changing the pre-tilt at the whole boundary of the LC-layer, only influences the region in the LC-layer above the electrode, but not the useful area.

The next figure shows when a pre-tilt of $90^\circ$ is defined in only a small region above the electrodes, in attempt to align the LC vertically above the electrodes.
Figure 3.33: $\alpha [\text{deg}]$ vs $x [\mu m]$ for G35 at 100 V. The blue curve corresponds with the original model of G35. When a pre-tilt of $90^\circ$ is defined in a region of $10 \mu m$ above the electrode, $[78 \mu m; 88 \mu m]$ and $[241 \mu m; 251 \mu m]$, the green curve is achieved. When a pre-tilt of $90^\circ$ is defined in a region of $20 \mu m$ above the electrode, $[73 \mu m; 93 \mu m]$ and $[236 \mu m; 256 \mu m]$, the red curve is achieved.

If the pre-tilt in the region above the electrode is changed, the deviation above the electrodes becomes larger. This is because the director configuration changes suddenly at the border of the region. By changing the pre-tilt, it is possible to change locally the angular profile. A close-up is shown in the next figure.
Figure 3.34: $\alpha[\text{deg}]$ vs $x[\mu\text{m}]$ for G35 at 100 V. A close-up of the region between the electrodes of figure 3.33 is shown.

The figure shows that the angular profile is not changed in the region between the electrodes, but close to the electrodes at 220 $\mu\text{m}$, the slope of the linear part of the curve is influenced. For the green curve (pre-tilt of 90° in a region of 10 $\mu\text{m}$) the slope is bigger than the blue original curve of G35. And the slope of the red curve (pre-tilt of 90° in a region of 20 $\mu\text{m}$) is bigger than the slope of the green curve. Although the useful area is not improved, a local pre-tilt may be used to optimize angular profile.

3.7 Comparison with previous work

In this section the results obtained from the simulations are compared with results from previous work, which are summarized in paragraph 2.3.2.

3.7.1 Simulated and analytical electrical potential G22

The simulated electrical potential in the LC layer is compared with the analytical electrical potential in the next figure.
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Figure 3.35: Simulated equipotential lines (multi-color) of GRIN lens G22 at 100 V and analytical equipotential lines (magenta dashed lines).

The analytical electrical potential is calculated for GRIN lens G22, without taking LC interaction into account, assuming the LC is a homogeneous material with dielectric constant $\epsilon = 10$. Instead of electrodes with finite dimensions, singular point sources were used. In this singular point, the value of the potential is undetermined. Because of this, the value of the analytical electric potential differs from the simulated electrical potential, for which the potential is defined, 100 V, at the source (the finite rectangular electrode). From the figure it is derived that the simulated potential is approximately a factor 30 larger than the analytical potential.

The shape of the equipotential lines are roughly the same, only above the electrodes there is a difference. The theoretic equipotential curve is a smooth one with a maximum right above the electrode. The simulated equipotential curve has a local minimum right above the electrodes. This is because the LC interactions were taken into account for the simulated electrical potential. The reason why the LC interaction has most influence above the electrode was explained in section 3.3.1. So roughly the two potentials are the same, the difference is explained by the differences in the sources that are used (point source and finite rectangular source) and the LC interaction with the electrical field that is taken into account for the
simulations.

3.7.2 The simulated and analytical refractive index of GRIN lens G22

The analytical refractive index is calculated based on the assumption that all directors are aligned with the electric field. From this LC configuration, the refractive index is calculated in the same way as explained in paragraph 2.2.2. The simulated refractive modulation index is compared with the analytical of G22 in the next figure.

![Graph showing refractive index vs x [µm] for GRIN lens G22.](image)

**Figure 3.36:** $n$ vs $x$ [µm] for GRIN lens G22. The refractive index is plotted for different voltages and the theoretic refractive index is plotted in magenta.

They coincide in the region between the electrodes but above the electrodes they differ. The analytical refractive index is calculated assuming that all directors are aligned with the electrical field. In the simulations, the LC interaction is taken into account. The LC interaction is less important at higher voltages, that is the reason why the green curve, corresponding with the highest voltage, differs less from the analytical refractive index. Above the electrode, the curves differ the most. This is because the LC interaction will dominate above the electrodes.

3.7.3 The simulated and analytical refraction angle of GRIN lens G22

In the next figure the simulated angular profile of G22 at 100 V is compared with the theoretic angular profile.
Figure 3.37: $\alpha [\text{deg}]$ vs $x [\mu \text{m}]$ for G22. The blue curve is the simulated angular profile when 100 V is applied. The magenta curve is the theoretic angular profile.

The differences between the two curves are a direct consequence of the differences in the refractive index modulation. The slope of the theoretic curve is bigger, because the refractive index has a narrower parabolic profile. The deviation above the electrodes of the simulated profile is due to the deviation of the refractive index profile above the electrodes.

3.7.4 The simulated and experimental refraction angle of GRIN lens G35

The simulated angular profile of G35 at 100 V is compared with the measured angular profile of G35 at 100 V. This measured angular profile was discussed in section 2.3.3.
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Figure 3.38: $\alpha [\text{deg}]$ vs $x [\mu \text{m}]$ for G35. The blue curve is the simulated angular profile when 100 V is applied on G35. The red curve is the measured angular profile.

The two angular profiles have a linear part between the electrodes, but the slope of the experimental curve is larger than the slope of the simulated curve. This difference is attributed to the difference in thickness of LC-layer, as will be explained in section 5. GRIN lens G35 seemed to have a bigger thickness of LC layer, 13 $\mu \text{m}$ instead of 12 $\mu \text{m}$. Increasing the thickness results in a bigger slope, making the difference between the simulated angular profile of G35 with 12 $\mu \text{m}$ and the measured angular profile of G35 with a thickness of 13 $\mu \text{m}$ smaller.

Above the electrodes the two curves differ a lot. This is because the measurements close to the electrodes are inaccurate. Taking into account the difference in the slopes and the inaccurate measurements above the electrodes, the two profiles have roughly the same shape.

3.8 Simulation program 2dimMOS

This software is designed to calculate the electro-optical properties of LCD’s. To calculate the electrostatic potential and the director, the finite element method is used. Therefore a mesh of triangles is defined in the GRIN lens. The director distribution is calculated using the Oseen-Frank expression for the free elastic energy density, see section 2.1.5. The potential calculation starts at the expression for the electrostatic free energy density 2.5. Because of mutual dependence both are calculated iteratively.

The input model is two dimensional and because it is not possible to take periodic boundaries into account with this program, several unit cells have to be modeled. This is in order to reduce the influence of the boundaries.
Simulation the GRIN lens with this program, led to inaccurate results. This is contributed to the accumulation of numerical faults by the calculation of the electrical potential through the dielectric layer. This leads to an inaccurate potential in the LC-layer and as consequence an inaccurate calculation of the LC configuration. This is illustrated by the next figure of the output model of 2dimMOS.

![Figure 3.39: Several unit cells of the GRIN lens: output model of 2dimMOS. The red lines are the equipotential lines. The directors are depicted in yellow, in the LC layer (green)](image)

This picture shows clearly that the equipotential lines (red) are different in different unit cells, which is not acceptable.

A plot of the refractive index modulation obtained from the director configuration simulated with 2dimMOS is shown in the next figure.
Figure 3.40: $n$ vs $x[\mu m]$ for GRIN lens G22 at 100 V. For this simulation an input model with 9 electrodes is used. The position of the center of the electrodes is indicated with the letter 'e'.

This plot shows a refractive index profile that is not symmetric between two electrodes. Also the profile of different unit cells differ from each other.

A plot of the angular profile derived from the refractive index modulation of figure 3.40 is shown in the next figure.
Figure 3.41: $\alpha [\text{deg}]$ vs $x [\mu m]$ for GRIN lens G22 at 100 V. The position of the center of the electrodes is indicated with the letter 'e'. Only the region between two electrodes is shown.

The angular profile has a lot of noise and in the middle of the unit cell (indicated in the plot with 'middle'), the refraction angle is not zero. This is not acceptable, because a completely symmetric input model is used.

Simulating the director configuration and the potential of the GRIN lens leads to inaccurate results using the program 2dimMOS. This is contributed to the large dielectric layer of the GRIN lens. The simulation program Shintech LCD master is recommended to simulate the GRIN lens.

3.9 Conclusions

In this chapter a simulation based model taking LC interaction into account is set up for the GRIN lens and extra investigation is done in the region above the electrodes.

The electrical potential, the electric field and the director configuration of the GRIN lenses G22 and G35 are determined. When 100 V is applied on the GRIN lens, only about 15% of the voltage is present over the LC layer. The equipotential lines are deformed in the region above the electrodes due to LC interaction.

This LC interaction causes bad alignment of the directors with the electric field in the region above the electrodes. Part of the directors stay horizontally oriented instead of rotating vertically. For voltages lower than 100 V, the alignment becomes bad. For voltages higher than 300 V, all LC molecules are aligned, but disclinations appear in the LC.
From the director configuration the refractive index variation and the refraction angle are derived. For samples G22 and G35, they predict a useful area (area with the desired lens profile) of respectively 65% and 77%. Above the electrodes there are undesired deviations due to bad LC alignment.

Increasing the voltage above 100 V does not improve the useful area. Therefore free parameters of the GRIN lens were varied. The useful area of G22 can be improved by decreasing the dielectric constant of the dielectric layer below the LC from 59 µm to 40 µm or increasing the dielectric constant of the dielectric layer below the LC from $\epsilon = 3.4$ to $\epsilon = 5$. The insight given by the simulations of all free parameters can be used to increase the useful area of GRIN G35.

Because a GRIN lens is not a conventional model for simulation programs of liquid crystal, not all simulation programs give satisfying results. The software 2dimMOS led to inaccurate results, while the software Shintech LCD MASTER gave good results.
Chapter 4

Lens profile measurements

4.1 Introduction

An important characteristic of the GRIN lens is the refractive index modulation. This determines the refraction angle and so the quality of the 3D display. In the previous experiment, see section 2.3.3, the angular refraction pattern of the GRIN lens was measured. But it was not possible to characterize the GRIN lens close to the electrodes. As an attempt to determine the refractive index close to the electrodes, this experiment is done.

Two important profiles are calculated, which are resumed in the next scheme.

| 1 | \( n(x) \) \( \searrow \) \( \alpha(x) \rightarrow I(y) \) \\ \( f(x) \nearrow \) |
| 2 | \( I(y) \rightarrow \alpha(x) \) \( \nearrow n(x) \) \( \searrow f(x) \) |

First, the intensity profile \( I(y) \), where \( y \) is the position on the detector, is calculated via the angular profile \( \alpha(x) \), where \( x \) is the position along the lens sample, which is derived from the lens profile. This lens profile is for a classical lens \( f(x) \), the lens shape, and for a GRIN lens \( n(x) \), the refractive index variation. The calculated \( I(y) \) will be compared with the measured \( I(y) \).

Second, the lens shape \( f(x) \) for a classical lens and the refractive index \( n(x) \) for a GRIN lens are derived. These profiles are calculated from the angular profile \( \alpha(x) \) which is derived from the measured intensity profile \( I(y) \). These profiles will be compared with the theoretical lens profiles.
4.2 Experimental setup

A parallel laser beam with a spot of 2 mm falls in on the sample and the refracted pattern is projected on a screen as shown in the figure below.

![Experimental setup](image)

**Figure 4.1:** Experimental setup: The incident light is refracted by the sample and projected on the screen. A picture of the refracted pattern is shown (red) together its intensity profile (blue curve).

For this experiment, a He-Ne laser is used and different samples, the microlens and G22, are measured. The intensity plot is obtained by analyzing figure 4.1 with the software ImageJ, subtracting the background intensity and then rescaling the pixels to millimeters.

4.3 Theory and analyzing method

4.3.1 Calculation of the intensity profile from the lens profile

Derivation of the angular profile from the lens profile

The angular profile $\alpha(x)$ of the classical lens will be determined by the lens shape $f(x)$, while for a GRIN lens $\alpha(x)$ is determined by the refractive index $n(x)$. The difference is illustrated in the next figure.
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Figure 4.2: Refraction by a classic lens (left) and a GRIN lens (right)

The derivation of $\alpha(x)$ from $f(x)$, for a classical lens, and from $n(x)$, for a GRIN lens, is done in the next two paragraphs.

**Classical lens**

Figure 4.2 shows,

$$\alpha = \theta_2 - \theta_1,$$  \hspace{1cm} (4.1)

and $\theta_1$ depends on the lens shape $f(x)$,

$$\theta_1 = \phi$$  \hspace{1cm} (4.2)

$$\tan(\phi) = \frac{\partial f}{\partial x}$$

$$\theta_1 = \arctan\left(\frac{\partial f}{\partial x}\right).$$  \hspace{1cm} (4.3)

The expression for $\theta_2$ is derived using Snell’s law and expression 4.3 for $\theta_1$,

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\theta_2 = \arcsin \left[ \frac{n_1}{n_2} \sin \left( \arctan \left( \frac{\partial f}{\partial x} \right) \right) \right].$$  \hspace{1cm} (4.4)

Substituting formula 4.3 and 4.4 into 4.1 results in the expression for $\alpha(x)$.

$$\alpha(x) = \arcsin \left[ \frac{n_1}{n_2} \sin \left( \arctan \left( \frac{\partial f}{\partial x} \right) \right) \right] - \arctan \left( \frac{\partial f}{\partial x} \right)$$  \hspace{1cm} (4.5)
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with \( n_1 \) the refractive index of air and \( n_2 \) the refractive index of the lens material. This is the expression for \( \alpha(x) \) which is derived from \( f(x) \).

**GRIN lens**

For the GRIN lens, the relation between \( \alpha(x) \) and \( n(x) \) is derived in section 2.2.3 and is given by formula 2.16,

\[
\alpha(x) = \arcsin(h \frac{\partial n}{\partial x})
\]  

with \( h \) the thickness of the LC layer.

**Derivation of the intensity profile from the angular profile**

The intensity \( I(y) \) gives the intensity in a point at position \( y \) on the screen.

To derive the formula for \( I(y) \) from \( \alpha(x) \), two formulas are needed. The first one is a general formula, giving the relation between the incident intensity \( I_{\text{in}} \) at position \( x \) and outgoing intensity \( I(y) \) incident at position \( y \) on the screen. The second is the expression for \( y(x) \), relating for every position \( x \) of the lens, the corresponding position \( y \) on the screen.

The first required formula is derived from the next figure.

**Figure 4.3:** Refraction by the lens sample

Figure 4.3 illustrates that an equal amount of light in area \( dx \) incident at position \( x \), will reach the position \( y \) in an area \( dy \) on the screen:

\[
I_{\text{in}} |dx| = I(y) |dy|
\]

\[
I(y) = I_{\text{in}} \left| \frac{dx}{dy} \right|
\]  

So the light at position \( y \) comes from the light incident at position \( x \) of the lens. In case the light from two or more different positions \( x \) comes together in the same position \( y \) on the
screen, formula \[4.7\] is not valid. In the following figure a situation is shown where formula \[4.7\] is valid, and one where it is not valid.

**Figure 4.4:** Situation in which formula \[4.7\] is valid

![Diagram showing a situation where formula 4.7 is valid](image1)

**Figure 4.5:** Situation in which formula \[4.7\] is not valid. The blue circles indicate the positions \(y\) where more than one light beam come together

![Diagram showing a situation where formula 4.7 is not valid](image2)

So the condition for the validity of formula \[4.7\] is that \(y\) is a single-valued function of \(x\). This is when every \(y\) is related to only one \(x\), and every \(x\) is related to only one \(y\).

The second required formula follows from the next figure.
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Figure 4.6: Relation between $y(x)$ and $\alpha(x)$

\[ y(x) = L \cdot \tan(\alpha(x)) \]  
\[ (4.9) \]

Taking the derivative $\frac{dy}{dx}$ of formula 4.9 gives the expression of the unknown $\frac{dx}{dy}$ of formula 4.8:

\[ \frac{|dx|}{|dy|} = \frac{1}{\frac{dy}{dx}}. \]  
\[ (4.10) \]

Substituting equation 4.10 in 4.8 results in the expression for $I(y)$.

4.3.2 Calculation of the lens profile from the intensity profile

Derivation of the angular profile from the intensity profile

In this section the backwards calculation is done: the angular profile $\alpha(x)$ is derived from the measured intensity profile $I(y)$ on the screen.

The inverse formula $x(y)$ is derived from formula 4.7.

\[ dx = \frac{I(y)}{I_{in}} \, dy \]
\[ x = \int_0^y \frac{I(y)}{I_{in}} \, dy \]  
\[ (4.11) \]

The constant $I_{in}$ is calculated from the total integral of the intensity over the whole screen $[0:y_p]$, with $y_p$ the end-position of the light on the screen. The period of the lenticular $p$, is also known.

\[ I_{in} = \int_0^{y_p} \frac{I(y)}{p} \, dy \]  
\[ (4.12) \]

If $x(y)$ is known, the inverse function $y(x)$ is also known. From formula 4.9 the angular profile $\alpha(x)$ is derived.
4.4 Measurements

4.4.1 Calculation of the intensity profile from the lens profile

**Microlens**

For the microlens with lens shape \( f_s(x) \), where the subscript ‘s’ indicates that the lens shape is known in advance, the intensity profile \( I_s(y) \) is calculated according to the method explained in section 4.3.1. The subscript ‘s’ indicates that \( I_s(y) \) is derived from \( f_s(x) \) and not
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experimentally determined, as will be done in the next section.

The lens has the shape of a segment of a circle with radius 1.149 mm and has a refractive index $n=1.49$. A plot of the lens shape $f_s(x)$ is shown in the next figure.

![Graph showing $f_s(x)$ as a function of $x$ in mm.](image)

**Figure 4.7:** $f_s[mm]$ as function of $x[mm]$)

The curve is a segment of a circle, which would be more clear when the two axes have the same scale. The angular profile is calculated according to formula 4.5 and plotted in figure 4.8.

![Graph showing $\alpha_s[rad]$ as function of $x[\mu m]$.](image)

**Figure 4.8:** $\alpha_s[rad]$ as function of $x[\mu m]$]

Figure 4.8 shows the angular profile which is linear for the whole lens. Now that the angular profile $\alpha_s(x)$ is known, $y(x)$ is calculated with formula 4.9. Then the derivative of $y(x)$ is taken and substituted into expression 4.8 for $I(y)$. The intensity profile $I_s(y)$ is plotted in the next figure.
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Figure 4.9: $I_s [au]$ as function of $y [mm]$

Neglecting second order deviations, figure 4.9 is the rectangular function with a width of 30 cm.

**GRIN lens G22**

From the refractive index modulation profile $n_s(x)$, the angular profile $\alpha_s(x)$ is derived and from $\alpha_s(x)$, the intensity profile $I_s(y)$ is calculated.

For GRIN lens G22, $n_s(x)$ was obtained in section 3.4.1. All design parameters of GRIN G22 are shown in section 2.3 figure 2.12(a). The most important parameters are the ordinary refractive index, $n_o = 1.5271$, the extra ordinary index, $n_e = 1.7659$ and the thickness of the LC layer $h = 24 \, \mu m$. The lens width is $p = 166 \, \mu m$ and the positive and negative electrode are placed in the dielectric layer under the LC material at $x=0 \, \mu m$ and $x=166 \, \mu m$. A plot of the refractive index variation of GRIN lens G22 is shown in the next figure.

Figure 4.10: $n_s$ as function of $x[\mu m]$ for GRIN lens G22

The refractive index $n_s(x)$ is parabola-like between the electrodes but above the electrodes
the value of \( n_s(x) \) increases suddenly. According to formula 4.6, the angular profile \( \alpha_s(x) \) is calculated, see next figure.

![Figure 4.11: \( \alpha_s[\text{deg}] \) as function of \( x[\text{mm}] \)](image)

The angular profile is linear in the region between the electrodes, but again above the electrodes, the angle changes sign and the absolute value increases suddenly. Next step in deriving the intensity \( I_s(y) \), is calculating \( y(x) \) according to formula 4.9 and the derivative \( \frac{dx}{dy} \).

\[
\left| \frac{dx}{dy} \right| = \left| \frac{1}{\frac{y(x_2)-y(x_1)}{x_2-x_1}} \right|.
\] (4.18)

Formula 4.18 is calculated for or each part \( y_n(x) \) of \( y(x) \), with \( n \) the lowest number of single-valued functions of which \( y(x) \) consists in. Given \( y(x) \), \( n = 5 \) single-valued functions are defined as shown in the next figure of \( y(x) \).
Figure 4.12: $y[\text{mm}]$ as function of $x[\mu\text{m}]$ and the division in five single-valued functions

The five functions $\left| \frac{dx}{dy} \right|$ are calculated for each single-valued part of figure 4.12 and plotted in figure 4.13. The interpretation of the five derivatives $\left| \frac{dx}{dy} \right|$ follows from formula 4.8:

$$\left| \frac{dx}{dy} \right| = \frac{I_n(y)}{I_m}. \quad (4.19)$$

The five derivatives represent the normalized intensities coming from the incident light beam on the five different parts of the GRIN lens.
In this figure the intensity $I_1(y)$ corresponds to the first single-valued function $y_1(x)$ in the plot 4.12 and similar for the other $I_n(y)$. Function $y_3(x)$, defined in the central, linear part of the lens in figure 4.12, corresponds to the intensity $I_3(y)$ in the middle, having the largest intensity. The other parts of the lens are responsible for the light spread out away from the center.

When interference is not taken into account, the refraction pattern on the screen, $I_s(y)$, is the sum of the different intensities $I_n(y)$,

$$I_s(y) = \sum_{n=1}^{5} I_n(y). \quad (4.20)$$

The sum in formula (4.20) is done by interpolating the numerical functions $I_n(y)$ in Matlab and then summing them. The sum is plotted in the next figure.
Very roughly, $I_s(y)$ is a rectangular function, with a width of 20 cm. At the edges of its definition interval, the intensity increases fast. The light spread out away from the central part has a very low intensity.

### 4.4.2 Calculation of the lens profile from the intensity profile

**Microlens**

The microlens (lens pitch $p=631 \ \mu m$, refractive index $n=1.49$, radius of curvature $R=1.149 \ mm$) is placed before the laser at a distance $L=1.027 \ m$ from the screen. The intensity is measured and plotted in the next figure.
fringes. These are not only caused by background noise but are the effect of interference. This is because the incident laser beam with a width of 2 mm, falls in on different microlenses with a pitch of 631 \( \mu m \). Every time the path length \( s \) of the refracted laser beam differs \( \Delta s = n \lambda \), an intensity maximum appears, as illustrated in the next figure.

\[ \text{Figure 4.16: Interference at the microlens} \]

If the fringes are not considered, the intensity profile has an approximately constant value in the interval \([-150 \text{ mm}; 150 \text{ mm}]\). At the edges of the interval, the intensity increases a little.

In order to derive the lens profile \( f(x) \), the calculations of paragraph 4.3.2 are done. The function \( x(y) \) is derived by taking the integral of \( I(y) \) according to formula 4.11, where the constant \( I_{in} \) is calculated with formula 4.12 for the microlens with lens pitch \( p = 631 \mu m \). A plot of the position \( x \) (lens) as a function of the position \( y \) (screen) towards which the light is bent, is shown in the next figure.

\[ \text{Figure 4.17: Position } x[\text{mm}] \text{ of the lens as function of the position } y[\text{mm}] \text{ on the screen} \]

The function \( x(y) \) is approximately linear.
Chapter 4. Lens profile measurements

The inverse function \( y(x) \) is taken and substituted into formula 4.1 for the angular profile of the lens. The plot of \( \alpha(x) \) is shown in the next figure.

The function \( \alpha(x) \) is approximately linear. This is because the relation ’arctan’ as used in formula 4.13 is linear for small angles.

Knowing \( \alpha(x) \), the lens shape \( f(x) \) is derived. First the derivative \( \frac{\partial f}{\partial x} \) is calculated from formula 4.14. This is done by solving 4.14 to \( \phi \) with a macro in Excell. Then \( f(x) \) is calculated according to formula 4.15. A plot of the experimental lens shape \( f(x) \) is shown in the next figure.

This \( f(x) \) is derived from the measured intensity \( I(y) \). If the coordinates are equally scaled, it is clear that this lens has the shape of half a circle.
GRIN lens G22

The GRIN lens G22 (lens pitch $p = 166 \, \mu m$, $n_o = 1.5271$, $n_e = 1.7659$) is placed before the laser at a distance $L = 1.206 \, m$ from the screen. The intensity is measured and plotted in the next figure.

![Figure 4.20: $I[au]$ as function of $y[mm]$](image)

This plot shows that the intensity is defined in an interval with a width of 30 cm. The fringes in the plot are caused by interference. This interference is caused by light incident at different GRIN lenses, in the same way as for the classical lens, and also by light incident at different positions of the same GRIN lens, for which the light is deflected to the same position on the screen. In the interval where $I(y)$ exists, the mean value is approximately a constant. At the edges of the interval, the intensity increases before dropping to zero.

In order to derive $\alpha(x)$ and $n(x)$, the method as explained is paragraph 4.3.2 is followed. First the function $x(y)$ is calculated by taking the integral of $I(y)$, according to formula 4.11. The constant $I_m$ is calculated using formula 4.12 with lens pitch $p = 166 \, \mu m$. A plot of $x(y)$ is shown in the next figure.

![Figure 4.21: $x[mm]$ as function of $y[mm]$](image)

An important remark about this plot is that it is calculated based on equation 4.7, under the assumption that $x(y)$ is a single-valued function. Since $n(x)$ of the GRIN lens is unknown...
close to the electrodes, it is questionable whether this assumption is true. Figure 4.21 is linear for almost its whole range, only close to the edges, it is not. Next step to determine $n(x)$ is calculating $\alpha(x)$ according to formula 4.13. A plot is shown in the next figure.

![Figure 4.22: $\alpha[\text{deg}]$ as function of $x[\text{mm}]$](image)

This plot is similar to the inverse plot of figure 4.21 because the tangent relation, between $y$ and $\alpha$, is approximately linear for small angles.

Now the derivative of the index refraction profile $\frac{dn}{dx}$ is calculated according to formula 4.16, with $h = 0.024$ mm, the thickness of the LC layer, and $L = 1206$ mm, the distance from the sample to the screen. The refractive index $n(x)$ is calculated by integration according to formula 4.17 and a plot is shown in the following figure.

![Figure 4.23: $n$ as function of $x[\text{mm}]$](image)

The refractive index has a parabolic shape in the center but at the edges it tends to a linear profile.
4.4.3 Comparisons of the lens profiles and the intensity profiles

**Microlens**

In this section the calculated lens shape \( f(x) \) is compared with the theoretical \( f_s(x) \) and the calculated intensity profile \( I_s(y) \) is compared with the measured intensity profile \( I(y) \).

**Lens shape**

In the next figure \( f(x) \) is compared with \( f_s(x) \).

![Figure 4.24: comparison of \( f[mm] \) (blue) and \( f_s[mm] \) (pink) as function of \( x[mm] \)](image)

In this figure, the plotted \( f(x) \) is calculated with more accuracy by taking transmission effects into account when calculating \( I(y) \). Therefore the Fresnel formula for the transmission coefficient is used \([6]\).

\[
T = \frac{1}{2} \frac{n_2 \cos(\theta_2)}{n_1 \cos(\theta_1)} \left[ \left( \frac{2n_1 \cos(\theta_1)}{n_2 \cos(\theta_1) + n_1 \cos(\theta_2)} \right)^2 + \left( \frac{2n_1 \cos(\theta_1)}{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)} \right)^2 \right] \tag{4.21}
\]

with \( \theta_1, \theta_2, n_1, n_2 \) as indicated in figure 4.2.

Figure 4.24 shows that \( f(x) \) coincides with \( f_s(x) \). For this circle-shaped lens, \( f_s(x) \) is known in advance. From \( f_s(x) \) follows that \( \alpha(x) \) and \( y(x) \) are single-valued functions. Under this condition, the method of paragraph 4.3.2 can be followed and so it is possible to derive \( f(x) \) from \( I(y) \).

**Intensity profile**

In the next figure \( I_s(y) \) is compared with \( I(y) \).
Chapter 4. Lens profile measurements

Figure 4.25: $I_{s}[au]$ and $I[au]$ as function of $y[mm]$

The model used to calculate $I_s(y)$ did not take interference effects into account, as a consequence there are no fringes in the profile of $I_s(y)$. But when leaving the fringes out of consideration, the show a satisfactory correspondence. So it is possible to derive $I_s(y)$ out $f_s(x)$.

**GRIN lens G22**

In this section the calculated refractive index profile $n(x)$ is compared with $n_s(x)$ from the simulations and the calculated intensity profile $I_s(y)$ is compared with the measured intensity profile $I(y)$.

**Refractive index profile**

The next figure shows $n(x)$ together with $n_s(x)$.

Figure 4.26: Refractive index $n$ (blue) and simulated refractive index $n_s$ (green) as function of $x[mm]$
The two curves have both a parabolic shape between the two electrodes located at position 0 and 0.166 mm. Above the electrodes the two curves differ, $n_s(x)$ increases when $n(x)$ decreases. The refractive index $n(x)$ can only be derived correctly from $I(y)$ if $n(x)$ leads to a single-valued function $\alpha(x)$. This is not the case for $n_s(x)$ and for the real $n(x)$ of the GRIN lens, it is not known beforehand. But because only a small part of the light is refracted to the same positions on the screen, the curve obtained for $n(x)$ gives a good estimation of the refractive index profile of the lens. The deviation of the calculated $n(x)$ from the simulated in the middle of the GRIN lens ($x = 0.83 \, \mu m$) is $1.782 - 1.7659 = 0.0161$. This is only a deviation of 1%. The deviation in the region above the electrode, [0 mm; 0.015 mm] and [0.155 mm; 0.166 mm] is larger. So for the GRIN lens, $n(x)$ could be derived from $I(y)$, with a precision of 1% for the region between the electrodes. The region above the electrodes could not be characterized accurately.

**Intensity profile**

The next plot shows $I(y)$ together with $I_s(y)$.

![Figure 4.27: Comparison of $I[au]$ (blue) and $I_s[au]$ (green) as function of $y[mm]$](image)

The two curves are both defined in a compact range and are, very roughly, both rectangular functions. The calculated profile $I_s(y)$ does not have fringes because the interference effects were not taken into account as discussed in section 4.4.2. The width of the intensity profile $I_s(y)$ is smaller. This is explained by the inaccuracy in thickness of the LC-layer. The thickness $h$ of the LC-layer, $h = 24 \, \mu m$ for sample G22, is very difficult to determine.
exactly. This is because the GRIN lens consists of different dielectric layers. The dielectric material is soft in comparison with the spacers used in the LC. This causes the spacers to penetrate in the dielectric layer, resulting in an unknown thickness of the LC. Measuring the thickness of the LC layer with a microscope is very difficult, because it is difficult to distinguish the different layers.

The thickness of the LC $h$ is related with $I(y)$ as follows:

$h$ is related with $\alpha(x)$ according to formula 4.6. $\alpha(x)$ is related with $y(x)$ according to formula 4.9 and this $y(x)$ is related with $I(y)$ according to formula 4.8.

Approximating the sine and tangent function as a linear function in formula 4.6 and 4.9, the relation between $y$ and $h$ is made,

$$y \approx L \cdot h \frac{dn}{dx}. \quad \text{(4.22)}$$

To let the two curves of figure 4.27 coincide, $y$ has to be rescaled with a factor 1.4.

Using formula 4.22, the corrected thickness $h^*$ is estimated by multiplying $h$ with 1.4,

$$h^* = 1.4 \cdot h = 1.4 \cdot 24 \mu m = 33.6 \mu m.$$

The angular profile $\alpha_s^*(x)$ for a GRIN lens with $h^* = 33 \mu m$ is simulated in the program Shintech and the same calculations as in paragraph 4.3.1 are done to obtain $I_s^*(y)$. $I_s^*(y)$ is plotted in the next figure, together with $I_s(y)$ calculated for a GRIN lens with $h = 24 \mu m$ and the experimental $I(y)$.

![Graph showing comparison of light intensity](image)

**Figure 4.28:** comparison of $I[au]$, $I_s[au]$ and $I_s^*[au]$ with $h^* = 33 \mu m$ as function of $y[mm]$

For the corrected thickness $h^*$, the profile of $I_s^*(y)$ coincides very good with $I(y)$. 
A last remark about figure 4.27 is that there are no experimental values outside the region of $[-200\, \text{mm}; \, 200\, \text{mm}]$. This is because the camera can not measure low intensities, without going in saturation for the high intensities in the middle. The light with low intensity that is refracted by the lens at large angles and projected on the screen far away from the middle, is seen by the eye, but not measured by the camera. It may be worthwhile to measure this light, since it contains information on what happens above the electrodes. This is because the intensity in the middle of figure 4.27, with the highest intensity, comes from the part of the GRIN lens between the electrodes. The light with low intensity, spread out away from the middle, comes from the region of the GRIN lens above the electrodes as illustrated in figure 4.13.

### 4.5 Conclusions

This experiment measures the intensity $I(y)$ projected on a screen by an incident parallel laser beam on different samples: a microlens and a GRIN lens. For a microlens it is possible to calculate the intensity $I(y)$ if the lens shape $f(x)$ is known. Also backwards $f(x)$ can be calculated if $I(y)$ is measured. So the lens shape can be calculated accurately from the measured intensity.

For a GRIN lens it is possible to calculate the projected intensity $I(y)$ if the refractive index modulation $n(x)$ is known, for example from simulations. Comparing the calculated with the measured intensity, shows that the simulated $n_s(x)$ leads to a similar intensity profile. Backwards $n(x)$ can be derived from the measured $I(y)$, with a precision of 1% for the region between the electrodes, but above the electrodes this method leads to inaccurate results. This is because the light refracted by the GRIN lens G22 at position $y$ of the screen, can come from the light incident at two or more different positions $x$ on the GRIN lens.
Chapter 5

Experimental thickness of the liquid crystal layer in the GRIN lens

5.1 Introduction

In this chapter the thickness of the LC-layer in the GRIN lens is derived from the voltage-transmission curve. The thickness of the LC-layer is important because it determines the refractive index of the lens. In a previous experiment, the thickness of the LC-layer was measured by looking through a microscope, leading to inaccurate results. From the transmission-voltage curve, the retardation is derived and from the retardation, the thickness of the LC layer in the GRIN lens can be determined.
5.2 Experimental setup

The GRIN lens is placed under a polarization microscope, as shown in the next figure.

![Diagram of polarization microscope and the GRIN sample](image)

**Figure 5.1:** Polarization microscope and the GRIN sample

The light passes through the polarizer, the filter, the diaphragm, the GRIN lens, the analyzer and finally is measured with a photomultiplier. The GRIN lens sample G35 and G22 are placed under the microscope. The voltages applied on the electrodes and the polarization of the incident light are different than in the usual GRIN configuration. The electrode above the LC stays connected to the ground, but on the electrodes under the LC the same voltage is applied. Now the LC acts like a homogeneous material: all the directors have in good approximation the same inclination angle theta. This is because the electric field is in good approximation oriented in the vertical direction over the whole GRIN lens, certainly in the region above the electrode. Therefore the transmission is measured above the electrode. The
inclination angle will depend on the strength of the electric field. The set-up is illustrated in the next figure.

![Diagram of GRIN lens and directors](image)

**Figure 5.2:** The GRIN lens and the directors. The electrodes on which the same voltage is applied are indicated with the orange circles and the thickness of the LC layer with a purple arrow.

In the usual GRIN lens configuration, the perpendicular incident light is linearly polarized in the direction of the long axis of the LC molecule. Then all light is refracted with the effective extra-ordinary index, only the extra ordinary wave propagates through the LC. For this experiment, the linearly polarized light incides perpendicular on the GRIN lens, but the polarization makes an angle with the long axis of the LC molecules, see next figure.

![Diagram of polarization](image)

**Figure 5.3:** Polarization of the incident light

Now, the extra-ordinary and ordinary wave travel through the LC. In the experiment, the voltage applied on the electrodes increases slowly from 0 V to 110 V and then decreases from 110 V back to 0 V, meanwhile the transmission is measured.
Chapter 5. Experimental thickness of the liquid crystal layer in the GRIN lens

5.3 Theory

The directors will align more vertically with the electric field when the applied voltage increases. The effective refractive index changes with the inclination angle of the director. Therefore the retardation, the phase difference between the ordinary and extra ordinary wave, will change. The relation between the refractive index and the retardation will be discussed. Finally, the relation between the retardation and the transmission is used to determine the thickness of the LC layer.

5.3.1 The electrical field and the directors

The next three figures show the possible orientations of the liquid crystal for the LC layer in the GRIN lens, when increasing the voltage. In the next figures the propagation of light and the electrical field are assumed in the z-direction.

![Figure 5.4: LC configurations at different voltages](image1.jpg)

Figure 5.4(a) shows the directors when the voltage is below the threshold voltage $V_{th}$. The threshold voltage is the voltage necessary to start the rotation of the directors. All directors are horizontally aligned, as a consequence of the pre-tilt. The inclination angle $\theta$ is defined as the angle between the propagation direction of the light and the director, which in this case is $\theta = \frac{\pi}{2}$.

In figure 5.4(b) the electrical field is higher than the threshold voltage, but lower than the saturation voltage $V_{sat}$. The saturation voltage is the lowest voltage necessary to have the directors completely aligned with the electrical field. The directors tend to align with the electrical field and start to rotate with an inclination angle $\theta$, $\frac{\pi}{2} > \theta > 0$. Above the electrodes, the field is assumed homogeneous and all directors have the same inclination angle $\theta$. The exact function $\theta(V)$ can be derived from simulations.

When the applied voltage is above the saturation voltage $V_{sat}$, all directors are aligned with the electrical field, as shown in figure 5.4(c). The directors are vertically orientated with $\theta = 0$. 
**5.3.2 Refractive index**

The incident light is perpendicular on the GRIN sample and linearly polarized. Depending on the LC configuration the light has different refractive indices in the LC. The refractive indices are represented by the index ellipsoid, as shown in figure 5.5 [1]. The index ellipsoid is a diagram of an ellipsoid that depicts the orientation and relative magnitude of refractive indices in a crystal [7].

![Index ellipsoid of the LC](image)

**Figure 5.5:** Index ellipsoid of the LC

Figure 5.5(a) illustrates the index ellipsoid of the LC. Light propagating in the z-direction has two components, one along the ordinary axis and one along the extra-ordinary axis. The extra-ordinary wave sees the refractive index $n_e$ and the ordinary wave sees $n_o$.

Figure 5.5(b) illustrates the index ellipsoid of the LC with the extra-ordinary axis in the xy-plane. The azimuth angle $\phi$ is the angle between the polarization direction of the incident light and the optical axis. The extra-ordinary wave sees the refractive index $n_e$ and the ordinary wave sees $n_o$. Because the rubbing-direction of the LC determines the projection of the optical axis on the XY-plane, $\phi$ changes by rotating the sample under the microscope. This situation corresponds with the directors of figure 5.4(a) with the c-axis equivalent to the y-axis.

Figure 5.5(c) illustrates the index ellipsoid of the LC with the extra-ordinary axis not in the xy-plane. The inclination angle, $\theta$, the angle between the direction of propagation and the c-axis, is the same angle as used in figure 5.4(b). The inclination angle $\theta$ causes a change in the extra-ordinary index. The extra-ordinary wave will see $n_{eff}$ given by formula 2.2:

$$n_{eff} = \frac{1}{\sqrt{\frac{\cos \theta^2}{n_0^2} + \frac{\sin \theta^2}{n_e^2}}}$$

(5.1)
Chapter 5. Experimental thickness of the liquid crystal layer in the GRIN lens

This figure corresponds with the configuration in figure 5.4(b).

5.3.3 Retardation

The retardation is the phase-shift between the ordinary and extra-ordinary wave. The retardation $\Gamma$ is defined for a LC-layer with thickness $d$ and for light with wavelength $\lambda$ as

$$\Gamma = \frac{(n_{\text{eff}} - n_o) \, d \, 2 \, \pi}{\lambda}. \quad (5.2)$$

$\Gamma$ is linear in $\Delta n = n_{\text{eff}} - n_o$, the difference in refractive indices.

**If** $V < V_{th}$

The directors are horizontally aligned, $\theta = \frac{\pi}{2}$. Substituting $\theta$ in formula (5.1) gives the effective extra-ordinary index $n_{\text{eff}} = n_e$. Substituting $n_{\text{eff}}$ in formula (5.2) gives the retardation $\Gamma$.

$$\Gamma = \frac{(n_e - n_o) \, d \, 2 \, \pi}{\lambda}. \quad (5.3)$$

**If** $V_{sat} < V$

The directors are vertically aligned, $\theta = 0$. Substituting $\theta$ in formula (5.1) gives the effective extra-ordinary index $n_{\text{eff}} = n_o$, resulting in $\Delta n = 0$ and $\Gamma = 0$ according to formula (5.2). The linearly polarized wave, incident perpendicular on the LC layer, does not see the anisotropy of the LC. As a consequence, the wave does not decompose in an ordinary and extra-ordinary wave and the retardation $\Gamma = 0$.

**If** $V_{sat} > V > V_{th}$

The next scheme gives a short resume of what happens when the voltage is decreased from $V_{sat}$ to $V_{th}$.

<table>
<thead>
<tr>
<th>Voltage $V$</th>
<th>$V &gt; V_{sat}$</th>
<th>$V_{sat} &gt; V &gt; V_{th}$</th>
<th>$V_{th} &gt; V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclination $\theta$</td>
<td>$0$</td>
<td>$0 &lt; \theta &lt; \frac{\pi}{2}$</td>
<td>$\frac{\pi}{2}$</td>
</tr>
<tr>
<td>Effective index of refraction $n_{\text{eff}}$</td>
<td>$n_o$</td>
<td>$n_o &lt; n_{eff}(V) &lt; n_e$</td>
<td>$n_e$</td>
</tr>
<tr>
<td>$\Delta n$</td>
<td>$0$</td>
<td>$\Delta n$</td>
<td>$n_e - n_o$</td>
</tr>
<tr>
<td>Retardation $\Gamma$</td>
<td>$0$</td>
<td>linear with $\Delta n$</td>
<td>maximal</td>
</tr>
</tbody>
</table>

If the high voltage is decreased from $V_{sat}$ to $V_{th}$, $\theta$ will increase from 0 to $\frac{\pi}{2}$. According to formula (5.1) $n_{\text{eff}}$ will increase from $n_o$ to $n_e$ and $\Delta n$ from 0 to $(n_e - n_o)$. The retardation increases linearly with $\Delta n$ reaching a maximum for $\Delta n = n_e - n_o$. The retardation $\Gamma$ for a voltage between the extreme values is given by formula (5.2) and for the threshold voltage by
5.3.4 Transmission

The transmission $T$ for the LC placed between two crossed polarizers is given by the next formula [8].

$$T = |\sin(2\phi)|^2 |\sin(\frac{\Gamma}{2})|^2$$  \hspace{1cm} (5.4)

$T$ is maximal for the azimuthal angle $\phi = \frac{\pi}{4}$. Each time $\Gamma = \pi + k \frac{2\pi}{2}$, a maximum in transmission occurs. This is explained by the following figure.

Figure 5.6: Retardation of the LC-layer

Because of the polarizer, the incident light is polarized along the x-axis. When entering the LC, the light is decomposed in an ordinary and extraordinary wave. If the optical axis has an angle $\phi = \frac{\pi}{4}$ with the x-axis, the ordinary and extra-ordinary wave have the same amplitude. When $\Gamma = \pi$, there is a phase shift of $\pi$ between the ordinary and extra-ordinary wave and, as figure [5.6] illustrates, the outgoing light is polarized along the y-axis. Because crossed polarizers are used all light will be transmitted, resulting in a maximum transmission. Each time $\Gamma = k \frac{2\pi}{2}$, a minimum in transmission occurs.
Again the incident light is polarized along the x-axis. When entering the LC, the light is decomposed in an ordinary and extra-ordinary wave in the same way as in figure 5.6. When $\Gamma = k \frac{2\pi}{\lambda}$, there will be no phase shift and the outgoing light is polarized along the x-axis. Because crossed polarizers are used, all light is absorbed by the analyzer.

5.3.5 Derivation of the thickness from the transmission-voltage curve

For voltages higher than the saturation voltage, corresponding with figure 5.4(c), the retardation $\Gamma = 0$. There is no phase-shift, resulting in a minimum in transmission. Decreasing the voltage will lead towards the first minimum in transmission when $\Gamma = 2\pi$. When decreasing more the voltage, the transmission will have the next minimum for $\Gamma = 2 \cdot 2\pi$, $\Gamma = 3 \cdot 2\pi$, ..., $\Gamma = k \cdot 2\pi$ ($k \in \mathbb{N}$). The retardation $\Gamma$ reaches his maximum at $\Gamma = r \cdot 2\pi$ ($r \in \mathbb{R}$) when $V = V_{th}$ and remains constant for $V < V_{th}$. The maximal retardation is known from how many times the transmission reaches a minimum and the thickness $d$ is derived from formula 5.3.
5.4 Measurements

5.4.1 GRIN G35

The next plot shows the transmission-voltage curve, measured in the middle of the GRIN lens.

![Figure 5.8: Transmission [au] vs Voltage [V] measured in the middle of sample G35](image)

The pink curve is the transmission measured when the voltage increases, the blue curve is the transmission when the voltage is decreasing. It is important that the curve shows all minima, so that $\Gamma$ is derived correctly from the figure. Before this sample was measured, it was examined by looking at the transmission through the microscope. This showed that the transmission remained constant when increasing the voltage above 110 V so that $V_{\text{sat}} = 110$ V.

The threshold voltage $V_{\text{th}}$ is derived from figure 5.8 as the voltage for which the first change in transmission occurs. This is when $V_{\text{th}} = 17$ V.

The transmission at 0 V can be every percentage of the total transmission. This is because the GRIN lens acts as a retarder when no voltage is applied and the transmission will depend on the thickness of the LC layer.

The maxima and minima do not reach every time respectively 100% and 0% of the total transmission. This can be a consequence of the birefringence in the foil that is used in the dielectrical layer of the GRIN lens.

When $U \approx 20$ V, the curve shows hysteresis.

At $U \approx 75$ V the transmission becomes saturated. This can be avoided by calibration of the microscope.
To obtain the retardation, the minima that occur in the transmission are counted, as illustrated in figure 5.9.

![Figure 5.9: Transmission [au] vs Voltage [V] and the transmission minima](image)

The counting starts from the transmission when $\Gamma = 0$, this is for $V_{sat} = 110 \text{ V}$, and ends with the transmission when the retardation is maximal, this is for $V_{th} = 17 \text{ V}$. The retardation $\Gamma = 6.5 \, 2\pi$ is counted. The 0.5 comes from the fact that below $V_{th}$ the transmission is still close to maximum. Substituting $\Gamma$ and the constants, $\lambda = 480 \times 10^{-9} \text{ m}$, $n_e = 1.7659$ and $n_o = 1.5271$, in formula 5.3 gives the thickness $d$,

$$d = 13.1 \mu \text{m}.$$  

To investigate if the thickness of the LC layer is the same through the whole GRIN lens, the transmission-voltage curve is measured at different positions of the GRIN lens and the thicknesses are derived. The thickness of GRIN lens G35 seemed to be somewhat smaller (less than 7%) close to the edges of the sample.

### 5.4.2 GRIN G22

When looking through the microscope, GRIN lens G22 seemed damaged. Most of the line electrodes were not continuously straight and the lens showed irregular structures. Unfortunately it was not feasible to measure the transmission-voltage curve.

### 5.5 Microscopic view of GRIN lens G35

The next picture shows the transmission of GRIN lens G35 in the normal GRIN configuration, when an opposite voltage is applied at the electrodes.
Figure 5.10: Microscopic view of the GRIN lens at low voltage (left) and high voltage (right). The position of the line-electrode is indicated with the red arrow.

In figure 5.10 (left) the voltage is lower than $\approx 50$ V. The thick dark line in the middle indicated with the red arrow is the low transmission above the line-electrode. The closer to the line electrodes, the more dark lines appear, indicating the minima in transmission. This is because close to the electrode, the field and the directors change the most, resulting in a fast change in retardation and transmission.

In figure 5.10 (right), the voltage is higher than $\approx 50$ V. The transmission above the electrodes is not invariant along the line electrodes, but a zig-zag pattern appears. The voltage at which this zig-zag pattern appears can also be higher or lower than 50 V. For example when the voltage is decreased from 100 V, this zig-zag pattern disappears only at $\approx 30$ V. The simulations done in chapter 3 assumed that along the line electrodes the director configuration did not change. Because the transmission depends on the director orientation, it is derived from this picture that the director configuration does change along the line-electrodes. The zig-zag configuration of the LC is contributed to the fact that the energy needed for twist (constant $K_2$) is smaller than the energy needed for bend (constant $K_3$) and splay (constant $K_1$). In paragraph 3.3.2, the minimal energy is obtained for a director configuration with only bend and splay. This bend and splay are reduced by twisting the molecules, resulting in a lower minimal energy for a director configuration which has a zig-zag pattern along the line electrode.
5.6 Conclusions

In this experiment the transmission-voltage curve is measured of GRIN lenses G35 and G22. For GRIN lens G35 a thickness of 13.1 $\mu m$ is found. Looking through the microscope at the transmission above the line electrodes, the quality of the sample can be estimated. Almost all line electrodes in sample G35 were functioning, while for sample G22 most of them were damaged.
Chapter 6

Conclusions

A model of the GRIN lens based on simulations, with LC interaction is given. The electrical field and the director configuration in the LC layer are determined. The model predicts that the directors align between the electrodes as desired. Above the electrodes the directors are oriented horizontally at voltages of 100 V or lower. At voltages of approximately 300 V there is complete alignment.

From the director configuration the refractive index and the refraction angle of the GRIN lens are derived. Between the electrodes, the refractive index is parabolic and the refraction angle is linear. Above the electrodes, the refractive index and the refraction angle deviate from respectively the parabolic and linear profile. At voltages of 100 V or lower, the deviations are very large due to bad alignment of the LC. At voltages higher than 100 V, by complete alignment of the LC, the refractive index and the refraction angle are not as desired because of the non-optimal shape of the electric field.

To optimize the shape of the electrical field, in order to increase the useful area of the GRIN lens, the influence of free parameters of the GRIN lens are studied. The useful area of G22 can be improved by decreasing the dielectric constant of the dielectric layer below the LC from $59 \, \mu m$ to $40 \, \mu m$ or increasing the dielectric constant of the dielectric layer below the LC from $\epsilon = 3.4$ to $\epsilon = 5$. The insight given by the simulations of the free parameters of the GRIN lens is useful to increase the useful area of GRIN G35.

The refractive index of conventional lenses is determined very accurately with an experiment. For GRIN lenses the refractive index was measured with a precision of 1% for the region between the electrodes, but above the electrodes this method could not determine the refractive index.

The problem of the inaccurate thickness of the LC layer has been solved. The thickness of the LC layer is determined experimentally by measuring the transmission as function of the voltage of the GRIN lens. For GRIN lens G35 a thickness of $13.1 \, \mu m$ is found.
Bibliography


