Design and Advanced Control of a Process with Variable Time Delay

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Master’s thesis submitted to obtain the degree of Civil Engineer in Mechanics and Electrotechnics

Academic Year 2005-2006
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1 June 2006
Preface

One year ago, the word ‘thesis’ still had something mythical, but now, here it is. For months we have been shaping it, questioning it and fostering it. Today, we are proud of it.

It was a pleasure to work on this subject as it challenged us on various fields. From the control engineering point of view, since variable time delay problems are not easily addressed with today’s most common controllers. From the practical point of view, as the process had to be designed and realized. And, finally, from the personal point of view, since it took a lot of effort and perseverance to take the thesis to what it is now. Practice never lies.

We aimed to shape and structure this thesis to lead you along the different phases we walked through, the numerous problems and challenges we experienced, and the solutions we contrived. At the same time we hope to contribute our mite to the research on and improvement of controllers for processes with variable time delay.

Despite our enthusiasm, the final realization would not have been possible without the help of a few persons, to whom we owe our thanks:
our promoter, Prof. dr. ir. R. De Keyser, for giving the opportunity to do research on such an interesting subject, for his proposals and suggestions to solve the more difficult problems we experienced;
our thesis guide, ir. Mihaela Sbarciog for her daily guidance, her effort, her support and her enthusiasm, for providing us with the tools we desperately needed, for the useful tips she gave us and for the correction of the text;
and Prof. dr. S. Cristea, for her interest in our thesis.

We also have to thank a lot of other people who made a contribution in one or another way. In particular our special thanks to Mr. Pascal Ghyselbrecht for helping us with the design and the selection of appropriate devices, for his crucial tips and for guiding us through the practical implementation; and to Mr. Marc Deboeck from Lameco NV for his patience even after numerous phone calls, for his support in the search for a suitable pump and for his special effort to make it affordable.

And finally, to our parents, our family and our friends: thanks a lot for your support!
Survey

Design and Advanced Control of a Process with Variable Time Delay

by

Steven Himpe and Vincent Theunynck

Master’s thesis submitted to obtain the degree of Civil Engineer in Mechanics and Electrotechnics, option Control Engineering and Automation

Academic Year 2005-2006

University of Ghent
Faculty of Engineering

Promoter: Prof. dr. ir. Robin De Keyser
Thesis Guide: ir. Mihaela Sbarciog

Summary

In this thesis a test setup for a process with variable time delay is designed and implemented. The process under consideration is a heating tank whose water temperature has to be controlled. A constant heat input is provided by a heater which causes the water to warm up. The actual control is done via an adjustable inflow of cold tap water while an equal outflow of hot water is removed from the tank in order to have a constant volume. The variable time delay is obtained by measuring the temperature in the outlet tube, at a certain distance from the tank instead of in the tank itself. After the construction of the plant, an advanced controller of the MPC-type (Model Based Predictive Control) is programmed.

The first part of this thesis discusses the design and implementation of the setup. The second part handles the advanced controller and research is done on how to adjust the MPC-controller to deal with the variable time delay problem.

Keywords: Variable time delay, non-linear process, Model Based Predictive Control (MPC), Non-linear Extended Prediction Self-Adaptive Control (NEPSAC), Smith predictor, PI-controller
Extended abstract

(English and Dutch version)
Abstract—This article handles on one hand the design and implementation of a setup to create a process with variable time delay, and on the other hand an advanced control strategy for it, using a model predictive control (MPC) approach developed at Ghent University: EPSAC (Extended Prediction Self-Adaptive Control).

Keywords—variable time delay, MPC, EPSAC, Smith predictor

I. INTRODUCTION

Variable time delay processes pose a challenging problem in control engineering. A time delay of the order of the time constant of the system, combined with a variation of it, requires an inventive control strategy to fulfil certain performance requirements. As there are many practical problems involving variable time delay in industry (e.g. steel rolling processes or paper manufacturing), it would be interesting to have an experimental setup with variable time delay, to test and develop new control strategies on it.

The proposed setup is a water tank whose temperature has to be controlled. Inside the tank there is a heater which spreads a constant amount of heat (so no manipulation of the heat input), and causes the water to warm up. A certain temperature value can be reached by an outflow of (hot) water, which is compensated with an equal inflow of (cold) tap water. In this way, the volume of water within the tank remains constant (Fig. 1). The variable time delay is obtained by placing the temperature sensor not inside the tank, but in the outlet tube, at a certain distance. Due to the varying flow, the time the water needs to reach the sensor is also variable.

Fig. 1. Process with variable time delay: a heating tank system

After the implementation of this setup, a controller of the MPC type will be programmed, with adaptations to accommodate the variable time delay. Additionally, the effects of controller parameters on the system’s performance will be investigated.

II. DESIGN AND IMPLEMENTATION OF THE SETUP

A. Requirements for the setup

The time delay of the process should be of the same order of magnitude as the time constant. The closed loop time constant has to be small enough (≈ 1 min) and an input change should result in a significant output change. These requirements limit the range in which the parameters that are involved (vol. V, heat input $Q_h$, outlet tube vol. $L_S$, max. flow $q_{max}$) can be chosen.

A float switch-guarded level provided the best outcome to obtain a constant water volume. With the several components known (float switch, 1100 W heater and mixer), a minimum built-in tank volume of 1.13 l was constructed out of plexiglas. The outlet tube (9.5 m long, $L_S=1.02$ l) is insulated to reduce heat loss. With a maximum flow of $±2$ l/min, a temperature range between $±25$ and $45°C$ is obtained.

B. Implementation

The best way to manipulate the flow is using a peristaltic pump, because it is an active actuator and the flow can be calculated via the pump’s rotation speed. A pump with DC-motor provides the easiest way to control the pump speed: the drive that is used to control the motor has IxR feedforward compensation to obtain an approximative linear relation between control voltage and motor speed (and thus flow). Besides the pump and motor drive, DC-voltage sources are needed for the pump and for the measurement circuits. To read in the data of the measurement, an appropriate DAQ-board is chosen.

C. Temperature Measurement

The temperature is measured at three points: in the inlet tube, in the tank, and at the end of the outlet tube. The commonly used in industry Pt100 serves as sensor. A wheatstone bridge and amplifier convert the temperature dependent resistance to a 0-10 V range. Passive RC filters are used to remove noise from the signals, and a digital filter is implemented on the computer. A calibration of the temperature measurements results in a maximum error of 0.4°C for the outlet temperature, and 0.8°C for the inlet temperature.

III. ADVANCED CONTROL

A. Modelling and Identification

A.1 Water tank

An energy balance results in a non-linear first order differential equation for the temperature dynamics in the water tank:

$$V \frac{dT(t)}{dt} = \frac{Q_h}{\rho c_p} + q(t) (T_{in}(t) - T(t))$$  \hspace{1cm} (1)
Discretization of this equation with respect to the sampling period $T_s$ provides a good model to predict the tank temperature.

A.2 Outlet tube

A theoretic approach leads to a model that is too complex to be used in the controller, therefore it is assumed that the outlet tube can be approximated by a first order transfer function

$$\frac{K}{RCs + 1}$$

with gain $K$ and time constant $RC$ (combination of thermal resistance and capacitance). Due to the insulation there is almost no heat loss, resulting in a gain of $\pm 0.99$. Via graphical identification the time constant $RC$ is estimated as 29 sec. With this simple model, acceptable results are obtained, although not ideal.

A.3 Variable time delay

The flows of the past are stored in the computer and used to calculate the actual time delay $d$ via

$$\int_{t-d}^{t} q(\tau) d\tau = LS$$

The obtained result is adjusted to incorporate the reaction time of the sensors.

B. MPC via the EPSAC algorithm

In MPC, the models are used to predict future outputs over a prediction horizon. With an appropriately chosen noise model, predictions of future disturbances are calculated and added to the output predictions. The obtained result is used in an optimal control input calculation by minimizing a cost function.

Besides the default noise model which can only deal with steady state errors, an improved one was implemented resulting in even less overshoot ($0.25^\circ C$). The closed loop time constant is not longer than 1 minute.

Finally, as a reference, a genuine PI-controller and a PI-controller in the Smith predictor strategy were also implemented. The genuine PI-controller has severe problems in controlling the process, while the Smith predictor is only comparable to EPSAC within its linearization region.

IV. CONCLUSIONS

This article presented first the design and implementation of an experimental setup for a system with a long and variable time delay. Hereby, the key principles and devices for a good working system were briefly discussed. The second part stated how an MPC-based controller can be adapted in order to deal efficiently with this complex control problem. The advanced control technique has proven its superiority over the genuine PI-controller and the PI-controller in the Smith predictor strategy.

ACKNOWLEDGMENTS

The authors would like to acknowledge the suggestions of many people, but in particular Prof. dr. ir. R. De Keyser and ir. M. Sbarciog.

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Ontwerp en Geavanceerde Regeling
van een Proces met Variable Dode Tijd

Steven Himpe, Vincent Theunynck
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Abstract—Dit artikel behandelt enerzijds het ontwerp en de implementatie van een testopstelling voor een proces met variabele dode tijd, en anderzijds een geavanceerde regelstrategie ervoor, gebruik makend van een ‘Model Predictive Control’ (MPC) methode ontwikkeld aan de Universiteit Gent: EPSAC (Extended Prediction Self-Adaptive Control).

Keywords—variabele dode tijd, MPC, EPSAC, Smith predictor

I. INLEIDING

Processen met een variabele dode tijd vormen een grote uitdaging in de regeltechniek. Een dode tijd van dezelfde grootteorde als de tijdconstante van het systeem, gecombineerd met een variatie ervan, vergen een inventieve regelstrategie om te voldoen aan bepaalde performantie-eisen. Gezien de vele praktische toepassingen in de industrie (walsprocessen of papierproductie bvb.) zou het interessant zijn om een experimentele opstelling met variabele dode tijd te bouwen om regelstrategieën op te testen en te ontwikkelen.

De opstelling bestaat uit een watertank waarvan de temperatuur moet geregeld worden. In de tank bevindt zich een verwarmer die zorgt voor een constante warmtetoevoer (deze wordt dus niet gemanipuleerd), waardoor het water wordt opgewarmd. Een zekere temperatuur kan worden bereikt door het warme water te laten wegstromen, en evenveel koud leidingwater te laten binnenstromen. Op deze manier blijft het watervolume in de tank constant (Fig. 1). De variabele dode tijd wordt verkregen door de temperatuursensor niet in de tank, maar in de uitgaande leiding op zekere afstand van de tank te plaatsen. Aangezien het debiet varieert, is de tijd die het water nodig heeft om naar de sensor te stromen eveneens variabel.

II. ONTWERP EN IMPLEMENTATIE VAN DE OPSTELLING

A. Vereisten voor de opstelling

De dode tijd van het proces moet van dezelfde grootteorde zijn als de tijdconstante. De gesloten kring tijdconstante moet klein zijn (≈ 1 min) en een ingangsverandering moet resulteren in een voldoende uitgangsverandering. Deze vereisten limiteren de keuze range van de betrokken parameters (tankvol. V, warmtetoever Qh, leidingvol. LS, max. debiet qmax).

Een mechanische niveauregelaar bleek de beste uitkomst voor een constant watervolume. Na keuze van de componenten (niveauregelaar, 1100 W verwarmer en mixer) werd een minimum inbouwvolume van 1.13 l gehaald voor de plexiglazen tank. De uitgaande leiding (lengte 9.5 m, vol. 1.02 l) is geïsoleerd om warmteverlies te reduceren. Met een maximaal debiet van 2 l/min wordt een temperatuur range van ±25-45 °C gehaald.

B. Implementatie

De beste manier om het debiet te manipuleren is via een peristaltische pomp. Met deze actieve actuator kan het debiet berekend worden uit de rotatiesnelheid. Een pomp met DC-motor is het makkelijkst om de snelheid te regelen: de motordrive is voorzien van IxR compensatie om benaderend een lineair verband te verkrijgen tussen regelspanning en motorsnelheid (en dus debiet). Naast de pomp en drive zijn ook DC spanningsbronnen nodig voor de pomp en de meetcircuits. De meetdata worden ingelezen met een geschikt DAQ-bord.

C. Temperatuurmeting

De temperatuur wordt gemeten op drie plaatsen: in de ingaande leiding, in de tank, en op het eind van de uitgaande leiding. De industriële veelgebruikte Pt100 dient als sensor. Een wheatstone bridge met versterker zet de temperatuurschommeling weerstand om in een 0-10 V range. Passieve RC filters verwijderen de ruis op de signalen. Een extra digitale filter wordt in de pc geïmplementeerd. Een calibratie van de temperatuurmeting resulteert in een maximale fout van 0.4 °C voor de uitlaattemperatuur, en 0.8 °C voor de inlaattemperatuur.

III. GEAVANCEERDE REGELING

A. Modellering en identificatie

A.1 Watertank

Een energiebalans resulteert in een niet-lineaire eerste orde differentiaalvergelijking voor de temperatuurdynamica:

\[ V \frac{dT(t)}{dt} = \frac{Q_h}{\rho c_p} + q(t) (T_{in}(t) - T(t)) \]  

Discretisering van deze vergelijking met sampling periode Ts leidt tot een goed model om de tanktemperatuur te voorspellen.

Fig. 1. Proces met variable dode tijd: een verwarmingstank

Na de implementatie van deze opstelling wordt een regelaar van het MPC-type geprogrammeerd, met aanpassingen om tege-
A.2 Uitgaande leiding

Een theoretische behandeling leidt tot een voor de regelaar te complex model. Daarom wordt aangenomen dat de uitgaande leiding kan benaderd worden met een 1ste orde transferfunctie

$$\frac{K}{R C s + 1} \quad (2)$$

met statische versterking $K$ en tijdsconstante $RC$ (combinatie van een thermische weerstand en capaciteit). De versterking bedraagt ±0.99 door het beperkte warmteverlies. Via grafische identificatie wordt de tijdsconstante geschat op 29 sec. Dit eenvoudige model levert aanvaardbare resultaten, hoewel niet ideaal.

A.3 Variabele dode tijd

De debieten uit het verleden worden opgeslagen in de computer en gebruikt om de actuele dode tijd $d$ te berekenen via

$$\int_{t-d}^{t} q(\tau) \, d\tau = LS \quad (3)$$

Dit resultaat wordt gecorrigeerd door de reactietijd van de sensoren in te caluleren.

B. MPC via het EPSAC algoritme

In MPC worden de modellen gebruikt om toekomstige uitgangen te voorspellen over de ‘prediction horizon’. Met een gepaste keuze van ruismodel worden voorspellingen van de toekomstige ruim hieraan toegevoegd. Hieruit volgt een berekening van de optimale regelingang door een kostfunctie te minimaliseren.

De variabele dode tijd brengt complexe problemen met zich mee indien de normale strategie wordt toegepast. Toekomstige dode tijden zouden dan ook moeten voorspeld worden, en bepaalde parameters worden ook variabel. Verder zijn er vreemde effecten als het ‘overslaan’ van samples of eenzelfde voorspelling over meerdere samples. Om tegemoet te komen aan de variabele dode tijd wordt een regelschema gebaseerd op de Smith predictor gebruikt (Fig. 2). Door de dode tijd en de dynamica te ontkoppelen verdwijnen de bovenstaande problemen.

Een sampling tijd van 4 seconden bleek een goed compromis tussen nauwkeurigheid en berekeningstijd. Onderzoek naar de invloed van de ‘prediction horizon’ $N_2$ (Fig. 3) toonde aan dat $N_2 \cdot T_e$ een voldoende deel van de dynamica moet bestrijken, en dus gerealiseerd is aan de (hoge) tijdsconstante. Aangezien het proces vrij lineair is binnen kleine temperatuur ranges blijft het positieve effect van niet-lineaire EPSAC zeer beperkt.

De auteurs wensen hun dank te betuigen aan vele mensen voor hun suggesties, en in het bijzonder aan Prof. dr. ir. R. De Keyser en ir. M. Sbarciog.

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Chapter 1

Introduction

In control engineering, systems with time delay have always required extra effort to design an adequate controller. A delayed reaction to the applied inputs might cause a control output which is too fierce, resulting in a high overshoot after the time delay has passed. The best example one could give to illustrate the difficulties in controlling such processes comes from every-day life: a person who wants to take a shower. The person will manipulate the hot water tap in his attempt to reach the optimal temperature; however the effects of his control actions will only be experienced after the water has flowed through the water tube (transport delay), and touches his skin, which acts as a sensor. The person, who obviously wants to reach the optimal temperature as soon as possible, will therefore be inclined to turn the valve too fiercely, with too cold or too hot water as a result.

Another example of a system with time delay is a steel rolling process (Figure 1.1). The hot rolling mill consists of two opposing rollers that are used to flatten hot steel into uniform sheets. At a certain distance from the rollers, an optical sensor measures the thickness of the outcoming plate.

![Figure 1.1: Schematic representation of a steel rolling process](image)

In order to have a correct thickness, the control uses this measurement to adjust the force applied on the roller that is not fixed. The effect of this pressure change will however only be noticed after a certain time, depending on the roller speed. Hence, we face again a problem
with time delay. Considering the high quality requirements of steel plates, there is a lot to gain from an adequate control.

A last example is that of pharmacokinetics, a discipline in which the time course of the drug concentration throughout the body, and its relation to the organism or system, is investigated and modelled. When medicine is administered to a patient, it takes a certain time before an organ experiences the influence of the drug, while that might be faster or slower for another organ. It is obvious that the time delay is an extremely important factor here, for example in the case of anaesthesia, where an error can have life-threatening consequences. Moreover, it is an extremely difficult parameter to model, since it might change from person to person, from organ to organ, and even be dependent on the moment.

**Trends in controlling systems with time delay**

Problems with time delay require in a sense a certain form of patience and/or knowledge of the process. During years, many solutions have been postulated to achieve good control performances, amongst which the most important ones are:

- ‘De-tuning’ of a PID-controller:
  In this strategy the sensitivity of the PID-controller is being diminished. This is done mainly by reducing the integrator-action: the integrator is responsible for a continuous increase of the controller output as long as there is a difference between setpoint and process output. This result is reinforced by the slow response due to the time delay. By ‘de-tuning’, this influence is reduced, with a positive effect on the controlled process. In a famous paper from 1942 [24], Ziegler and Nichols suggested that the best option is to reduce the integration constant by a factor of $1/d^2$, where $d$ represents the time delay of the process. The proportional action should be reduced with a factor $1/d$, while the differential action should not be changed.

- Smith predictor:
  In the Smith predictor control strategy [17] - proposed by Otto Smith in 1957 - the controller is provided with ‘foreknowledge’ of the time delay. First, one tries to develop a mathematical model of the process, in which the dynamics are decoupled from the time delay (Figure 1.2). Then the output of the model (with time delay) is compared to the real process output in order to have an estimation of the noise. This noise is added to the output of the model without time delay, the result is fed back and compared to the actual setpoint. The error is then supplied to a simple controller (a PID for instance), tuned as if there was no time delay at all.
• Model Based Predictive Control (MBPC):
  This approach - which is also based on a mathematical model of the process - has proved its effectiveness in problems with time delay as well. As this control strategy will be explained extensively in the following chapters, only the main ideas are given in this introduction [3]: using a process model, predictions of the future process output are calculated based on past inputs and outputs and postulated future inputs. A cost function is minimized, in order to determine the future optimal control sequence. The receding-horizon control mechanism is used to introduce feedback into the optimization problem. Out of the various MBPC-algorithms, the non-linear EPSAC-method (Extended Prediction Self-Adaptive Control) - developed at the Control Engineering department of the University of Ghent - will be implemented.

The problems with time delay (and their solutions) mentioned above get an extra dimension when the time delay varies in time. This brings us to the real aim of this thesis: the design and construction of a plant with variable time delay, and the implementation of a NEPSAC-based real-time controller, adapted to cope with the variable time delay. The effectiveness of this controller will be investigated, by comparing it to a de-tuned PID controller and a Smith predictor.

Figure 1.2: Schematic representation of the Smith predictor approach
CHAPTER 1. INTRODUCTION

Process with variable time delay: a heating tank

The process dealt with in this thesis consists of a tank, filled with water. The purpose is to control the temperature of the water. The water volume the tank holds remains constant in time. Inside there is a heater which spreads a constant amount of heat. It is important to stress that the power of the heater is not manipulated. This constant heat input causes the water to warm up, while a mixer takes care of a uniform temperature throughout the tank. The manipulation of the temperature is done by varying the outflow of water at tank temperature, while an equal amount of cold tap water is flowing in (since the volume in the tank has to remain constant).

The variable time delay is now introduced by measuring the temperature in the outlet tube, at a certain distance from the tank. As the flow varies - because it is manipulated - the time the water needs to flow from the tank to the measurement point will also vary. Figure 1.3 presents a schematic of the plant:

![Figure 1.3: Schematic representation of the process](image)

The water temperature is measured in three points:

- just before entering the tank, to register the temperature of the incoming tap water $T_{in}(t)$;
- inside the tank, mainly as a reference for the actual temperature in the tank; this measurement however will not be used by the controller;
- in the outlet tube at a distance $L$ from the tank; this measurement represents the process output $T_d(t)$.

A theoretical model of this plant will be derived later on (see chapter Modelling and identification). We confine ourself in this introduction to the system equations, which are discussed briefly.
The temperature dynamics of the water in the tank satisfy the following non-linear first order differential equation, which is obtained by applying an energy balance on the system:

\[ V \frac{dT(t)}{dt} = \frac{\dot{Q}_h}{\rho c_p} + q(t)(T_{in}(t) - T(t)) \]  

(1.1)

where

- \( V \): the volume of water in the tank;
- \( T(t) \): the temperature of the water in the tank;
- \( \dot{Q}_h \): the amount of heat coming from the heater (constant);
- \( \rho \): the density of water;
- \( c_p \): the specific heat coefficient of water;
- \( q(t) \): the flow which enters/leaves the tank;
- \( T_{in}(t) \): the temperature of the incoming tap water.

The time constant of this process is defined as:

\[ \tau = \frac{V}{q(t)} \]  

(1.2)

By measuring the temperature in the outlet tube with length \( L \) and section \( S \), a time delay \( d \) is introduced, hence the temperature at the measurement point (assuming no heat loss and no dynamics in the tube) can be written as:

\[ T_d(t) = T(t - d) \]  

(1.3)

where the variable time delay depends on the flows applied to the process. If the flow does not change \((q(t)=\text{constant})\), this time delay is given by:

\[ d = \frac{LS}{q(t)} \]  

(1.4)

In order to increase the complexity of the process, which makes it more challenging from the control point of view, the outlet tube should be designed in such a way that the time delay is approximately equal to the time constant of the process.
CHAPTER 1. INTRODUCTION

Content and structure of the thesis

Several subjects will be treated in this thesis. Roughly, the structure can be divided into two main parts: first the Design and implementation of the entire setup, and second the Advanced control of the process.

The first topic in Design and implementation handles the Requirements for the setup (Chapter 2). This chapter introduces conditions the design should fulfill as well as possible implementation solutions. Functional relations between several important parameters are derived, where the parameters are chosen in such a way that the controlled temperature range is maximized and setpoint changes within that region can be reached in a short time. Several designs will be investigated and tested against feasibility, cost and effectiveness.

Next, the Mechanical and electrical implementation (Chapter 3) of the plant is discussed. A first section handles the way the flow is manipulated, discussing the type of actuators that are needed and methods to efficiently control them. Further on, a brief word will be spent on the measurements and the data acquisition. In the end, an overview of the whole setup will be given.

As it is a crucial part for the success of the control, a separate chapter deals with the Temperature measurement (Chapter 4), discussing the type of sensors and requirements for the measurement, the circuitry to convert it to a useful signal and the calibration of the circuits.

With the design and implementation of the plant done, we arrive at the part handling the Advanced Control. In Modelling and identification (Chapter 5) theoretical models of the water tank, the heat loss, and the outlet tube will be derived. Afterwards the results will be tested against experimental data, to tune the models in the identification section.

Having obtained reliable models, Controllers (Chapter 6) can be designed in order to improve the performance compared to the open loop response. In the first section the NEPSAC-algorithm will be explained, with the adjustments to deal with the variable time delay. Afterwards, measurements are performed and the influence of several parameters is examined. In order to have a point of comparison, a genuine PID-controller, and a PID-controller in the Smith predictor strategy are also implemented in the last two sections.

In the last chapter the main results will be summarized and the final Conclusions of this thesis will be drawn (Chapter 7).

Appendix A contains a GUIDE for the Graphical User Interface that is programmed to manage the different controllers and perform experiments in an easy way. Instructions for the correct use of the setup are also given.
Part I

Design and Implementation
Chapter 2

Requirements for the setup

2.1 Choice of parameters

2.1.1 Crucial design parameters

When the idea of the heating tank system was presented, three important demands were to be satisfied:

1. A unit change in the input produces a significant change in the output.
2. The closed loop time constant equals roughly one minute.
3. The time constant of the tank is more or less equal to the transport delay \( t_1 \).

\[
\frac{V}{q} \approx \frac{L}{qS} \quad (2.1)
\]

The first two demands both require the process to be ‘sufficiently sensitive’. This means a change in the flow - which is the controlled variable - should result quite rapidly in a change of the temperature. The third demand relates the volume of the tank to the volume of the outlet tube: both volumes have to be approximately equal.

Intuitively, it is clear that only few parameters are determinant for the process dynamics. For instance, a high power heater will heat the water in the tank very fast, while applying large flows to the process will cause a large temperature drop. The volume of the tank is also important: taking it too large will result in modest changes in temperature. Hence the choice of parameters \( \dot{Q}_h \) (the heat input), \( q_{\text{max}} \) (the maximum flow) and \( V \) (the controlled water volume) influences the sensitivity of the process and, consequently, determines whether or not the first two design requirements are met. Under these considerations, it is obvious that for a good design a more detailed mathematical analysis is necessary.

\(^1\)This equation assumes constant flow \( q \). \( L \) denotes the length of the outlet tube while \( \frac{q}{S} \) is the velocity of the outflow.
2.1.2 Functional relations

To derive functional relations between parameters $\dot{Q}_h$, $q_{\text{max}}$ and $V$, we start from the equation describing the process dynamics:

$$V \frac{dT(t)}{dt} = \frac{\dot{Q}_h}{\rho c_p} + q(t)(T_{\text{in}}(t) - T(t))$$  \hspace{1cm} (2.2)

As the flow $q(t)$ will be the manipulated variable in our control problem, the sensitivity of the process must be interpreted as the net effect a change in the flow has on the tank temperature. Subsequently, the relationship between $q(t)$ and $\left| \frac{dT(t)}{dt} \right|$ has to be examined more closely.

Assuming a tank temperature $T_c$ - where $c$ denotes the current operating point - the maximum temperature drop is reached when applying the maximum flow (the incoming temperature $T_{\text{in}}$ is considered to remain constant):

$$\left( \frac{dT(t)}{dt} \right)_{\text{min}} = \frac{\dot{Q}_h}{\rho c_p V} + \frac{q_{\text{max}}}{V} (T_{\text{in}} - T_c)$$  \hspace{1cm} (2.3)

while zero flow accounts for maximum temperature increase:

$$\left( \frac{dT(t)}{dt} \right)_{\text{max}} = \frac{\dot{Q}_h}{\rho c_p V}$$  \hspace{1cm} (2.4)

By defining a coefficient $\xi$, the maximum cooling rate (2.3) can be related to the maximum heating rate (2.4) at the current operating point:

$$\xi = - \frac{\left( \frac{dT(t)}{dt} \right)_{\text{min}}}{\left( \frac{dT(t)}{dt} \right)_{\text{max}}} = - \frac{\frac{\dot{Q}_h}{\rho c_p V} + \frac{q_{\text{max}}}{V} (T_{\text{in}} - T_c)}{\frac{\dot{Q}_h}{\rho c_p V}}$$  \hspace{1cm} (2.5)

This allows to determine an operating range of temperatures $T_c$ - given by $\xi \approx 1$ - where warming up or cooling down take about the same time. Consequently, temperatures within this range - above or below the current operating point $T_c$ - can be reached in limited time. This is important considering the limited closed loop time constant. Simplifying (2.5) a linear relation between $\xi$ and $T_c$ appears:

$$T_c = \frac{\dot{Q}_h}{\rho c_p q_{\text{max}}} (1 + \xi) + T_{\text{in}}$$  \hspace{1cm} (2.6)

The central operating point is defined as the temperature at which maximum cooling rate (2.3) and maximum heating rate (2.4) have equal absolute values, expressed by $\xi = 1$. It corresponds to the steady state temperature $T_{\text{cc}}$:

$$T_{\text{cc}} = T_{\text{in}} + \frac{\dot{Q}_h}{\rho c_p q_{\text{max}}/2}$$  \hspace{1cm} (2.7)

Thus, the heating power $\dot{Q}_h$ and the maximum flow $q_{\text{max}}$ both determine the position of the central operating point and the slope of the linear characteristic describing the operating range. It is interesting from the control point of view to maximize this range. On one hand, this means maximizing the slope of the linear characteristic, while on the other hand the
central operating point must be lowered to reduce ambient heat loss. The choice of \( q_{\text{max}} \) comes with yet another requirement, that of the minimum time delay.

\[
d_{\text{min}} = \frac{L}{\frac{q_{\text{max}}}{S}}
\]  

(2.8)

It is clear that a small maximum flow will result in huge time delays. However, these time delays have to be limited taking into account the second requirement (closed loop time constant of approximately one minute). As a result, a restriction for \( q_{\text{max}} \) applies:

\[
\frac{LS}{1\text{min}} << q_{\text{max}}
\]  

(2.9)

This formula, together with formulas (2.1), (2.6) and (2.7) provides functional relations between the process’s crucial parameters: the choice of the heat input \( \dot{Q}_h \) appears to be dependent on the choice of \( q_{\text{max}} \), which is in turn related to the volume of the outlet tube \( LS \) and therefore dependent on the volume of the tank \( V \).

### 2.1.3 Conclusion

Figures 2.1 and 2.2 illustrate the use of these functional relations. They show the relative cooling capacity \( \xi \) and the static characteristic for two sets of parameters.

![Figure 2.1](image)

(a) \( \xi \) as a function of \( T_c \)

(b) Static characteristic

**Figure 2.1:** Effect of a not tuned set of parameters on the operating range

Figure 2.1 corresponds to a set of parameters which are chosen independently one of another. Taking into account the huge specific heat of water, the heating power \( \dot{Q}_h \) is very small compared to the volume it heats. This leads to a very small temperature range, as the static characteristic of the system defined by equation (2.2) indicates. Moreover, the variable time delay can not be less than one minute, violating the requirement for the closed loop time constant.
CHAPTER 2. REQUIREMENTS FOR THE SETUP

Changing the parameters with respect to the functional relations presented, a tuned set can be obtained. The operating range is broadened while the minimum time delay is lowered to 30 seconds.

![Figure 2.2: Effect of a tuned set of parameters on the operating range](image)

(a) ξ as a function of $T_e$  
(b) Static characteristic

Table 2.1 summarizes the parameter values we have in mind while designing the setup:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Approximate value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat input $\dot{Q}_h$</td>
<td>1100 W</td>
</tr>
<tr>
<td>Controlled volume $V$</td>
<td>1.2 l</td>
</tr>
<tr>
<td>Maximum flow $q_{max}$</td>
<td>$\approx (2 \cdot V)/\text{min} = 2.4 \text{l/min}$</td>
</tr>
</tbody>
</table>

**Table 2.1**: Estimated parameter values of the setup

It is not clear whether this strategy of tuning parameters will eventually lead to limited closed loop time constants. Everything depends on the control strategy. However, it appears to be a useful tool that allows the design of a process whose input changes cause significant changes in the process temperature.

To conclude, it is important to remark that we must not only focus on the values of the parameters as derived from these functional relations. We must answer also other key questions such as: Which components are on sale and at what price? Does the volume of the tank provide enough space to install the heater and the mixer, and possibly some extra components if necessary?

It turns out the value of the design lies in fact in the harmonization of sometimes contradictory considerations, in order to satisfy the design requirements as much as possible.
CHAPTER 2. REQUIREMENTS FOR THE SETUP

2.2 Design types for constant water volume

2.2.1 Preliminaries

The previous section treated the design from a control point of view. Functional relations between several parameters were presented in order to maximize the operating range. From a practical point of view however, an important assumption has been made: the volume of water is constant, irrespective of the amount of water flowing into or out of the tank. This of course strongly reduces the complexity of the process dynamics.

The question now is how can we do this in practice. The most ‘natural’ approach would be to use an overflow system as illustrated in Figure 2.4.

![Overflow system](image)

**Figure 2.3:** Overflow system

Any inflow of cold water would lead to an overflow of exactly the same amount of hot water to be collected by the outlet tube. The position of the tube attached to the tank’s wall would then determine the level of the water and therefore its volume $Ah$ ($A$ denotes the area of the tank while $h$ refers to the water level).

There is however a problem, which comes with the temperature measurement at the outflow. The outlet tube would only be partly filled with water - and partly with surrounding air. In order to perform reliable measurements of the water temperature, it is necessary to mount the sensor in the way Figure 2.4 suggests$^2$: the cavity would be completely filled with water, avoiding any contact between the sensor tip and the air. As the red arrows indicate though, the water in the cavity is trapped and it is very unlikely to be refreshed. Hence, measurements biased by the presence of air would be excluded at the cost of poor temperature measurements of the outflow. Moreover, not mounting the sensor in counterflow would lead to an increased reaction time of the sensor.

$^2$The type of sensor will be discussed in section 4.1.
In fact, the use of an overflowing tank appears to have serious consequences regarding the temperature measurements. Therefore, it is necessary to think of other designs which combine two important issues:

- The water level is constant, irrespective of the amount of cold water flowing in.
- The outlet and inlet tubes are completely filled at all times.

The following sections deal with several designs that succeed to meet both requirements. The cost and the feasibility of every design are, of course, of great importance. The ones that eventually fulfill all demands will be treated more in depth afterwards.

### 2.2.2 The pressurized tank

**Description**

This design relies on the incompressibility of water to keep the water volume constant.

![Figure 2.5: Pressurized tank](image)

In a closed and sealed tank, the inflow of cold water takes place under supply pressure. Once the tank is fully filled an equal amount will flow out, which leaves the volume of the water to be determined by the tank’s dimensions only. The curb of the outlet tube at the measurement point evacuates all air, leaving just water inside. Thus both requirements are met.

**Discussion**

A closed and pressurized tank provides the advantage of insulating it as a whole, minimizing ambient heat loss. Nevertheless, it is the closing and sealing itself which is the huge drawback of this design. For the mixer, a radial shaft seal must be used, which is subjected to wear and tends to leak at some time. The heater in turn has its own sealing difficulties. If its shape is complex, it becomes very difficult to install it, since the opening in the tank wall has to be made big enough to put it through. Sealing it afterwards is quite hard, leaving aside the problems caused by the high temperature it can reach. Thus, the simple basic principle this design relies on pales at the complexity of its implementation. Therefore, it is rejected.
2.2.3 The communicating vessels

Description

This type of level control uses the time-honoured principle of communicating vessels. The configuration is the same as the one introduced while presenting the overflowing tank system, though it differs in curbing the outlet tube at the measurement point. This insures all air to be evacuated from the tube and creates two communicating water levels: one in tank and one above the sensor. Consequently, an inflow of cold water makes the system react in such a way that it keeps the water levels constant.

By easily repositioning the end of the outlet tube, different levels - and thus volumes - can be obtained. This provides a buffer against wrongly chosen parameter values, when taking the design into practice.

Discussion

The underlying principle of this design is again very simple. Its effectiveness however depends entirely on the dynamics of the communicating vessels. To meet the design requirement of a constant water level, these additional system dynamics should have a time constant of at least an order less in magnitude than the time constant of the heating tank system itself. As the practical implementation of this design does not really impose extra difficulties, its success depends entirely on the behavior of these additional dynamics. Thus further investigations are required.

2.2.4 The relay-guarded level

Description

Electronically controlling the water level does not evoke any physical principle and can therefore be classified as an active level control.

Two electrodes are mounted in the tank wall (Figure 2.7). Underneath the water level, they make contact to connect the NC-relay\(^3\) to the control voltage. As a result the relay interrupts

\(^3\)Normally Closed
the voltage across the electromechanical valve, which in turn closes the supply. No more water flows into the tank. The opposite happens when the level lowers again to interrupt the electrode connection.

This type of - active - control separates level control from flow control. As the level control is situated at the inlet of the process, the flow control has to be at the outlet and active as well.

Still, level control and flow control could be held together when replacing the electrodes with a user-controlled switch. This approach would lead back to the principle of the communicating vessels. The active level control is therefore the main difference between both approaches.

Discussion

This way of controlling has several drawbacks. The first disadvantage is the increase in number of components, which in turn increases the overall cost of the plant. The second one is situated at the level control itself: only if the electrodes were mounted infinitely close to each other a constant level would be insured. As this is practically impossible, some hysteresis must be allowed. Limiting this hysteresis effect introduces a more devastating drawback: the more it is narrowed, the more the activation frequency of the valve is increased, causing water shocks in the supply circuit. Of course, this can not be tolerated. Consequently, these significant drawbacks lead to the rejection of the design.

2.2.5 The float switch-guarded level

Description

As in the previous design, level control is again separated from flow control. The supply tube is connected to a mechanical float switch, which is installed inside the tank. When water flows in, the level rises, causing the float to experience an upward force proportional to the submerged volume. This is the well-known principle of Archimedes. Consequently, the
float chokes the supply through a lever mechanism to eventually cut it off. Every inflow of fresh water would cause more upward force and choke the supply even more. Thus, a stable equilibrium point is reached.

![Figure 2.8: Float switch guarded level](image)

**Discussion**

The effectiveness of this type of level control comes with some requirements that are needed to minimize hysteresis. First of all, the supply must be pressurized. If not, the opening of the choking valve has to be too large before any fresh water flows in (due to head loss). Secondly, the float switch itself should be made in such a way that even a tiny change in the water level causes the float to open the choking valve. Though such a switch is widely used and consequently very cheap, it is still the question whether it can guarantee a constant water level, irrespective of the flow.

A drawback of this design is the built-in volume required. The bigger the float switch, the larger the tank’s volume must be to install the heater, the mixer and the switch. This of course has its consequences on the choice of all other parameters of the process.

The success of the design depends on the supply pressure, the effectiveness of the float switch and its size. Still, implementation is very simple, therefore it deserves to be investigated more thoroughly.

### 2.2.6 Conclusion

Several designs were presented and tested against feasibility, cost and effectiveness. Out of these, only the communicating vessels approach and the float switch-guarded level have a chance to succeed and will be treated in depth in the next sections.

### 2.3 Communicating Vessels

The design relying on the principle of the communicating vessels was presented in the previous section. It was found out that a prerequisite for its success is the time constant of the dynamics...
introduced. When this time constant would not be negligible compared to the time constant of the heating system itself, the dynamics of the communicating vessels would intervene, violating the design constraint of constant water level.

A theoretical model has to be derived in order to establish the relationship between the time constants. The analytical results will be compared to results obtained experimentally. Then conclusions regarding this design can be drawn.

### 2.3.1 Theoretical approach

**Introduction**

Referring to the previous section, Figure 2.9 shows the variables of interest: elevation, velocity and pressure.

- \( p_1, p_2 \): static pressure at both surfaces
- \( w_1, w_2 \): stream velocities
- \( y_1, y_2 \): elevations of the water surface

These quantities may be directly converted to produce mechanical energy. For instance, a certain elevation \( \Delta y \) of the outlet tube end provides the system with potential energy \( \rho g \Delta y A_{tube} \) to trigger off flow in the tube. Hence, water will travel from one side to the other trying to re-establish the static equilibrium. This is exactly what is meant by communicating water levels.

In this application however, the build up of potential energy is not achieved by repositioning the end of the outlet tube, but it is done by adding water to the tank. This increase in the volume must be instantaneously levelled by the reaction of the system.

![Figure 2.9: Streamline for the Bernoulli equation](image)
A streamline is considered to start from the surface of the tank to the end of the outlet tube, indicated by indices 1 and 2 respectively. Along this streamline the famous Bernoulli equation applies:

\[ g\Delta y + \frac{\Delta w^2}{2} + \frac{\Delta p}{\rho} = 0 \]  

(2.10)

where \( \Delta y = y_1 - y_2, \Delta w = w_1 - w_2, \Delta p = p_1 - p_2 \). Bernoulli’s idealistic assumption of constant internal energy is however never achieved in practice. Therefore this equation has to be corrected for total hydraulic head loss \( h_L \):

\[ g\Delta y + \frac{\Delta w^2}{2} + \frac{\Delta p}{\rho} + \frac{h_L}{\rho} = 0 \]  

(2.11)

Head losses are induced to overcome hydraulic resistance. They generally vary with the square of stream velocity and are expressed as a fraction \( \zeta \) of the dynamic pressure:

\[ h_L = \zeta \rho \frac{w^2}{2} \]  

(2.12)

Referring to the stockpile of books that can be found about hydraulic resistance (see e.g. [11]), it is in fact very difficult to identify and estimate all the effects that cause head loss. Every setup has its own geometrical configuration and uses different types of fittings and tube material. Moreover, to prevent the theoretical model to be lost in complexity, some simplifying assumptions have to be made. Hence it is not expected that the model will provide the exact results. Still, it is a useful tool to identify the parameters that particularly affect the dynamics of the system.

The next section identifies two types of head loss:

- head loss at the intake;
- head loss caused by friction in the outlet tube.

First, some general considerations are given to allow the estimation of the corresponding loss factors.

**Estimation of head losses**

*General considerations*

The calculation of head losses in a specific setup requires knowledge of the different dimensions involved. This means that the length and cross section of the outlet tube must be available. Another important parameter in all general flow problems - and especially in loss calculations - is the Reynolds number, defined as:

\[ Re = \frac{w \cdot D_h}{\nu} \]  

(2.13)
CHAPTER 2. REQUIREMENTS FOR THE SETUP

The hydraulic diameter $D_h$ is for circular tubes - as in this case - equal to the tube diameter\(^4\). \(\nu\) denotes the dynamic viscosity of water and \(w\) the stream velocity [23].

At this stage, the dimensions of the outlet tube are themselves parameters of the problem: they must be chosen in such a way that they minimize the hydraulic resistance. Yet, from previous section it is known that head losses decrease with the square of the stream velocity. Furthermore, the hydraulic resistance of a tube varies proportionally with its length. This leads to the choice of a short tube with large cross sectional area.

It is however important to point out that the tube diameter cannot be increased unbounded. In fact this diameter should not be larger than 20 mm, which counts as a practical limit when it comes down to flexibility. Even this extreme value leads to a long tube. For instance, a controlled volume of one liter already needs 3.2 m of tube. Thus, it is recommended to wind the tube in a helical coil, irrespective of its dimensions.

Another important issue is the roughness of the tube. This depends on the type of material and its manufacturing. As plastic tubes will be used in this setup, which are smooth enough, the roughness effect can be ignored.

**Head loss at the intake**

The exact shape of the inlet fitting is not known at this point. Still, an estimation of its loss factor $\zeta$ can be made by seeing it as an abrupt expansion. This type of modelling is motivated by the fact that the fitting is slided into the tube, creating an enlargement of the cross section downstream (Figure 2.10).

The loss factor due to diffusion effects appears as a function of this enlargement [11]:

$$\zeta = \left(1 - \frac{A_0}{A_1}\right)^2$$

where $A_0$ and $A_1$ denote the cross sectional area of the inlet fitting and the tube respectively.

For instance, assuming a cross sectional diameter enlargement of about 20% - which can be the case in practice - the loss factor becomes:

$$\zeta = \left(1 - \left(\frac{1}{1.2}\right)^2\right)^2 = 0.093$$

\(^4\)The hydraulic diameter is defined as $\frac{4A}{\Pi}$, $A$ and $\Pi$ denoting the cross sectional area and the perimeter respectively.

\[\text{Figure 2.10: Cross section enlargement at the fitting}\]
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Resistance of the helical coil

Bending of the flow in curved tubes introduces centrifugal forces directed from the center of the curvature to the outer wall of the tube. They disturb the flow pattern and cause diffusion effects. This increases the hydraulic resistance compared to straight tubes. The resistance will therefore not only be dependent on the Reynolds number, but also on the relative curvature radius $\frac{R_0}{D_0}$ (Figure 2.11) [11]:

$$\zeta = n\lambda \frac{2\pi R_0}{D_h}$$

(2.16)

$n$ equals the number of turns while $\lambda$ introduces the Reynolds dependence:

$$\lambda = f \left( Re, \frac{R_0}{D_0} \right)$$

(2.17)

The function $f$ depends on the range of the parameter $Re \sqrt{\frac{D_0}{2R_0}}$.

![Figure 2.11: Curvature of the outlet tube](image)

Comparison of the time constants

For this particular problem, the extended Bernoulli equation (2.11) can be simplified taking into account the following assumptions:

- $p_1$ and $p_2$ are static pressures equal to the atmospheric pressure, thus they cancel.
- $y_1 - y_2$ expresses the difference in elevation, now denoted as $\Delta y$.
- $w_1$ is the stream velocity at the surface of the tank and therefore equal to zero.
- $w_2$ is the stream velocity at the output and varies in time. It will be referred to as $w(t)$.

With the introduction of the loss factors the model equation becomes:

$$g \Delta y = (1 + \zeta_{\text{intake}} + \zeta_{\text{tube}}) \frac{w(t)^2}{2}$$

(2.18)

The most important parameters governing the problem are the Reynolds number $Re$, the relative curvature $\frac{R_0}{D_0}$ and the dimensions of the tube and the fitting. More complexity is
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introduced as the Reynolds number depends on the stream velocity, which in turn varies with the relative elevation $\Delta y$. This grade of complexity precludes a general analytic treatment of the problem. Nevertheless, with an estimation of the orders of most important parameters, useful conclusions about the behavior of the system can be drawn.

An experiment is considered, in which water is added to the tank at a constant rate. The governing parameters and the estimation of their order of magnitude (denoted by $\Theta$) are given next:

- $D_0 \approx \Theta(0.02 \text{ m})$ with $R_0 \approx 5D_0$. This results in $\sqrt{\frac{D_0}{2R_0}} \approx 0.32$.
- $w \approx \Theta(0.1 \text{ m/s})$ and $D_0$ lead to $Re \approx \Theta(2 \cdot 10^3)$
- $L \approx \Theta(3 \text{ m})$ is the estimated order of the tube length
- The number of turns $n$ is, with these orders of $R_0$ and $L$, approximative 5.

With these orders, the parameter $Re\sqrt{\frac{D_0}{2R_0}} \approx \Theta(600)$ leads to a loss factor $\zeta$ of the helical tube [11]:

$$\zeta_{\text{tube}} = n \lambda \frac{2\pi R_0}{D_0} = n \left( \frac{2}{Re^{0.65}} \left( \frac{D_0}{2R_0} \right)^{0.175} \right) \frac{2\pi R_0}{D_0}$$

(2.19)

$\lambda$ varies with the stream velocity $w$ due to its Reynolds dependence. These variations are however small, thus $\lambda$ is taken constant and equal to 0.01. This leads to $\zeta_{\text{tube}} \approx 1.5$. For the head loss at the intake, equation (2.15) calculates $\zeta_{\text{intake}} = 0.093$ for a cross sectional diameter enlargement of 20%.

Taking into account these losses, equation (2.18) simplifies to:

$$g \Delta y = 2.6 \frac{w(t)^2}{2}$$

(2.20)

Furthermore, the law of mass conservation applies to the tank which acts as a capacitor:

$$A_{\text{tank}} \frac{d\Delta y}{dt} = q_{\text{in}} - q_{\text{out}} \quad \text{or} \quad \frac{d\Delta y}{dt} = \frac{q_{\text{in}}}{A_{\text{tank}}} - \frac{A_{\text{tube}}}{A_{\text{tank}}} w(t)$$

(2.21)

Differentiating equation (2.20), and substituting $\frac{d\Delta y}{dt}$ into (2.21), a non-linear first order differential equation appears:

$$\frac{q_{\text{in}}}{A_{\text{tank}}} - \frac{A_{\text{tube}}}{A_{\text{tank}}} w(t) = 2.6 \frac{w(t)}{g} \frac{dw(t)}{dt}$$

(2.22)

Linearizing this equation around $w_0$ allows an estimation of the time constant of the system:

$$2.6 \frac{w_0}{g} \frac{d\delta w(t)}{dt} - \frac{A_{\text{tube}}}{A_{\text{tank}}} \delta w(t) = 0 \quad \text{or} \quad \tau = \frac{2.6w_0}{g \frac{A_{\text{tube}}}{A_{\text{tank}}}}$$

(2.23)

Different parameters affect the magnitude of this time constant and consequently determine the reaction time of the system.
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The time constant is:

- proportional to the overall head loss, which was expected intuitively.
- proportional to the area of the tank. This is expected too because a large area requires more inflow in order to build up static pressure $\rho g \Delta y$.
- inverse proportional to the area of the tube. This parameter also causes a reduction in head loss of the helical tube.

For example, a cross sectional tank diameter of ten times the diameter of the inlet tube, leads to a time constant $\tau = 2.7 \text{ sec}$ when $w_0 = 0.1 \frac{m}{\text{sec}}$. Comparing this time constant to the one of the heating tank system$^5$, we obtain

$$\frac{\tau_{cmv}}{\tau_{hts}} \approx \frac{V}{q_{out}} \frac{LS}{w_0 S} = \frac{2.7}{3/0.1} = 9\%$$  \hspace{1cm} (2.24)

This ratio indicates that both time constants have a different order of magnitude.

2.3.2 Results

The only way to verify whether the design requirements are met, is to take the design into practice. A setup was built to test the dynamics for two different tube diameters: 10 and 15 mm. For practical reasons, the length was limited to 4 m.

There is a constant inflow of water into the tank. The tube ends with an overflow, eliminating the build up of any counterpressure. The absolute volume, which is a measure of the change in height of the water level, is plotted against time. Figures 2.12 and 2.13 illustrate the theoretical results (in red) and experimental results (in blue):

Figure 2.12: Comparison of results for $D_0 = 10 \text{ mm}$

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$^5$Under the assumption of constant water volume $V = LS$
Analyzing these figures, there appears to be poor matching between theory and experiment. The theory predicts in fact much better results than the ones obtained in practice. Main reasons are:

- Not all types of losses seem to be identified. Besides, the presented formulas estimating the hydraulic resistances are just approximations of the real occurring head loss.
- The theory assumes fully developed laminar flow, which is not the case. This leads to an underestimation of the friction factor.

Even tests with larger diameters - up to 20 mm - were not able to convince.

2.3.3 Conclusion

Based on these results, it is clear that the system does not manage to stabilize the water level of the tank. The total hydraulic resistance requires too much static pressure to be overcome. That is why the design based on the communicating vessel principle is rejected.

2.4 Float switch-guarded level

Rejecting previous design leaves the float-switch guarded level as the only remaining option. This design will now be discussed in depth. First, the most important parameters governing the dynamics of this type of level control are extracted theoretically. The time constant is derived as well. These results will be used to select an appropriate float-switch and finally, an appreciation of the theory is given.

2.4.1 Theoretical approach

As presented in section 2.2, the float-switch relies on the principle of Archimedes to control the water level. When water flows out of the tank, the float goes down, gradually lifting the choking valve at the inlet which is connected to the float via a lever mechanism. Intuitively, the amount of water flowing in depends on the elevation of the valve and the inlet pressure. Figure 2.14 shows the variables of interest:
Again, the Bernoulli equation corrected for head loss (2.11) can be applied along the streamline.

**Estimation of head loss**

Only the head loss of the choking valve has to be estimated. It consists of two contributions [11]:

- a resistance depending on the exact shape of the valve
- a variable resistance depending on the degree of opening

To estimate the corresponding loss factors $\zeta$, the choking valve is modelled as a disk valve without bottom guides (Figure 2.14). The elevation $\Delta h$ is variable and depends on the relative position $\Delta H$ of the float:

$$\Delta h = -n\Delta H$$  \hspace{1cm} \text{(2.25)}$$

where $n$ denotes the ratio of the lever. At the steady state level ($H_0$), $\Delta H$ and thus $\Delta h$ are zero.

The variable resistance coefficient $\zeta_{\text{var}}$ is estimated first. It varies with the square of the elevation:

$$\zeta_{\text{var}} = \frac{0.155}{(\Delta h/D_0)^2}$$  \hspace{1cm} \text{(2.26)}$$

where $D_0$ is the diameter of the opening. Substituting equation (2.25), this coefficient is related to the relative position of the float:

$$\zeta_{\text{var}} = \frac{0.155}{(n\Delta H/D_0)^2}$$  \hspace{1cm} \text{(2.27)}$$
The parameters determining the shape-dependent resistance of the valve are the width of the tray flange $b$ and the opening diameter $D_0$:

$$\zeta_{fix} = 0.55 + 4 \left[ \frac{b}{D_0} - 0.1 \right]$$  \hspace{1cm} (2.28)

With these estimations, the total head loss of the choking valve appears as a function of $\Delta H$:

$$\zeta_{tot} = \zeta_{fix} + \zeta_{var} = 0.55 + 4 \left[ \frac{b}{D_0} - 0.1 \right] + \frac{0.155}{(n \Delta H/D_0)^2}$$  \hspace{1cm} (2.29)

**Comparison of the time constants**

The Bernoulli equation applied along the streamline illustrated in Figure 2.14 can be simplified considering:

- $\Delta p$ as the overpressure of the inflow
- $\Delta y = -(H_0 + \Delta H)$ as the oriented difference in height between the two end points of the streamline
- $\Delta w^2 = w_1^2$ as the difference in dynamic pressure since $w_2$ is approximately zero at the surface of the tank. $w_1$ varies in time and will be expressed as $w(t)$.

Introducing the loss factor, we obtain:

$$\frac{\Delta p}{\rho} + \frac{w(t)^2}{2} = (\zeta_{fix} + \zeta_{var}) \frac{w(t)^2}{2} + g (H_0 + \Delta H)$$

or

$$\frac{\Delta p}{\rho} = (\zeta_{fix} + \zeta_{var} - 1) \frac{w(t)^2}{2} + g (H_0 + \Delta H)$$  \hspace{1cm} (2.30)

Recalling equation (2.21), the change in the tank volume now becomes:

$$A_{tank} \frac{d\Delta H}{dt} = (\pi D_0 \Delta h) w(t) - q_{out}$$  \hspace{1cm} (2.31)

Solving equation (2.30) for $w(t)^6$:

$$w(t) = \sqrt{\frac{2}{\zeta_{tot} - 1} \left( \frac{\Delta p}{\rho} - g (H_0 + \Delta H) \right)}$$  \hspace{1cm} (2.32)

and substituting it into (2.31), with (2.25) taken into account, a non-linear differential equation results:

$$A_{tank} \frac{d\Delta H}{dt} = - (\pi D_0 n \Delta H) \sqrt{\frac{2}{\zeta_{tot} - 1} \left( \frac{\Delta p}{\rho} - g (H_0 + \Delta H) \right)} - q_{out}$$  \hspace{1cm} (2.33)

More complexity is introduced as $\zeta_{tot}$ also depends on $\Delta H$, relationship given by equation (2.29). Nevertheless, to get an idea about the time constant of the system, some simplifying assumptions are made:

---

$^6$Under the assumption of $\zeta_{tot} > 1$ which is certainly the case for small values of $\Delta H$ as equation (2.26) indicates.
1. \( \Delta p \) corresponds to the supply pressure of the water distribution system, typically \( \approx \Theta(2.10^5 \text{ Pa}) \). Hence, \( H_0 + \Delta H \approx \Theta(0.1 \text{ m}) \) is small compared to \( \frac{\Delta p}{\rho} \approx \Theta(20 \text{ m}) \).

2. Estimating the dimensions of the choking valve, \( \zeta_{\text{fix}} \) (2.28) and \( \zeta_{\text{var}} \) (2.26) can be compared to each other, simplifying \( \zeta_{\text{tot}} \). The diameter of the opening is very small, even \( D_0 \approx \Theta(3 \text{ mm}) \) is probably an overestimation. Referring to (2.26), small fluctuations on the water level cause high loss factors \( \zeta_{\text{var}} \). Therefore, taking \( \Delta h \approx D_0^7 \) leads to an overestimation of the loss factor \( \zeta_{\text{var}} \). As this is a conservative approximation, it can be made though.

Hence, the \( \Delta H \)-dependence in \( \zeta_{\text{tot}} \) disappears:

\[
\zeta_{\text{tot}} = \zeta_{\text{fix}} + \zeta_{\text{var}} \approx 0.55 + 4 \left[ \frac{b}{D_0} - 0.1 \right] + \frac{0.155}{n^2}
\]  

(2.34)

and (2.33) simplifies to a linear differential equation:

\[
A_{\text{tank}} \frac{d\Delta H}{dt} = - \left( \pi D_0 n \Delta H \right) \sqrt{\left( \frac{2}{\zeta_{\text{tot}} - 1} \frac{\Delta p}{\rho} \right) - q_{\text{out}}}
\]  

(2.35)

whose time constant can be derived immediately:

\[
\tau = \frac{A_{\text{tank}}}{\pi D_0 n \sqrt{\left( \frac{2}{\zeta_{\text{tot}} - 1} \frac{\Delta p}{\rho} \right)}}
\]  

(2.36)

The most important parameters governing the dynamics are:

- The area of the tank. Reducing this area will intensify the sensitivity to level fluctuations, and thus leads to faster dynamics.
- The diameter of the opening. Given the elevation of the choking valve, more water will flow in when the diameter is large.
- The ratio of the lever mechanism. If \( n \) is large, even a small change in level will already lead to a significant lift of the valve.
- The supply pressure of the water distribution system, which is maybe the most important parameter. If the pressure is low, this will result in a large time constant due to the small amount of energy available to overcome head loss. Moreover, very low pressure can even cause a permanent decrease of the water level when too much liquid flows out of the tank.

Except for the area of the tank, these parameters will also determine the steady state decrease of the water level for a given outflow:

\[
\Delta H^* = \frac{-q_{\text{out}}}{\pi D_0 n \sqrt{\left( \frac{2}{\zeta_{\text{tot}} - 1} \frac{\Delta p}{\rho} \right)}}
\]

(2.37)

To give an appreciation, it is interesting to look at some numerical values. Taking for instance:

\(^7\text{If } D_0 = 3 \text{ mm and } n = 3, \text{ the decrease } \Delta H \text{ of the water level is only 1 mm!}\)
• $D_0 = 3 \, mm$, $b = 2 \, mm$ and $n = 3$ as characteristic dimensions of the float switch valve and lever mechanism

• $\Delta p = 2$ bar as the supply pressure

• and $A_{tank} = 0.01 \, m^2$

the overall loss factor results from (2.34):

$$\zeta_{tot} \approx 0.55 + 4 \left[ \left( \frac{2}{3} \right) - 0.1 \right] + \frac{0.155}{3^2} = 2.83$$

With this expression, the time constant is calculated as in (2.36):

$$\tau = \frac{0.01}{0.003 \cdot 3\pi \sqrt{\left( \frac{2}{2.83 - 1 \cdot 10^3} \right)}} = 0.02 \, sec$$

which is extremely small, partly due to the chosen values, partly due to the assumption of constant head loss (independent of $\Delta H$).

Finally, assuming an outflow of $2l/min$, the steady state decrease of the water level is:

$$\Delta H^* = \frac{-2/60}{0.003 \cdot 3\pi \sqrt{\left( \frac{2}{2.83 - 1 \cdot 10^3} \right)}} = -0.08 \, mm$$

which is hardly visible.

### 2.4.2 Conclusion

Though these numeric values result from the simplified linear system, they indicate that the time constant and steady state level decrease can be very small in the real setup. The theoretical results show that only a few parameters are determinant for the success of the float-switch level control. Fortunately, we have found a level switch which has the right characteristic dimensions. When the design was taken into practice, almost no fluctuation on the water level was visible, as predicted theoretically. Also, the setup is located on the ground floor to profit maximally from the supply pressure.
Chapter 3

Mechanical and electrical implementation

3.1 Flow Control

3.1.1 Types of actuators

To compare the performance of the three real-time controllers, it is important that the controlled actuator applies the desired flow accurately. This leaves the tank temperature only to be manipulated by the control actions themselves and not by the arbitrariness of the device. Hence a good basis for comparison is created.

Generally, flow controlling devices can be split up into two categories:

1. Passive actuators: for this type of devices the flow does not depend only on actuator actions, but also on external influences.

2. Active actuators: these type of devices force the liquid to flow, irrespective of the configuration of the setup.

The two types differ significantly in the accuracy they can achieve. Taking for instance an electromagnetic valve as a common example in the first category, it is clear that the opening of the valve itself does not trigger off flow. It is the pressure difference however that provides the energy needed to overcome hydraulic resistance. That is why an electromagnetic valve by itself can not impose a certain flow on the process.

An alternative solution is to switch from open loop to closed loop control. In this case, a sensor registers the flow continuously to provide feedback for a controller. There are however two major disadvantages of this approach. Firstly, most flow sensors are quite expensive and do not work well for small inputs. Secondly, electromagnetic valves in general provide only ON-OFF regulation. The reason lies in the electromagnetic principles the valve relies on. Hence in this case, only a relay controller can be used. This complicates flow measurement even more. Taking into account all these considerations, valves seem not fit to be used for our purpose.
Somewhere between active and passive actuators are the non-volumetric pumps. The relation between pump speed and flow still depends on the discharge head. That is why non-volumetric pumps are typically used in large flow applications and are not well suited for our application (small flows). A solution is the use of a choking valve at the outlet increasing the discharge head and consequently influencing the static characteristic of the pump. However, this artificial measure of dumping energy is in fact not very elegant from the engineering point of view.

Another drawback comes with the speed regulation. In standard applications - for instance the circulation pump of the central heating - these non-volumetric pumps are equipped with a single phase electric motor, designed to operate at the network frequency. Compared to DC-motors, strategies for controlling this type of motors are less evident. Finally, non-volumetric pumps in combination with a choking valve can suffer from back flow when suddenly decreasing the speed. This is not desired either.

On the contrary, volumetric pumps belong to the category of active actuators and therefore offer a lot of advantages. Peristaltic pumps are the most widely used in practice (e.g. dosing applications). Due to roller compression of the tube, a constant volume of liquid is transferred from the pump inlet to the outlet at every rotation, without the risk of backflow. This makes the characteristic between flow and speed perfectly linear and only determined by volumetric properties. Compared to membrane pumps, they do not have valves, thus no maintenance is required. All these advantages explain why in this application a peristaltic pump was chosen. For more information about pumps, we refer to [16] and [9].

### 3.1.2 Controlling the flow

With the linear relation between flow and speed, the problem of flow control shifts to speed control. In this application a DC-motor is most suitable because it profits from high starting torques and offers a simple relation between armature voltage and speed [13]:

\[
V = R_a I_a + L_a \frac{dI_a}{dt} + E_a
\]

with

- \(V\): the armature voltage;
- \(I_a\): the current through the motor;
- \(R_a\) and \(L_a\): the resistance and inductance of the armature respectively;
- \(E_a\): the back emf generated in the armature.

For a permanent magnet DC-motor the flux \(\phi\) remains constant, resulting in:

\[
E_a = k\phi\Omega = K\Omega
\]
Substitution of this equation into (3.1) leads to:

\[ V = R_a I_a + L_a \frac{dI_a}{dt} + K\Omega \]  

(3.3)

Thus, changes in the motor speed can be effectuated by changing the armature voltage. Taking into account the peristaltic property of linearity between flow and speed, flow control is easily achieved.

This simple relation is however compromised by current dependence. For instance, an increase in flow requires the motor to provide higher torque because the load increases correspondingly:

\[ \Delta T = k\phi \Delta I_a = K\Delta I_a \]  

(3.4)

Another more important influence is produced by the peristaltic pump head whose friction of roller compression tends to vary with the speed. Especially at low speeds this influence is pronounced, hence relatively more torque is needed. Formula (3.3) shows that if a change in current occurs, the rotary speed is affected as \( V \) is preset. When the speed is supposed to vary a lot - as in this application - elimination of the load dependence is desired.

The solution to this problem lies in a feedforward compensation of the resistive voltage drop \( R_a I_a \), which is the dominant component, because the inductance \( L_a \) is very small\(^1\). By measuring the current \( I_a \) and estimating the total resistance \( \hat{R} \), the DC drive applies an additional voltage \( \hat{R} I_a \) to the motor. In literature this is known as ‘IxR compensation’. Equation (3.3) now becomes:

\[ V + \hat{R} I_a \approx R_a I_a + K\Omega \]  

(3.5)

With a good estimation of the resistance, an approximative linear relation between \( V \) and \( \Omega \) results:

\[ V \approx K\Omega \]  

(3.6)

This linearity is marginally affected by the duty cycle of the pump. Changes in current due to changes in the load lead to different resistive losses \( R_a I_a^2 \). The dissipation of this energy causes a change of the internal temperature of the motor. This influences in turn its overall resistance. For copper windings, \(^2\) gives an approximate formula which describes the temperature dependence:

\[ \Delta R = \Delta T \frac{T_a}{234.5 + T_a} \]  

(3.7)

For instance, with an ambient temperature \( T_a \) of 20 °C and a relative temperature increase of 50%, the resistance increases with about 4%. Hence, to achieve good IxR compensation - and good flow control - it is best to estimate \( \hat{R} \) when the motor has reached its average operating temperature.

Figure 3.1 illustrates the variance introduced by load dependence and the effectiveness of the compensation method. The ratio between the flow and the voltage is plotted against the control voltage that is sent to the drive. With accurate compensation this ratio is expected to be constant. A trend line is added as a reference. The data is collected from the flow control

\(^1\)This assumption holds for permanent magnet DC motors as no field winding is present.

\(^2\)The formula is given for a constant ambient temperature \( T_a \).
unit as used in the setup.

![Figure 3.1: Influence of IxR compensation on the linearity of flow control.](image)

The influence of friction at low speeds is obvious. Also the influence of higher torque at higher speed is present, but compared to the friction effect it is of less importance. All these observations lead to the conclusion that the combination of an IxR compensated drive and a peristaltic pump provide accurate flow control without any external feedback\(^3\).

The motor of the selected pump is a 24 V DC permanent magnet motor, with a power of 100 W and a nominal current of 4 A. The pump is connected to the motor via a gear wheel box, resulting in a maximum rotary speed for the pump of 290 rpm when the motor is supplied with 24 V.

Concerning the drive for the pump, we already determined that IxR compensation is required. Some other properties would be suitable however, especially those regarding the safety and protection measures for the motor. Therefore a 24 V motor drive with soft-start (to avoid too high peaks in armature current when the motor is starting up), and a current limiter was chosen. The nominal current is 4 A (as the one for the pump), but short peaks up to 9 A are allowed. The drive itself is a first quadrant chopper. A second quadrant chopper is not necessary as the pump can rapidly slow down by itself due to the high friction with the peristaltic tube.

### 3.2 DC-sources for the temperature measurement

As there are many items involved in the temperature measurement, chapter 4 will be dedicated entirely to this topic.

\(^3\)By measuring the rotary speed with a tachometer or an optical encoder feedback could also be applied. This is however an expensive solution, and very difficult to realize in practice as it is almost impossible to mount a sensor on the motor-pump combination.
For the measurement circuits, fixed DC-sources are required: +15 and -15 V to feed the operational amplifiers (op amps), 5 V for the excitation of the sensor and 9 V for the mixer. In order to obtain these voltages, a 2x15 V AC print transformer is used. Thus, in amplitude values, the transformer supplies approximately +21 and -21 V AC (using the center tap as ground). A rectifier bridge is used together with capacitors to smooth the ripple, resulting in a 19 V DC. This DC source is used to feed +15 V, -15 V, +5 V and +9 V voltage regulators, integrated circuits (ic) that supply a very stable DC voltage. Extra capacitors are added to the output of the ic’s to insure even more stability (Figure 3.2).

![Figure 3.2: DC-sources for temperature measurement](image)

### 3.3 Data acquisition

#### 3.3.1 Requirements

In the search for a data acquisition board (DAQ), attention must be payed to three issues. The most important one is that it should have enough input and outputs, analog as well as digital. The second one concerns the maximum sampling rate that can be achieved, while the third one regards the resolution.

In the setup, three temperature measurements have to be taken simultaneously. These are voltages in the 0-10 V range, sampled by the DAQ-board and converted into a bit sequence of a certain length, depending on the resolution. For instance, if the board has a 12-bit resolution, the minimum voltage difference it can distinguish is:

$$ (\Delta V)_{min} = \frac{10 V}{2^{12}} = 2.44 \cdot 10^{-3} V $$

(3.8)

corresponding to a temperature change of:

$$ \Delta T = 50 ^\circ C \cdot \frac{\Delta V}{10 V} = 0.0122 ^\circ C $$

(3.9)

This resolution outreaches the overall temperature accuracy by far, hence, a 12-bit DAQ-board is surely good enough for this application.
The second requirement concerns the maximum sampling rate of the board. Fast changing inputs may be left undetected if this sampling rate is too low. Worse, they can be mixed up with underlying slow changes of the input. This is exactly what the Nyquist sampling theorem points out.

In order to perform reliable data acquisition, the maximum sampling rate should be at least two times bigger than the input frequency of interest. Thus one has to take a look at the smallest time constant of the system. In this case, the smallest time constant corresponds to the maximum flow that can be applied:

$$\tau_{\text{min}} = \frac{V}{q_{\text{max}}}$$  \hspace{1cm} (3.10)

The minimum time constant of the system will be approximately 30 seconds. Thus the bandwidth of the system at this maximum flow rate is limited to 0.004 Hz. This will not cause any difficulty for a DAQ-board.

Another thing to consider is the number of analog inputs and outputs. There are three analog inputs needed for the temperature measurement, while only one analog output is needed for the DC-drive control. The drive itself works at a 0-5 V control voltage range, which is typically covered by most DAQ-boards. The input voltage range is also not restrictive.

In the setup, the wires connecting the measurement circuits to the DAQ-board are a few meters long, thus it is required to have a high input impedance. If not, the current drawn from the circuit cause too much resistive voltage drop, resulting in distorted temperature measurements.

### 3.3.2 Selection

Taking into account all these considerations, the things that play an important role in the search for a suitable DAQ-board are the resolution, the number of in- and outputs, and a high input impedance. For the setup, the USB-6008 DAQ-board from National Instruments was chosen (Figure 3.3). It has a $144\, \text{k}\Omega$ input impedance, offers a 12-bit resolution and can handle the voltage ranges of the in- and output.

![Figure 3.3: The National Instruments USB-6008 Data Acquisition Board](image)
3.4 Overview

Now that the design principle, measurement circuits, flow control and data acquisition are discussed, a detailed overview can be given of the entire setup.

3.4.1 Process tank

Description

A major challenge was to create the shape of the tank in such a way that the heater, the mixer and the level switch could be installed within a minimum volume. Besides, to insure a homogeneous temperature, attention had to be payed that the internal flow propelled by the mixer reached every water particle in the tank and especially provided constant refreshment at the heating point. At last, it had to rule out any chance of shortcut between inlet and outlet. Even when the level switch is adjusted to the minimum, it still has a minimum height. That is why a rather slender shape was chosen.

The tank is made of plexiglas (PMMA) because - and above all - it is water resistant, easy to cut up and to glue together. It also offers the advantages of transparency and a relatively low thermal conductivity. Based on the shape of the tank and the pressure differences the mixer causes, Figure 3.4 shows the internal flow in red.

The heater is installed diagonally opposite to the mixer in order to insure good refreshment. The blue arrows mark the inlet and outlet. The inlet of cold water happens just in the center of the internal flow. Hence, there is no chance of shortcut.

Exact parameters

With all devices installed in the tank, the water volume is approximately 1.13 liter. After measurement of the main supply and the current through the heater, the heat input is calculated as 1105 W. Considering a minimum heat loss, the 1100 W can be maintained. The current through the DC-motor of the mixer is about 6 mA. With a supply voltage of 9 V, the power of the motor is about $9\,V \cdot 6\,mA = 0.054\,W$, thus the heat input caused by the mixer is practically zero.
3.4.2 Electrical supply

Different voltage levels are needed for the high power circuitry and the instrumentation. Table 3.1 gives an overview:

<table>
<thead>
<tr>
<th>High power circuitry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heater</td>
</tr>
<tr>
<td>DC-drive</td>
</tr>
<tr>
<td>Low power circuitry</td>
</tr>
<tr>
<td>Mixer</td>
</tr>
<tr>
<td>Opamp</td>
</tr>
<tr>
<td>Wheatstone supply</td>
</tr>
</tbody>
</table>

Table 3.1: Overview of voltage levels

Figure 3.5 illustrates the electric scheme of the setup. A distinction is made between low and high power devices. The heater (≈ 5A AC) and the DC-drive (≈ 2A AC) are the high power devices separately protected with a glass fuse. The DC-drive has a stand alone power supply which rectifies the 230 V AC grid voltage.
One rectifier bridge is installed to supply the measurement circuits and the mixer, which together account for approximately 10 mA. Because of these small currents, there is no fuse installed. Still, there is a transformer which provides galvanic protection against over voltage. If over voltage occurs, the transformer will fail before the measurement circuits are damaged.

All electric devices are placed in a wooden box with a plastic lid (Figure 3.6). The joints are siliconed to guarantee perfect sealing. These measures prevent the electric part from accidental water damage.
3.4.3 Water circuit

The peristaltic pump head is installed through the box (Figure 3.7). The pump can be controlled as soon as the switch is activated. The pump has a theoretic maximum flow of ±4.1 liters/min, but this is limited to 1.8 liters/min for two reasons:

- If the flow is too high, the float switch is no longer able to maintain the water level in the tank.
- Higher flows require more armature current, causing the motor to heat up, due to more resistive heat loss. To avoid the coils burning through, it is safer to limit the flow, especially during long duty cycles.
Due to the thickness of the insulation (about 1 cm), it is not easy to wind the ±9.5 m long outlet tube having an inner diameter of ±12 mm in a smooth helical form. Nevertheless, the pump has no problem to get the water through. The total volume in the tube amounts to 1.02 liters. The Pt100 sensor is mounted counterflow as it can be seen in Figure 3.8. Finally, the last picture (Figure 3.9) shows an overview of the entire plant.
Figure 3.9: Overview of the entire plant
Chapter 4

Temperature Measurement

4.1 Choice of the sensor

4.1.1 Different types of sensors

Many types of temperature sensors are available in industry. Out of those that are qualified for our application, three important types can be named [2], [19]:

1. Thermocouples (TC’s);
2. Thermistors;
3. Resistance Temperature Detectors (RTD’s).

We will first give a short description of these three types of contact temperature sensors and discuss their advantages and disadvantages. In the end, the best sensor suitable for our application is selected.

Thermocouples (TC’s)

Thermocouples are based on the principle that the junction between two different metals (e.g. iron and constantan) produces a voltage that increases with temperature. The reason is the difference in the binding energy of the electrons to the metal ions, which makes the generated voltage dependent on both the type of metals under consideration and the junction temperature. A thermocouple can only measure temperature differences. Subsequently, a reference junction - usually called the cold junction, Figure 4.1 - is added to allow absolute temperature measurements.

Thermocouples offer a large temperature range (-200 to 1800 °C) and are quite cheap. Their robustness and fast response time are the main advantages.

There are however major disadvantages too. Thermocouples are not very sensitive and non-linear. The generated voltage is very low (order of microvolts), thus strong amplification is required. They are not very stable in time (poor repeatability) and have a low accuracy.

Thermocouples are most frequently used in high temperature environments, like ovens, furnaces and for flue gas measurements.
CHAPTER 4. TEMPERATURE MEASUREMENT

(a) Principle

(b) The ‘cold’ junction

Figure 4.1: Thermocouple

Thermistors

Thermistors are made of certain metal oxides whose electrical resistance changes with temperature (the name is a contraction of ‘thermal’ and ‘resistor’). They can be categorized into two groups:

- NTC-thermistors (Negative Temperature Coefficient): an increase in the temperature causes a lower resistance;
- PTC-thermistors (Positive Temperature Coefficient): an increase in the temperature causes an increase in the resistance.

Thermistors are cheap and sensitive. Compared to thermocouples, they have smaller bandwidth, but offer better accuracy. The main disadvantage is a pronounced non-linearity. Their maximum temperature is limited to approximately 200 °C. They have a tendency to drift under alternating temperatures.

Thermistors are popular in low temperature applications, mainly to monitor a certain temperature that is supposed to remain constant.

Resistance Temperature Detectors (RTD’s)

The basic principle of RTD’s is the same as of thermistors: a change in temperature causes a change in the electrical resistance. There is an important difference however: they are made exclusively of metals, which makes them reasonably linear, at least over a limited range. Present sensors consist of a thin film wire-wound coil of a certain type of metal. Platinum and nickel are used the most.

Their linearity (especially in the case of platinum sensors), their high accuracy under all circumstances and their repeatability makes them very interesting and therefore widely used in industry.

The drawbacks are smaller temperature range (-200 to 800 °C) and lower reaction time due to thermal inertia. Unlike the thermocouples, they are not self-powered. According to Ohm’s
CHAPTER 4. TEMPERATURE MEASUREMENT

law, a current must be passed through the device to provide a voltage that can be measured. The current causes Joule heating within the RTD, changing its temperature.

The most frequently used sensor of this type is the Pt100, a platinum RTD with a nominal resistance of 100 Ω at 0 °C. This sensor has proved its quality in many industrial applications.

Conclusions

Table 4.1 gives an overview of the different properties of the three sensor types that were discussed, while Figure 4.2 compares their characteristics.

<table>
<thead>
<tr>
<th></th>
<th>Thermocouple</th>
<th>Thermistor</th>
<th>RTD</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost</td>
<td>+</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>linearity</td>
<td>−</td>
<td>−</td>
<td>++</td>
</tr>
<tr>
<td>accuracy</td>
<td>−</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>temperature range</td>
<td>++</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>repeatability</td>
<td>−</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>sensitivity</td>
<td>−</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>reaction time</td>
<td>+</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>robustness</td>
<td>++</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>risk of self-heating</td>
<td>−</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 4.1: Comparison of the different types of sensors

To finally make the best choice between these sensors, one must know the requirements and specifications of the measurement. In our case these are:

- good accuracy under all circumstances;
- good repeatability (thus measurements are the same at any time);
- fast reaction time (hence fast temperature changes are not left undetected);
- robustness;
• preferably a linear characteristic (although this is not insurmountable with today’s computer control);

• temperature range between 0 and 50°C.

The thermocouple is immediately excluded because it is more suited for high temperature applications. Moreover, its accuracy and repeatability are not ideal. This still leaves the choice between the thermistor and RTD. Taking into account the important thermistor disadvantage of poor accuracy at fast temperature changes, the RTD is the best option. Its accuracy and repeatability are good, the nearly linear characteristic is a useful attribute. A direct-contact version is chosen for the outlet temperature measurement because it has less thermal inertia and hence only a few seconds reaction time. For the inlet temperature monitoring, the cheaper non-direct contact version is satisfactory. Both RTD’s are of the Pt100-type.

4.1.2 The Pt100 sensor

Though nearly linear, especially over the interest temperature range of 0 to 50°C (Figure 4.3), best accuracy is obtained using the non-linear equation as given by the latest standard (ITS-90)[19]:

\[ R_t = R_0 (1 + At + Bt^2 + C(t - 100)^3) \] (4.1)

\( R_t \) is the resistance at temperature \( t \) in °C and \( R_0 \) the reference resistance at 0°C (100 Ω). The coefficients are:

\( A = 3.9083 \cdot 10^{-3} \)
\( B = -5.775 \cdot 10^{-7} \)
\( C = -4.183 \cdot 10^{-12} \) (below 0°C), or \( C = 0 \) (above 0°C)

![Figure 4.3: Resistance-Temperature Characteristic of Pt100](image)

Equation (4.1) is however not exact: some variation on the resistance can occur. Two tolerance classes for Pt100 sensors exist:

Class A with a tolerance of ±(0.15 + 0.002|t|) °C
Class B with a tolerance of ±(0.30 + 0.005|t|) °C
For a better accuracy, class A sensors were chosen for this setup.

Apart from this inherent inaccuracy, other errors on the resistance are caused by the self-heating and the resistance of the wires connecting the Pt100 to the measurement circuit. For currents up to 1 mA the self-heating error can be considered not too important\(^1\). The error introduced by the wire resistance can be eliminated through calibration.

As already has been discussed in section 2.2, further decrease in reaction time can be achieved when the sensor is mounted counterflow and concentric with the tube.

Now that the best suited temperature sensors are selected, next section deals with the circuits converting the Pt100 resistance into a voltage.

### 4.2 Measurement circuits

#### 4.2.1 Preliminaries

The main objectives to fulfill in designing the measurement circuits are:

- temperature range between 0 and 50°C, to be mapped to a voltage range from 0 to 10 V;
- small additional errors;
- low excitation current to avoid self-heating;
- high measurement resolution and low noise level.

We choose the excitation current to be 1 mA. This value is a trade-off between self-heating and output voltage magnitude \(v\), since low voltage signals require strong amplification, making the measurement more sensitive to all kinds of noise from external sources.

There are two ways to excite the sensor [19]:

1. via a fixed current source;
2. via the Wheatstone bridge.

These options are briefly discussed before the measurement circuits are presented.

#### 4.2.2 Excitation via a fixed current source

In this approach a current of 1 mA is sent through the sensor via a fixed current source. Hence a first problem is introduced: an ideal fixed current source does not exist. In practice circuits have been designed that approximate it quite good at the cost of increased complexity. An important advantage however can be seen in Figure 4.4.

\(^1\)For instance, a 1mA current through a 100 Ω resistance causes \(10^{-4}\) W Joule heating.
Due to separate excitation, it is possible to connect the Pt100 with four wires: two force leads to supply the 1 mA current, and two sense leads to amplify the output $v$. If an appropriate amplifier is chosen, like the instrumentation amplifier, the current through these sense leads is nearly zero. As a result, the error due to wire resistance is eliminated because no parasitic voltage is added to the sensor voltage $v$. However, in its structure lies also its shortcoming. Multiplying the 1 mA current with the respective resistances at 0 and 50°C, we obtain

$$v_{0°C} = I \cdot R_{0°C} = 1 mA \cdot 100 \Omega = 0.1 V$$
$$v_{50°C} = I \cdot R_{50°C} = 1 mA \cdot 119.4 \Omega = 0.1194 V$$

If these voltages are amplified to get 10 V at 50°C, an output voltage range of 1.625 V results\(^2\). Unless an offset voltage of 0.1 V is subtracted from the sensor voltage, it is impossible to obtain the full 10 V range after amplification. Any deviation of this offset voltage would cause considerable errors after amplification, leaving aside the difficulty to implement an accurate offset circuit.

Supposing no additional circuitry is implemented and given a DAQ-board resolution of 12-bit at 10V input range, the overall temperature resolution would be:

$$\frac{50°C}{1.625V} \cdot \frac{10 V}{2^{12}} = 0.075 °C$$

On the contrary, if the full 10V input range would be used, the resolution is:

$$\frac{50°C}{10 V} \cdot \frac{10 V}{2^{12}} = 0.0122 °C$$

which represents an increase in the accuracy of six times.

Considering on one hand this trade-off between resolution and difficult hardware implementation of an accurate offset circuit, and on the other hand the fixed current source, which can only be approximated in practice, the method of separate excitation is rejected\(^3\).

### 4.2.3 Excitation via the Wheatstone bridge

The other possibility is to excite the sensor via a Wheatstone bridge, whose structure is given in Figure 4.5.

---

\(^2\)The amplification factor is calculated as 83.75 V/°C, leading to 8.375 V at 0°C. Subtracted from 10 V, a 1.625 V range results.

\(^3\)This method is more useful when the temperature range is large.
CHAPTER 4. TEMPERATURE MEASUREMENT

Figure 4.5: The Wheatstone bridge

To eliminate any offset in the balanced bridge, the resistors have an accuracy of 0.1%\(^4\). With the values in the setup (Figure 4.5) an excitation current of approximately 1 mA is obtained. The Wheatstone bridge is very sensitive to its input voltage, thus it is very important that this voltage has a low noise level and shows no tendency to drift. This issue has been discussed in section 3.2.

The output voltage \(v\) can be calculated from the voltage division formula and results in:

\[
v = \left( \frac{R_{Pt100}}{4990\,\Omega + R_{Pt100}} - \frac{100}{4990\,\Omega + 100\,\Omega} \right) \times 5\,V \tag{4.2}\]

The bridge is in balance at 0\(^\circ\)C:

\[
v_{0^\circ\text{C}} = \left( \frac{100}{4990\,\Omega + 100\,\Omega} - \frac{100}{4990\,\Omega + 100\,\Omega} \right) \times 5\,V = 0\,V
\]

while for the 50\(^\circ\)C range under consideration, the maximum output voltage corresponds to 119.4\(\Omega\):

\[
v_{50^\circ\text{C}} = \left( \frac{119.4}{4990\,\Omega + 119.4\,\Omega} - \frac{100}{4990\,\Omega + 100\,\Omega} \right) \times 5\,V = 0.01861\,V
\]

To map this output voltage to a 0 to 10 V range, an amplification of approximately 537.3 is needed. That is why a low noise level at the input is required.

The disadvantage of this method is that a four-wire connection scheme can not be used. This leads to the addition of the parasitic wire resistance. Nevertheless, since this resistance does not change in time, calibration of the measurement circuit can eliminate this influence.

As equation (4.2) indicates, the output voltage of the bridge varies non-linearly with the sensor resistance. However, the contribution of \(R_{Pt100}\) in the denominator is very small compared to the 4990 \(\Omega\) resistance, thus linearity is almost achieved as shown in Figure 4.6:

\(^4\)This explains the odd 4990 \(\Omega\) value, as the number of resistance values for resistors with 0.1% accuracy is limited.
CHAPTER 4. TEMPERATURE MEASUREMENT

Figure 4.6: Characteristic Wheatstone bridge & Amplifier

To amplify this output voltage to the 10 V range, we consider two different amplifier circuits: the differential amplifier and the instrumentation amplifier.

4.2.4 The differential amplifier circuit

Figure 4.7 presents the electrical scheme of the differential amplifier.

Figure 4.7: Scheme Differential Amplifier

The circuit consists of a series connection of a differential amplifier and an inverting amplifier. The overall amplification factor is:

$$V_{out} = \frac{R_4}{R_3} \frac{R_2}{R_1} V$$  (4.3)

The resistance $R_4$ is split up into a fixed and variable part. This allows to adjust the amplification factor in such a way that the circuit can be perfectly tuned to the 10 V output range.

The variable resistance with the maximum value of 10 kΩ in the differential amplifier circuit is used to compensate for the offset of the op amp.

The amplification factor of the differential amplifier circuit is calculated as $\frac{R_4}{R_3}$, in the ideal case $\frac{R_1}{R_2} = \frac{R_1}{R_2}$. In reality however, these ratios are never perfectly equal, leading to the
complex amplification formula (4.4):

\[ V_{out} = \frac{R_4}{2 R_1} \left( \frac{1}{R_1} - \frac{R_1/R_2 + 1}{R_1/R_2' + 1} \right) (V_2 - V_1) + \frac{R_4}{R_3} \frac{R_2}{R_1} \left( \frac{R_1/R_2 - R_1'/R_2'}{R_1/R_2' + 1} \right) \frac{V_2 + V_1}{2} \]  \hspace{1cm} (4.4)

When rewriting this formula, the meaning of both terms in the right hand side becomes clear:

\[ V_{out} = A_D (V_2 - V_1) + A_C \frac{V_2 + V_1}{2} \]  \hspace{1cm} (4.5)

\( A_D \) represents the differential amplification ( \( v \) in the Wheatstone bridge) and \( A_C \) the common-mode amplification. Obviously, the circuit does not only amplify the voltage difference, but also the ‘common-mode’ voltage \( \frac{V_2 + V_1}{2} \). The amplification of this common-mode voltage is not desired. Therefore the ‘common-mode rejection ration’ \( CMMR = \frac{A_D}{A_C} \) must be maximized, which means in fact minimization of \( A_C \).

If \( x \) is the maximum relative error on the values of all resistors, the worst case scenario leads to [20]:

\[
\begin{align*}
R_1 &= R_1^0 \pm x R_1^0 \\
R_1' &= R_1^0 \pm x R_1^0 \\
R_2 &= R_2^0 \pm x R_2^0 \\
R_2' &= R_2^0 \pm x R_2^0
\end{align*}
\]

\hspace{1cm} (4.6)

Consequently, the ratios of interest result in:

\[
\begin{align*}
\frac{R_1}{R_2} &= \frac{R_1^0}{R_2^0} \pm 2 x \frac{R_1^0}{R_2^0} \\
\frac{R_1'}{R_2'} &= \frac{R_1^0}{R_2^0} \pm 2 x \frac{R_1^0}{R_2^0}
\end{align*}
\]

\hspace{1cm} (4.7)

The difference between the two ratios is then

\[ \frac{R_1}{R_2} - \frac{R_1'}{R_2'} = 4 x \frac{R_1^0}{R_2^0} \approx 4 x \frac{R_1}{R_2} \]  \hspace{1cm} (4.8)

which allows the \( CMMR \) to be expressed as:

\[
CMMR = \frac{A_D}{A_C} = \frac{1}{2} \left( \frac{R_1}{R_2} + \frac{R_1'}{R_2'} \right) = \frac{1}{2} \left( \frac{R_1}{R_2} \pm 4 x \frac{R_1^0}{R_2^0} + 2 \right)
\]

\[ \approx \frac{(1 \pm 2 x) + \frac{R_1^0}{R_2^0}}{4 x} \approx \frac{1 + \frac{R_2}{R_1}}{4 x} \]  \hspace{1cm} (4.9)

Taking the limit of zero relative error, the \( CMMR \) goes to infinite, as expected. After the discussion of the instrumentation amplifier, an appreciation will be given of this \( CMMR \) value.
4.2.5 The instrumentation amplifier circuit

Figure 4.8 presents the electrical scheme of the instrumentation amplifier circuit.

![Scheme Instrumentation Amplifier](image)

**Figure 4.8:** Scheme Instrumentation Amplifier

Assuming ideal op amps we can write

\[ \frac{V'_2 - V'_1}{R_3 + R_v + R_4} = \frac{V_2 - V_1}{R_v} \]  

(4.10)

In the second part of the scheme we recognize a differential amplifier again, which allows to reuse equation (4.4) after substituting equation (4.10):

\[ V_{out} = \frac{R_2}{2R_1} \left[ 1 + \frac{R_1/R_2 + 1}{R_v} \right] \frac{R_3 + R_v + R_4}{R_v} (V_2 - V_1) + \frac{R_2}{R_1} \left[ \frac{R_1/R_2 - R'_1/R'_2}{R'_1/R'_2 + 1} \right] \frac{V_2 + V_1}{2} \]  

(4.11)

If we do the same analysis for the $CMMR$ of the instrumentation amplifier, we now have

\[ CMMR = \frac{A_D}{A_C} = \frac{1}{2} \frac{R_3 + R_v + R_4}{R_v} \frac{R'_3 + R'_4}{R'_3 - R'_1} + 2 \frac{R'_1}{R'_2} \]  

(4.12)

\[ = \frac{1}{2} \frac{R_3 + R_v + R_4}{R_v} \frac{4R'_1}{R'_2} + 2 \]  

\[ = \frac{1}{2} \frac{R_3 + R_v + R_4}{R_v} (1 \pm 2x) + \frac{R'_3}{R'_1} \]  

(4.12)

4.2.6 Conclusions

Comparing both $CMMR$ values, the one of the instrumentation amplifier is a factor \( \frac{R_3 + R_v + R_4}{R_v} \) higher. This means the instrumentation amplifier will almost completely reject the common-mode voltage to merely amplify the Wheatstone bridge output voltage \( v \), while the differential amplifier will add an extra voltage.
Because of its better performance, the instrumentation amplifier is used for the crucial measurements: the output temperature and the tank temperature. These are signals that will be used for the modelling and identification of the process. The inlet temperature is of less importance and, because it is less complex to build, the differential amplifier will be used there.

Finally, Table 4.2 summarizes the values of the resistors used in both circuits:

<table>
<thead>
<tr>
<th></th>
<th>Differential Amplifier</th>
<th>Instrumentation Amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1 = R'_1$</td>
<td>1kΩ</td>
<td>1kΩ</td>
</tr>
<tr>
<td>$R_2 = R'_2$</td>
<td>10kΩ</td>
<td>10kΩ</td>
</tr>
<tr>
<td>$R_3$</td>
<td>1kΩ</td>
<td>27kΩ</td>
</tr>
<tr>
<td>$R_4$</td>
<td>55kΩ, variable</td>
<td>27kΩ</td>
</tr>
<tr>
<td>$R_v$</td>
<td>1kΩ, variable</td>
<td>1kΩ, variable</td>
</tr>
</tbody>
</table>

Table 4.2: Resistance values in both amplifiers

With these values, the amplification $A_D$ of both amplification circuits results in approximately 537.3. Table 4.3 contains the \textit{CMMR}'s for resistor tolerances $x$ of respectively 5%, 1% and 0.1%:

<table>
<thead>
<tr>
<th></th>
<th>Differential Amplifier</th>
<th>Instrumentation Amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% resistors</td>
<td>55</td>
<td>3025</td>
</tr>
<tr>
<td>1% resistors</td>
<td>275</td>
<td>15125</td>
</tr>
<tr>
<td>0.1% resistors</td>
<td>2750</td>
<td>151250</td>
</tr>
</tbody>
</table>

Table 4.3: \textit{CMMR}'s

Out of these values and with $A_D \approx 537.3$, the common-mode amplification $A_C$ can be calculated. The results are shown in Table 4.4.

<table>
<thead>
<tr>
<th></th>
<th>Differential Amplifier</th>
<th>Instrumentation Amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% resistors</td>
<td>9.77</td>
<td>0.178</td>
</tr>
<tr>
<td>1% resistors</td>
<td>1.95</td>
<td>0.0355</td>
</tr>
<tr>
<td>0.1% resistors</td>
<td>0.195</td>
<td>0.00355</td>
</tr>
</tbody>
</table>

Table 4.4: Common-mode amplifications

As a result, the error in °C due to the common-mode amplification is computed. The worst
case common-mode voltage is reached at 50 °C:

\[ V_2 = \frac{119.4 \Omega}{4990 \Omega + 119.4 \Omega} \times 5V = 0.1168V \]

\[ V_1 = \frac{100 \Omega}{4990 \Omega + 100 \Omega} \times 5V = 0.0982V \]

\[ \frac{V_2 + V_1}{2} = 0.1075V \]

Finally, Table 4.5 summarizes the errors for the respective tolerances:

<table>
<thead>
<tr>
<th></th>
<th>Differential Amplifier</th>
<th>Instrumentation Amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% resistors</td>
<td>5.252 °C</td>
<td>0.096 °C</td>
</tr>
<tr>
<td>1% resistors</td>
<td>1.048 °C</td>
<td>0.019 °C</td>
</tr>
<tr>
<td>0.1% resistors</td>
<td>0.105 °C</td>
<td>0.002 °C</td>
</tr>
</tbody>
</table>

Table 4.5: Common-mode amplifications

The use of 5% resistors could cause an error of more than 5 °C. That is why in both circuits 0.1% accuracies are chosen for \( R_1 \) and \( R_2 \). Consequently, the maximum error due to the resistor accuracy is 0.1 °C for the differential amplifier, and 0.002 °C for the instrumentation amplifier.

In the setup, both circuits are soldered on a printed circuit board and tuned to have an almost perfect range of 10 V, corresponding to a 50 °C temperature range.

### 4.3 Noise analysis and filtering

Figure 4.9 shows a typical measurement obtained without filtering.

![Figure 4.9: A typical first measurement](image)

Obviously, the measurement signal is highly disturbed, containing fluctuations of more than 1 °C and all kinds of lower and higher frequency harmonics. A spectral analysis of this signal...
was made, resulting in the noise spectrum of Figure 4.10.

Disturbances are clearly found at a frequency of approximately 1100 Hz (and its multiples), though high amplitudes are detected for lower frequencies as well. In a first attempt, a hardware lowpass RC filter with a cutoff frequency of approximately 6 Hz was implemented to eliminate the high frequency disturbance \([12]\), resulting in measurements as shown in Figure 4.11.

As expected, some disturbances are not filtered out and even the 10 Hz disturbance is not fully removed (Figure 4.12). The harmonics appear at multiples of 3.3 Hz. Apparently, these frequencies can be related to the rotational frequency of the pump. In this experiment, the pump was turning at 100 rpm, corresponding to \(\frac{100}{60} = 1.67\) Hz. Taking into account that the peristaltic pump uses two rollers for tube compression, we find that each compression causes disturbance, explaining the harmonics at 3.3 Hz.

\footnote{The mean value was subtracted first which explains the zero DC amplitude.}
CHAPTER 4. TEMPERATURE MEASUREMENT

Figure 4.12: Noise spectrum after filtering at 6 Hz

This relation was validated as further measurements were taken at different speeds. Figure 4.13 shows the noise spectrum at rotation speeds of 75 rpm (4.13 a) and 50 rpm (4.13 b), corresponding to frequencies of $2 \cdot 1.25 \, Hz = 2.5 \, Hz$ and $2 \cdot 0.83 \, Hz = 1.67 \, Hz$.

Figure 4.13: Noise spectra after filtering at 6 Hz for different pump speeds

Based on this information, the filter components can be dimensioned. First the bandwidth of the process is calculated, to avoid filtering within the bandwidth. The linearized transfer
function of the process is:

\[ \Delta T = \frac{(T_{in} - T^*)}{V} \left( \frac{1}{s + \frac{q^*}{V}} \right) \]

where * denotes the linearization point \([q^*, T^*]\).

The break frequency \( \frac{q^*}{2\pi V} \) is maximum at maximum flow. With a maximum flow of 1.8 l/min, this frequency is calculated as 0.004 Hz. It seems the bandwidth of the system will not be a problem for the filter.

However, a problem comes with the components of the RC filter. Very low cutoff frequencies, for instance 0.5 Hz, result in large capacitors, which cause practical problems. Finally, a cutoff frequency of approximately 1.1 Hz was chosen. Additionally, a digital filter will be applied by measuring at a 5 Hz sampling rate and averaging.

The transfer function of the lowpass filter is given by

\[ \frac{\omega_c}{s + \omega_c} \]

with the cutoff frequency \( \omega_c = \frac{1}{RC} \). Given \( f_c = \frac{\omega_c}{2\pi} = 1.1 \text{ Hz} \), this can be achieved with \( R \approx 1.5 \text{k}\Omega \) and \( C \approx 100 \mu\text{F} \). Figure 4.14 shows the filter and its Bode plot.

Finally, Figure 4.15 shows a temperature measurement after filtering at 1.1 Hz. As it can be seen, there is no longer significant noise on the signal.
4.4 Calibration

Several conversions are made before the voltage signal is processed to reconstruct the true temperature. All these conversions introduce random errors which bias the result. Therefore it is good to have an idea about the overall accuracy obtained.

Figure 4.16 shows how the real temperature is converted into resistance and voltage before being transmitted through the RC-filter and finally processed by the computer [21].
The non-linear Pt100-formula relates temperature to resistance. Referring to section 4.1, the sensors in this setup are of tolerance class A, having ±0.15° accuracy. For the other devices however, we cannot rely on any data. Therefore calibration is done through measurement and statistical reasoning.

Both measurement circuits are designed to have a linear characteristic between input-resistance and output-voltage. Transmitting these signals through the RC-filters would ideally not cause any voltage drop because the capacitor is fully charged. In reality however, a small current exists due to the capacitor’s parasitic resistance, causing voltage drop across the resistance of the filter. Higher input voltage will lead to higher resistive voltage drop. That is why both RC-filters are described by a linear characteristic as well. Hence, all circuits can be treated in the same way.

First, we start from a number of measurements at different input signals, equally distributed. Using the method of least squares, a linear characteristic of input vs. output is derived, which will be used in conversions later on. The measurement error is defined as the difference between this reference characteristic and the measurement. The errors are believed to be normally distributed because they are caused by a number of factors which are not related to each other, for instance the measurement apparatus and the neon lightning. The expectancy $E[\epsilon]$ of this distribution is zero, while the variance is given by:

$$\sigma[\epsilon] = \sqrt{\frac{\sum_{i=1}^{N} \epsilon_i^2}{N - 1}} \quad (4.15)$$

where $N$ denotes the number of measurements. As it can be seen from the formula, the more measurements taken, the more accurate the linear characteristic will be.

With the information about the variance, an estimation of the tolerance can be made. Taking for example one of the measurement circuits, the input resistance has to be reconstructed from the measured output voltage. This reconstruction will however not be exact, due to the statistical derivation of the linear characteristic. Based on the properties of the normal distribution, an interval is defined which contains the reconstructed signal with 97% certainty:

$$P \left[ \hat{R} - 3\sigma[\epsilon] \leq R_{true} \leq \hat{R} + 3\sigma[\epsilon] \right] \quad (4.16)$$

in which the estimated resistance $\hat{R}$ is calculated from the linear characteristic.
In the real measurement environment, only the voltages after the RC-filters are measured. Hence, the reconstruction of $\hat{R}$ is itself based on an uncertain input value, in this case the voltage before the filter. This series of inaccuracies leads to the estimation of the overall inaccuracy as Figure 4.17 illustrates. In fact, to get a rigorous estimation, the worst case scenario is sketched, in which all inaccuracies are added.

Four devices - two filters and two measurement circuits - are calibrated according to this method\(^6\) (Figure 4.18 and 4.19). To simplify calculations, the Pt100 characteristic is linearized\(^7\). The results are summarized in Table 4.6.

---

\(^6\)An overall inaccuracy estimation is not issued for the tank temperature circuitry because this measurement is only indicative.

\(^7\)This is a good approximation because the second order coefficient is three orders less in magnitude.
The outlet temperature circuit seems to be two times more accurate compared to the inlet circuit. As the inlet temperature is of less importance, its accuracy is acceptable. Notice however that the Pt100 sensor directly introduces an error of ±0.15°C or 35% of the total error of the outlet measurement.
Part II

Advanced Control
Chapter 5

Modelling and Identification

5.1 Preliminaries

In this chapter a model is derived for each part of the plant in a rigorous way. It is obvious that the success of this approach depends on the difficulty of the part under consideration. As a result, we will sometimes be obliged to make some considerable simplifications or assumptions in order to obtain a model that is useful for the predictive controller.

First the temperature dynamics of the water tank itself are discussed. These can be obtained in a rather straightforward way using the energy balance. Next, the heat loss of the tank and the outlet tube are investigated. Regarding these heat losses, mainly the results will be given while the theory behind it will not be examined deeply, since this is of minor importance. Finally, a model of the outlet tube and the variable time delay will be derived. In the last section of this chapter, handling the identification, these theoretic results will be compared to experimental data in order to check their accuracy and to see whether or not they need adjustment.

5.2 Water tank

A theoretical model of the water temperature dynamics can be derived departing from the first law for thermodynamic systems (energy balance, for more information on thermodynamic systems, see [14]). As control volume the tank itself is considered (Figure 5.1). We recall that in the tank there is a heater which is constantly turned on (thus no manipulation of the heat input), while a mixer guarantees a uniform temperature in all the regions of the tank. The water level is kept constant by use of the mechanical float switch.
The energy balance gives

\[
\frac{dE_{cv}(t)}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i (h_i + \frac{w_i^2}{2} + gz_i) - \dot{m}_e (h_e + \frac{w_e^2}{2} + gz_e)
\]

(5.1)

where

- \( E_{cv}(t) \) the total energy in the control volume;
- Since the potential and kinetic energy in the control volume do not change in time, the only variation in energy comes from the internal energy \( U_{cv}(t) \). Considering this, the left hand side of equation (5.1) becomes

\[
\frac{dE_{cv}(t)}{dt} = \frac{dU_{cv}(t)}{dt} = \rho V c_v \frac{dT(t)}{dt} = \rho V c_p \frac{dT(t)}{dt}
\]

with \( V \) the volume of water in the tank, \( \rho \) the density of water, \( c_v \) and \( c_p \) the specific heats of water and \( T(t) \) the time dependent temperature in the tank.

- \( \dot{Q}_{cv} \) the net heat transfer to the control volume;
  The heat transfer can be divided into heat supplied by the heater, and heat losses of the tank: \( \dot{Q}_{cv} = \dot{Q}_h - \dot{Q}_{loss} \). In the next section it will be proved however that these heat losses remain very small.

- \( \dot{W}_{cv} \) the net work delivered by the system;
  This includes the work done by the mixer on the control volume, which is practically zero.

- \( \dot{m}_i, \dot{m}_e \) the incoming respectively outgoing mass flow;
  Since the water level in the tank remains constant these are equal: \( \dot{m}_i = \dot{m}_e = \dot{m} = \rho q(t) \), with \( q(t) \) the flow (dependent on time);
• $h_i$, $h_e$ the enthalpy of the incoming respectively outgoing water;  
  This can be rewritten as  
  $$h_i = c_p T_{in}(t) \quad \text{en} \quad h_e = c_p T(t)$$

• $w_i$, $w_e$ the velocity of the incoming respectively outgoing water;  
• $z_i$, $z_e$ the elevation of the inlet respectively outlet tube.

With the results described above, equation (5.1) becomes  
$$\rho V c_p \frac{dT(t)}{dt} = \dot{Q}_h + \rho q(t) \left[ c_p (T_{in}(t) - T(t)) + \frac{w_i^2 - w_e^2}{2} + g (z_i - z_e) \right]$$  
(5.2)

Since the difference in velocity and elevation are negligible compared to the term containing the specific heat, this expression can be simplified to  
$$V \frac{dT(t)}{dt} = \frac{\dot{Q}_h}{\rho c_p} + q(t) (T_{in}(t) - T(t))$$  
(5.3)

This non-linear first order differential equation describes the temperature dynamics of the water in the tank.

Since we are dealing with computer control, it is necessary to discretize equation (5.3) with respect to the sampling period $T_s$. An important remark is that ‘$t$’ is used again, however it denotes the discrete time index now, and not the continuous time. After discretizing with respect to $T_s$, equation (5.3) becomes  
$$V \left( \frac{T(t) - T(t-1)}{T_s} \right) = \frac{\dot{Q}_h}{\rho c_p} + q(t-1) (T_{in}(t-1) - T(t-1))$$  
(5.4)

A few manipulations of equation (5.4) result in:  
$$T(t) - T(t-1) = \frac{\dot{Q}_h T_s}{\rho V c_p} + \frac{q(t-1) T_s}{V} (T_{in}(t-1) - T(t-1))$$  
(5.5)

$$T(t) = T(t-1) + \frac{T_s}{V} \left[ \frac{\dot{Q}_h}{\rho c_p} + q(t-1) (T_{in}(t-1) - T(t-1)) \right]$$  
(5.6)

This expression represents the discrete time model of the water tank and will be validated against experimental data.

### 5.3 Heat loss

#### 5.3.1 Of the tank

The following heat losses should be considered [22],[23]:  
• conduction through the walls of the tank;  
• natural convection at the side and bottom of the tank walls, and at the water surface.
For simplicity we will consider the tank to have a rectangular shape. Note that this will not have an important effect on the result. We will assume a worst case scenario of 40 °C water temperature, which will be approximately the maximum temperature for the system. The temperature of the surroundings is supposed to be 18 °C. For each type of heat transfer we will calculate the corresponding thermal resistance.

**Conduction**

The thermal resistance for conductive heat transfer through a straight wall is given by

\[
R_{\text{conduction}} = \frac{\Delta x}{k A}
\]

(5.7)

with \(\Delta x\) the thickness and \(A\) the area of the wall. For the tank, the walls are made of 5 mm thick plexiglass (PMMA) which has a thermal conductivity \(k\) of 0.18 \(\frac{W}{mK}\). Each side wall has an area of 0.0136 \(m^2\) resulting in a thermal resistance of:

\[
R_{\text{cond},\text{sidewall}} = 2.04 \frac{K}{W}
\]

For the floor wall (area 0.01 \(m^2\)) we obtain:

\[
R_{\text{cond},\text{floor}} = 2.78 \frac{K}{W}
\]

**Convection**

The thermal resistance for convective heat transfer at a straight wall is given by

\[
R_{\text{convection}} = \frac{1}{h A}
\]

(5.8)

with \(h\) the convective heat coefficient. This parameter is calculated out of semi-empiric formulas that can be found in specialized literature [22]. Here, only the results are given:

Natural convection at a vertical wall: \(h = 5.91 \frac{W}{m^2K}\)

Natural convection at the bottom: \(h = 2.55 \frac{W}{m^2K}\)

Natural convection at the water surface: \(h = 5.70 \frac{W}{m^2K}\)

With these results we calculate the corresponding thermal resistances:

\[
R_{\text{conv},\text{sidewall}} = 12.43 \frac{K}{W}
\]

\[
R_{\text{conv},\text{floor}} = 39.23 \frac{K}{W}
\]

\[
R_{\text{conv},\text{surface}} = 17.55 \frac{K}{W}
\]
Total Heat loss

Once we have all thermal resistances, the respective heat losses can be calculated:

\[ Q_{\text{sidewall}} = \frac{T_{\text{water}} - T_{\text{surrounding}}}{R_{\text{cond, sidewall}} + R_{\text{conv, sidewall}}} = 1.38 \, \text{W} \]

\[ Q_{\text{floor}} = \frac{T_{\text{water}} - T_{\text{surrounding}}}{R_{\text{cond, floor}} + R_{\text{conv, floor}}} = 0.48 \, \text{W} \]

\[ Q_{\text{surface}} = \frac{T_{\text{water}} - T_{\text{surrounding}}}{R_{\text{conv, surface}}} = 1.14 \, \text{W} \]

Hence, the total heat loss is obtained:

\[ Q_{\text{total}} = 4Q_{\text{sidewall}} + Q_{\text{floor}} + Q_{\text{surface}} = 7.14 \, \text{W} \]

Although in this calculation heat loss due to radiation and evaporation are not taken into account, it is clear that the heat loss will be small compared to the heat input of 1100 W.

5.3.2 In the tube

Heat loss without insulation of the tube

Here we identify conduction through and natural convection at the wall of the tube. The tube is made of PVC, which has a thermal conductivity \( k \) of 0.15 \( \frac{\text{W}}{\text{mK}} \). It is 9.5 m long, has an inner diameter of 12 mm and a wall thickness of 2 mm. In all these calculations we will consider the system in steady state, meaning a constant water temperature in the tank and a corresponding constant outflow \( q \).

The thermal resistance for conduction through a circular tube is

\[ R_{\text{conduction}} = \frac{\ln(r_o/r_i)}{2\pi k L} \tag{5.9} \]

with \( r_o \) and \( r_i \) the outer and inner radius of the tube, and \( L \) its length. Substituting the real values we obtain

\[ R_{\text{conduction}} = 0.032 \, \frac{K}{W} \]

After a number of calculations involving dimensionless units like Nusselt and Prandtl, the convective heat coefficient is: \( h = 20.96 \, \frac{W}{m^2K} \). Now, the thermal resistance for the convective heat transfer can be estimated:

\[ R_{\text{convection}} = \frac{1}{h 2\pi r_o L} \tag{5.10} \]

which results in

\[ R_{\text{convection}} = 0.10 \, \frac{K}{W} \tag{5.11} \]

The overall heat loss results in

\[ Q = \frac{T_{\text{water,avg}} - T_{\text{surrounding}}}{R_{\text{conduction}} + R_{\text{conv, surface}}} = 137 \, \text{W} \]
The average temperature of the water inside the tube is considered here. Note that this temperature is not known in advance since the heat loss determines the temperature drop over the tube and therefore the value of this average temperature. After a few iterations however (starting with the tank temperature as initial condition), the solution reaches a steady state.

The temperature drop resulting from a certain heat loss can be found considering an energy balance:

$$\Delta T = \frac{Q}{\rho q c_p}$$  \hspace{1cm} (5.12)

With a heat loss of 137 W, corresponding to a water temperature in the tank of 40 °C, the temperature drop over the tube amounts to 3.9 °C, a considerable temperature loss. The same calculations were done for different flows (and their corresponding tank temperatures). Figure 5.2 gives an idea of the temperature drop over the tube as a function of the flow:

![Figure 5.2: Tank and outlet temperature as functions of the flow (tube without insulation)](image)

As it can be seen in Figure 5.2, the heat loss has an important influence on the temperature. At this point we decided to insulate the outlet tube, in order to reduce the heat loss.

**Heat loss for the insulated tube**

As insulation material polyethylene foam was chosen, with a thickness of 1 cm and a thermal conductivity $k$ of 0.035 $\frac{W}{mK}$. The same calculations for a worst case temperature of 40 °C were done, to be able to compare the results.

Formula (5.9) is now to be applied two times: first for the wall of the tube itself (which results in the same thermal resistance) and secondly for the insulator. We obtain:

$$R_{\text{cond,tube}} = 0.032 \frac{K}{W}$$

$$R_{\text{cond,insulation}} = 0.388 \frac{K}{W}$$
Hence, the total thermal resistance for conduction is:

\[ R_{\text{cond,total}} = 0.418 \frac{K}{W} \]

Note that the resistance has increased a lot compared to the non-insulated tube.

After application of several semi-empiric formulas, the convection coefficient is estimated as 
\[ h = 12.97 \frac{W}{m^2K} \]. The thermal resistance for convection now becomes:

\[ R_{\text{convection}} = 0.072 \frac{K}{W} \] \hspace{1cm} (5.13)

As expected - due to the larger contact area with the surrounding air - this thermal resistance has slightly decreased compared to the non-insulated tube. However the decrease is much smaller than the increase of the thermal resistance for conduction. The overall heat loss is now:

\[ Q = \frac{T_{\text{water,avg}} - T_{\text{surrounding}}}{R_{\text{condaction}} + R_{\text{convection}}} = 47 \text{ W} \]

which is three times less than in the case of a non-insulated tube!

In Figure 5.3 the heat loss is plotted as function of the flow for the insulated and non-insulated case. As it can be seen, the insulation has a great impact. Using equation (5.12), the temperature drop is now only 1.3 °C for a tank temperature of 40 °C.

![Figure 5.3: Heat loss as function of flow for insulated and non-insulated tube](image)

Yet, the tank and outlet temperature as functions of the flow are compared again. Figure 5.4 shows that the temperature drop due to heat loss is much smaller in this case, and that the insulation serves well to our purposes.
An important consequence is that, by adding the insulation, the thermal capacitance of the tube has increased significantly. It can be expected that this will introduce additional dynamics to the system. That is why a model of the outlet tube itself is required.

5.4 Outlet Tube

5.4.1 Dynamics

The dynamics of the outlet tube are governed by the thermal resistance and capacitance of the insulation. The ability to absorb heat decreases the temperature of the outflowing water downstream. As the tank temperature is variable in time, we face a difficult modelling problem.

In a first approach, the insulated tube is split up into small sections, each section having a certain temperature (Figure 5.5).

\[ T_i(x) \] is the temperature of the insulation, while \( T_w \) denotes the water temperature. \( A_i \) and \( A_w \) are the cross sectional areas of the insulation and the tube respectively. Taking \( \Delta x \) to the
limit, temperatures of adjacent sections can be approximated by linearization:

\[
T_i(x - \Delta x, t) \approx T_i - \Delta x \frac{\partial T_i(x, t)}{\partial x}
\] (5.14a)

\[
T_i(x + \Delta x, t) \approx T_i + \Delta x \frac{\partial T_i(x, t)}{\partial x}
\] (5.14b)

The same type of equations can be derived for \(T_w(x - \Delta x, t)\) and \(T_w(x + \Delta x, t)\).

Heat transfer takes place in two ways: from the fluid to the insulation and from the insulation to the surroundings at temperature \(T_a\) [14]. The respective thermal resistances are taken from \(r_w\) to \(r_0\) and from \(r_0\) to \(r_i\). Partial differential equations result when applying thermal laws of heat transfer. For a fluid element, the energy balance is:

\[
\rho_c p_w \left[ A_w \frac{\partial T_w(x, t)}{\partial t} + q(t) \frac{\partial T_w(x, t)}{\partial x} \right] = \frac{2\pi k_i}{\ln(r_0/r_w)} \left[ T_i(x, t) - T_w(x, t) \right]
\] (5.15)

and for an insulation element:

\[
A_i \rho c p_i \frac{\partial T_i(x, t)}{\partial t} = \frac{2\pi k_i}{\ln(r_i/r_w)} \left[ T_w(x, t) - T_i(x, t) \right] - \frac{2\pi k_i}{\ln(r_i/r_0)} \left[ T_i(x, t) - T_a \right] + k_i A_i \frac{\partial^2 T_i(x, t)}{\partial x^2}
\] (5.16)

These complex equations can not be integrated in a closed form. A solution can only be obtained through finite element programming. Considering the predictive nature of the Smith and EPSAC-based controllers, it is obvious that this approach is far too complex to be implemented in a real-time environment. A compromise has to be found between model accuracy and simplicity.

It is expected that the insulation will reduce ambient heat loss significantly, hence heat loss is not considered anymore. Moreover, even with small losses, a uniform temperature of the tube water can be assumed, averaged between the tank temperature \(T_{\text{tank}}(t)\) and the (unknown) outlet temperature \(T_{\text{out}}(t)\). Similarly, a uniform profile for the insulation is supposed. Under these assumptions, the \(x\)-dependence disappears. Little accuracy is lost while the reduction in complexity is huge.

Expressing the heat loss of the water as the total heat transferred to the insulation, the following equation is obtained:

\[
\rho_w c_{p,w} q(t) \left[ T_{\text{tank}}(t) - T_{\text{out}}(t) \right] = \frac{2\pi k_i}{\ln(r_0/r_w)} \left[ \frac{T_{\text{tank}}(t) + T_{\text{out}}(t)}{2} - T_i(t) \right]
\] (5.17)
This transferred heat leads to an increase in insulation temperature only because no ambient heat loss is considered:

\[
\frac{2\pi k_i}{\ln(r_0/r_w)} \left[ \frac{T_{\text{tank}}(t) + T_{\text{out}}(t)}{2} - T_i(t) \right] = \rho_i c_{p,i} A_i \frac{\delta T_i}{\delta t} \quad (5.18)
\]

The time dependence of the flow - and subsequently the tank temperature - still makes it impossible to integrate these equations in closed form. It is expected though that due to thermal inertia the rate at which flows are changed - the sampling frequency in fact - will be much bigger than the inverse time constant of the insulation. Hence these fast changes in flow will not have a significant effect on the insulation temperature. Only the averaged flow over a time interval approximately equal to the time constant of the insulation is important. A certain steady state temperature \(T_{av}\) in the tank corresponds to this averaged flow \(q_{av}\). As a result, the equations (5.17) and (5.18) can be integrated in closed form.

Introducing the following functional parameters:

\[
\alpha = \rho_w c_{p,w} q_{av} \quad (5.19a)
\]

\[
\beta = \frac{2\pi k_i}{\ln(r_0/r_w)} \quad (5.19b)
\]

\[
\gamma = \rho_i c_{p,i} A_i \quad (5.19c)
\]

the temperature at the outlet becomes:

\[
T_{\text{out}}(t) = T_{av} + [T_i(t = 0) - T_{av}] \frac{2\alpha}{2\alpha + \beta} \exp \left( -\frac{2\alpha\beta}{\gamma (2\alpha + \beta)} t \right) \quad (5.20)
\]

It is clear that the assumptions of an averaged flow and an averaged tank temperature are strong simplifications of reality and, even then, only valid for a certain operating range. Still, the result indicates that a linear first order model is acceptable. Both the time constant and the gain seem to be dependent on the flow. For the gain there is even an extra dependence on the initial insulation temperature.

As no accurate data about the insulation is provided, the parameters of this model will be estimated experimentally. Moreover, they will be kept constant for the whole range of temperatures.

### 5.4.2 Variable Time Delay

The measurement point for the water temperature is located in the outlet tube, at a distance \(L\) from the tank. Due to the variable flow, the time the water needs to get from the tank to the sensor will vary as well. For this duration (which is actually the variable time delay) the following expression was given in the introduction:

\[
d = \frac{LS}{q} \quad (5.21)
\]

This formula is obtained easily, considering that time is in fact distance divided by velocity.

If however an elementary mass particle is followed from its departure in the tank until it passes the sensor, formula (5.21) would only be valid if this mass particle traveled the whole
distance at the same fixed speed (or flow, which is equivalent). In reality this is not the case: the particle will - during the sampling time $T_s$ - flow at a certain speed and thereby bridge only a fraction of the total distance. The next sampling period, the flow (and thus the speed) might have changed, meaning a different fraction of the total distance will be covered (Figure 5.7). After a few sampling times the mass particle will eventually pass the sensor, where its temperature is registered. The number of sampling periods needed to reach the measurement point is in fact the discrete time delay, denoted by $N_d$.

![Figure 5.7: Position at each sampling time of the mass particle flowing through the outlet tube](image)

Therefore, the variable time delay $d$ will not only depend on the current flow $q(t)$, but also on flows that have been applied previously. For continuous time we calculate the variable time delay as:

$$d = \frac{LS}{\int_{t-d}^{t} q(\tau) \, d\tau} \quad (5.22)$$

where $\int_{t-d}^{t} q(\tau) \, d\tau$ is the averaged flow that the mass particle experienced during its passage through the tube. Equation (5.22) can be simplified:

$$\int_{t-d}^{t} q(\tau) \, d\tau = LS$$

(5.23)

From a physical point of view, this formula is equivalent to filling the tube at varying flow rates.
The continuous description of the variable time delay can be translated to discrete time, with $T_s$ as sampling period. Equation (5.23) then becomes:

$$T_s \sum_{i=1}^{N_d} q(t - i) = LS$$

(5.24)

where $t$ now denotes the discrete time index, and $N_d$ the discrete time delay.

At each sampling instant, $N_d$ is calculated as the number of flows to be summed before the total sum exceeds $\frac{LS}{T_s}$. The reason to exceed $\frac{LS}{T_s}$ can be seen from the definition of a discrete time delay, shown in Figure 5.8:

![Figure 5.8: Definition time delay](image)

The discrete time delay calculated in this way is however not expected to be completely exact: there can be a difference between the real physical flow and the calculated flow, and more important, the temperature sensor has a certain time delay as well (due to its thermal inertia). Obviously, equation (5.24) will need some corrections.

### 5.5 Identification

Now that the different theoretical models have been derived, it is necessary to validate them against experimental data. For the dynamics of the outlet tube there are even no numerical values obtained yet. Unfortunately common identification techniques like correlation and spectral analysis, or linear parameter estimation techniques like least squares method, instrumental variables method and prediction error method are difficult to apply. They require the linearization of the non-linear system dynamics around a certain operating point $q^*$ [6]. With this $q^*$ applied, an uncorrelated sequence of flow variations excites the process. Analysis of
the measured temperatures according to one of the previous estimation techniques, leads to
the estimation of parameters which are only valid for the linearization region. As a result,
different parameter sets for different linearization regions are obtained. For slowly reacting
systems however - which is the case for thermodynamic processes - it is unlikely that these
small and relatively fast changes in input will result in useful temperature changes. For
the outlet tube these methods would even be more difficult to apply as the input (the tank
temperature in this case) can only be indirectly manipulated.

It seems that the easiest way to perform the validation is to use graphical methods like step
responses. Moreover, in relatively simple processes like this one, it is expected that these
graphical methods will be sufficient to have an idea of the accuracy of the theoretical models.
First the dynamics of the tank will be discussed, followed by the dynamics of the outlet tube,
and - last but not least - the variable time delay.

5.5.1 Water Tank

Here the third measurement of the tank temperature (besides the tap water temperature and
tube outlet temperature) finds its relevance. Simple step responses were performed, changing
the flow from a steady state value \( q_1 \) to a new constant flow \( q_2 \). The tank dynamics (5.3) are
linearized around \( q^* = q_2 \) and written as a transfer function:

\[
V \frac{d\Delta T(t)}{dt} = (T_{in}(t) - T^*)\Delta q(t) - q^*\Delta T(t)
\]

(5.25)

\[
(Vs + q^*)\Delta T(s) = (T_{in} - T^*)\Delta q(s)
\]

(5.26)

\[
\frac{\Delta T(s)}{\Delta q(s)} = \frac{T_{in} - T^*}{Vs + q^*}
\]

(5.27)

\[
\frac{\Delta T(s)}{\Delta q(s)} = \frac{(T_{in} - T^*)/q^*}{Vs + q^*}
\]

(5.28)

where the steady state temperature \( T^* \) corresponds to the applied flow \( q^* \), and \( \Delta T \) and \( \Delta q \)
are variations around the steady state values \( T^* \) and \( q^* \). The static gain is given by

\[
\frac{T_{in} - T^*}{q^*}
\]

(5.29)

while

\[
\frac{V}{q^*}
\]

(5.30)

defines the time constant. With \( \Delta q \) just a step, the solution for this system is an exponential
of the form

\[
T_{tank}(t) = T_{init} + (T_{final} - T_{init})\left(1 - e^{-t/\tau}\right)
\]

(5.31)

with \( \tau \) the time constant corresponding to \( q^* \). The purpose now is to link the measurements
of the tank temperature to such a first order curve, by graphical identification. Thus, an
experimental time constant and gain are derived, as is shown in Figure 5.9.
Assuming a perfect first order process, the time constant is estimated via the temperature corresponding to 63% of the process gain:

$$T_{63\%} = 0.63(T_{\text{final}} - T_{\text{init}}) + T_{\text{init}}$$

(5.32)

The time to reach this temperature defines the time constant corresponding to the newly applied flow $q^\ast$.

This procedure was used for different flows, and for both heating up and cooling down, with the results as shown in Figure 5.10. As it can be seen there is a clear correspondence between the theoretical results - given by $\tau = \frac{V}{q}$ - and the experimental ones, although the latter seem to be a bit higher. The average relative error is 13.1%. This deviation could be explained by the fact that there are other devices in the tank, which are also heated and have a small influence on the time constant. We also note that there is no clear difference between heating up and cooling down.

Figure 5.9: Step response
Figure 5.9 also shows an exponential approximation (green curve) of the measurement, using the theoretical time constant. Apparently, the result is acceptable.

Having a look at the static gain, the correspondence is quite good as well. Figure 5.11 compares the theoretic results with the experimental ones for both heating up and cooling down.

The averaged relative error in this case amounts to 9.2%. Finally, to give a true validation
of the theoretical model, Figure 5.12 presents an experiment, comparing the measurement of the tank temperature (blue curve) with its estimation (red curve), using the discretized non-linear system equation (5.6).

The time delay between the measurement and the estimation is caused by the reaction time of the sensor, which is a few seconds. There is also a difference in the steady state value, but this kind of error can be easily overcome by the controller. The dynamics of the estimation - which uses the theoretical model - match the real measurement very well. Therefore, it is decided to use further the theoretical model with the nominal parameter values for controller design.

5.5.2 Outlet Tube

In section 5.4 it was proved that under certain assumptions and simplifications the outlet tube could be approximated as a first order system:

\[
\frac{K}{RCs + 1}
\]

(5.33)

where \( R \) represents the thermal resistance and \( C \) denotes the capacitance of the tube. Multiplied they define the time constant of the tube \( RC \). \( K \) represents the static gain.

To estimate the time constant \( RC \), the discretized version of equation (5.33) was compared with real measurements. By trial and error, \( RC \) has been changed until a reasonable match between the estimation and the measurement was achieved. However, as already mentioned, a linear first order model of the outlet tube does not perfectly describe the real dynamics. Therefore the model accuracy may vary a bit through the operating range. An averaged value of 29 for the time constant \( RC \) turns out to give the best results.

The static gain \( K \) is determined by the heat loss of the tube. As the heat loss is reduced significantly by the insulation, this value should be smaller but still close to 1, which is confirmed in Figure 5.13.
For the normal temperature range, a value of $K = 0.99$ could be taken as an average.

To compare the model with the reality, again an experiment is performed (Figure 5.14). The model of the tank and the tube are combined here, in order to estimate the outlet tube temperature.

### 5.5.3 Variable Time Delay

In section 5.4.2, a formula was derived to calculate the discrete time delay $N_d$. It was mentioned that the obtained value should be adjusted with a few samples to incorporate the reaction time of the temperature sensor. While checking the adjustments, there seemed to be
a discrepancy between heating up and cooling down: for heating up about 12 seconds were to be added (or $\frac{12}{s}$ in discrete time), while for cooling down the formula did not need any adjustment. A possible explanation could be that for a large step down in flow, the conversion factor between control voltage and flow might not be that accurate anymore (due to friction, see IxR compensation in section 3.1.2). After adjusting the formula for the variable time delay, the results are satisfactory.
Chapter 6

Controllers

The setup as it was designed and implemented in the previous chapters, could in fact be a scale model of a larger industrial application, which shares the problems of non-linearity and huge variable time delay. Such processes are very difficult to control with today’s most common industrial controllers (PID’s). However, with all knowledge packed in process models describing its dynamics, advanced control methods can be implemented. One of these advanced control methods uses on-line predictions of the future plant output to compute optimal future control actions. This control methodology is known as Model Based Predictive Control (MBPC, also called MPC).

In fact, MBPC is a family of control methods that rely on intuitive and easy to understand principles. The various MBPC-algorithms differ in the type of model used for prediction and the criterion they minimize [3]. One of the algorithms that have been used with success in practice is EPSAC\(^1\) [7]. Its inherent dead time compensation and ability to handle non-linear process dynamics makes it particularly suited to control the heating tank system. Simulation results for a similar system, which show the effectiveness of this control strategy can be found in [5].

This chapter deals with the implementation of the EPSAC-strategy for our setup. Besides the difference in scale compared to the system presented in [5], our control problem is more complex:

- The tube introduces dynamics into the system.
- Real-life implementation always brings up difficulties that can not be experienced in a simulation environment.

The most important performance-oriented tuning parameters are presented and discussed\(^2\).

\(^1\)Non-Linear Extended Prediction Self-Adaptive Control

\(^2\)It is however not the purpose to go into a detailed description of the EPSAC-methodology. For a detailed description of EPSAC, we refer to [7]. In this case, the controller will be gradually tuned to the process needs through an analysis of the measured data.
CHAPTER 6. CONTROLLERS

6.1 Non-linear EPSAC controller

6.1.1 General approach

As in all MBPC-methods, non-linear EPSAC calculates the optimal control input based on predictions of the output over a certain horizon, called the prediction horizon $N_2$. This is a well known procedure which does not need any adaptation in order to control the heating tank system with its variable time delay. Before the control loop layout and the influence of tuning parameters are presented, this procedure will be briefly discussed, though specifically in the interest of the heating tank system.

The future process output, as it is seen by EPSAC, comes from two contributions:

$$ y(t + k|t) = y_{base}(t + k|t) + y_{opt}(t + k|t) \quad k = 1..N_2 $$ (6.1)

1. $[y_{base}(t + k|t), k = 1..N_2]$ is the basic output which originates from several effects:

   - past control inputs $[u(t - 1), u(t - 2), u(t - 3), ...]$. These inputs have been applied to the process and hence, their effect on the output can be predicted based on the process model and its state $[x(t - 1), x(t - 2), x(t - 3), ...]$. Obviously, for a system with a large time delay, many past inputs will have to be taken into account.

   - the basic future control scenario $[u_{base}(t + k|t), k = 0..N_2 - 1]$, which is a postulated input sequence. This is part of the optimal control scenario to be calculated by EPSAC. The choice of $u_{base}$ is irrelevant for linear systems: any choice will lead to the same optimized control input sequence. For non-linear systems though, a good ‘guess’ of $u_{base}$ has to be made in order to reduce computation time.

   - predicted disturbances $[n(t + k|t), k = 1..N_2]$. These predictions are calculated via a noise model which captures - ideally - all effects in the measured output $y(t)$ that are not included in the process model itself. Out of these effects, the most obvious are the modelling errors, measurement noise and random disturbances.

   A smart choice of the noise model can enhance the controller’s performance significantly since more information about the process is provided to the controller. However, deriving a good noise model is hard, precisely due to the random nature of the disturbances. Most of the time, a default noise model of the form $\frac{1}{1-\sigma^2}$ is implemented. As this default model can only deal with steady state errors, effort will be taken to derive a more accurate model. This must be seen as a part of the controller tuning as the improved noise model is only relevant for the process under consideration.

2. $[y_{opt}(t + k|t), k = 1..N_2]$ is the result of the future optimizing control actions $[\delta u(t + k|t), k = 0..N_2 - 1]$. These optimizing control actions are the second part of the optimal control scenario, which is obtained by adding them to the a priori defined basic input:

$$ u(t + k|t) = u_{base}(t + k|t) + \delta u_{opt}(t + k|t) \quad k = 0..N_2 - 1 $$ (6.2)
CHAPTER 6. CONTROLLERS

The optimizing control actions result from minimization of a cost function of the form:

\[ J = \sum_{k=N_1}^{N_2} [r(t + k|t) - y(t + k|t)]^2 + \lambda \sum_{k=0}^{N_2-1} [u(t + k|t)]^2 \]  

(6.3)

where \( r(t + k|t) \) denotes the reference trajectory (by default equal to the setpoint, see Figure 6.1). This cost function takes into account both the deviation of the process output from the desired output, and the control effort. It introduces two parameters: \( \lambda \) and \( N_1 \). \( \lambda \) is called the weighting factor and defines the importance of the control effort compared to the deviation of the process output. In this control problem however, there are no reasons to include the control effort in the cost function, hence \( \lambda \) will be taken equal to zero. The other parameter, \( N_1 \) is by default equal to the time delay of the process. This makes good sense, since the effect of the optimal control input will only be visible after the time delay. This parameter is particularly interesting when studying variable time delay systems.

In this general procedure, there are many tuning parameters. Several of them have already been presented. Amongst the others is also \( N_u \), called the control horizon, which has a role to play in the structuring of the future control sequence \([u(t + k|t), k = 0..N_2 - 1]\). Basically, \( N_2 \) control inputs have to be postulated, introducing as many degrees of freedom as the number of components in the control sequence. With \( 1 \leq N_u \leq N_2 \), the control input remains constant after \( N_u \), reducing the degrees of freedom and favourizing stability. In the control of the heating tank system, \( N_u \) is taken equal to one, as this simplifies computation thoroughly. Figure 6.1 shows the concept of EPSAC with \( N_u = 1 \).

![Figure 6.1: EPSAC control concept for \( N_u = 1 \)](image)

Though common to both linear and non-linear EPSAC, the control strategy as it was presented is specifically derived for linear systems for which the superposition principle holds. However, extensions have been made for non-linear system control: at each sampling instant the optimizing control sequence is calculated in an iterative way, adjusting the basic future control scenario \( u_{base} \) in such a way that the optimizing control increments \( \delta u_{opt} \) gradually
go to zero. In this way, the superposition principle is no longer involved and the basic control scenario $u_{\text{base}}$ converges to the optimal solution. To limit the number of iterations and hence the computation time, it is important to make a good initial ‘guess’ for the basic future control scenario. An effective choice for $u_{\text{base}}$ is the previously applied optimal control input: $u_{\text{base}}(t + k|t) \equiv u(t - 1)$. This approach will also be implemented in the control of the heating tank system. The number of iterations will be referred to as $N_i$.

As already mentioned, the optimal control actions are calculated by minimizing the cost function (6.3), rewritten in matrix notations, with $Y$ and $R$ the future process output and the reference trajectory respectively:

$$J = \sum_{k=N_1}^{N_2} [r(t + k|t) - y(t + k|t)]^2 = (R - Y)^T (R - Y) \tag{6.4}$$

In the specific case of $N_u = 1$, the sequence of optimizing control actions is simplified to $[\delta u(t + k|t) \equiv \delta u(t|t)]$. This can be seen as a step input with magnitude $\delta u$, applied at time $t$. Hence, $y_{\text{opt}}$ can be calculated as a step response.

For non-linear systems the step responses at every iteration can be calculated in two ways: through simulation based on the non-linear process model, or through local linearization. This last calculation procedure is implemented in this case: every iteration, the process is linearized around $u_{\text{base}}$ (which is in turn iteratively adapted) to extract its step coefficients $g_k$. As a result, $y_{\text{opt}}$ is calculated as:

$$y_{\text{opt}} = \begin{pmatrix} g_{N_1} \\ g_{N_1 + 1} \\ \vdots \\ g_{N_2} \end{pmatrix} \delta u = G \cdot \delta u \tag{6.5}$$

Substituting this expression into equation (6.4), the cost function appears as a function of $\delta u$:

$$\sum_{k=N_1}^{N_2} [r(t + k|t) - y(t + k|t)]^2 = ((R - Y_{\text{base}}) - G \cdot \delta u)^T ((R - Y_{\text{base}}) - G \cdot \delta u) \tag{6.6}$$

The minimum cost index is obtained for

$$\delta u = (G^T G)^{-1} G^T (R - Y) \tag{6.7}$$

### 6.1.2 Control loop layout

For output prediction, two model configurations are possible: one is the parallel model, the other one is the series-parallel model. The first one can only be used for stable processes, while the second can also be used for unstable processes. Since the process under consideration is stable, the implementation of the parallel model as depicted in Figure 6.2 is used.
Figure 6.2: Parallel model implementation

Estimation of the current process output

The process model consists of the non-linear tank dynamics, the dynamics of the tube and the model of the variable time delay. At each sampling instant, the delay-free model output $x(t|t)$, which results from the process dynamics only, is estimated using the stored values $[x(t-1), x(t-2), ..., u(t-1), u(t-2), ...]$. Also at the same sampling instant, the variable time delay is computed from equation (5.24), with the corrections presented in section 5.5.3:

$$T_s \sum_{i=1}^{N_d} q(t - i) = LS$$

Once $N_d$ is known, $x(t - N_d)$ can be selected out of the database of $x$-values, such that $z(t|t) = x(t - N_d)$. Figure 6.3 illustrates this procedure and reveals the odd effect that appears when dealing with a variable time delay.
Figure 6.3: Estimation procedure for the outlet temperature
First of all, when the time delay has rapidly decreased, some samples do not appear in the output. Physically, this means that a high flow has been applied to the process, able to remove a lot of water from the outlet tube within just one sampling period. Subsequently, the water which acted as a memory for the tank temperature (because its temperature is the result of previous tank temperatures), is pumped out of the tube without even being measured! This effect is illustrated in Figure 6.3 at the transition from \((t - 5)\) to \((t - 4)\). The opposite occurs when suddenly a small flow is applied. As a result, the time delay increases. Because measurements are taken every sampling instant only, it appears that one sampling period later exactly the same temperature is measured again! Physically, this is of course only possible if the flow is set to zero. That is why this effect is merely caused by estimating \(x\) every \(T_s\) seconds only: no continuous temperature profile is available beforehand and thus approximations have to be made. This effect is shown on Figure 6.3 between \((t - 3)\) and \((t - 2)\). Once the model output at time \(t\) has been calculated, it is subtracted from the real measured output \(y(t)\) to estimate the noise \(n(t)\).

It is worthwhile noticing that, on a physical basis, the maximum increase in time delay cannot be more than just one sampling period. On the contrary, temperature of the water that has already been removed from the outlet tube is measured, which makes of course no sense. Formula (5.24) respects this physical truth as can been seen in Figure 6.4.

\[
\text{Figure 6.4: Example of the discrete time delay evolution according to (5.24)}
\]

**Prediction of the future process output**

Now that the parallel estimation procedure has been presented and the effects of variable time delay have been pointed out, the prediction procedure is to be set about. The complexity of this prediction procedure is of a higher order than the estimation procedure. In systems with constant time delay, the manipulated temperatures will occur once the time delay has passed. This is expressed as \(N_1 = N_{t_d}\) hence only the manipulated outputs are included in the cost function. In systems with variable time delay however, the choice of \(N_1\) is not obvious anymore, especially when this time delay depends on the controlled input and, more particularly, when the control horizon \(N_u\) is not equal to one. Predictions of the future time
delay should be made in order to determine the temperature profile at the output to be able to extract only that part which results from the future flows. Hence, it is not straightforward what the value of $N_1$ should be.

In many practical applications however, the simplification of $N_u = 1$ has lead to amazingly good results [7]. Therefore it is not expected that this assumption will significantly affect the controller’s performance. Additionally, in this case it would reduce complexity greatly. That is why only $N_u = 1$ will be considered.

Basically, the prediction procedure goes in the same way as the estimation procedure. At each sampling instant, predictions $[x(t+k|t), k = 1..N_2]$ are made, using the postulated future control sequence $q_{\text{base}}$ as input and the previously estimated values $[x(t), x(t-1), ...]$ as the current state of the system. Then, the variable time delay is computed to select the appropriate $x((t+k) - N_d)$ as the prediction of the outlet temperature $z(t+k|t)$. Only the predictions $[z(t+k|t), k = N_1..N_2]$ are included in the cost function because they are manipulated by $q_{\text{base}}$. This procedure is illustrated in Figure 6.5.

This strategy is very time consuming due to the number of calculations involved. First of all, the states $x(t+k|t)$ must be predicted; secondly, for each single state prediction the algorithm to calculate the variable time delay has to be called in order to obtain the future time delays; and thirdly, appropriate selections must be made to compose the future temperature profile at the output. Only a part of this profile is used, namely from $N_1$ (which varies as well) to $N_2$. Enough predictions must be included in the cost function to have a stable closed loop and reasonable performance (see section 6.1.4). Consequently, the prediction horizon $N_2$ must be chosen with respect to $N_1$, which makes it variable as well. This in turn means that the number of predictions $x(t+k|t)$ is not just a straightforward value. That is why the prediction procedure is far more complex than might be assumed initially. It requires a lot of calculations and is difficult to implement.

---

$^3$Taking $N_u = 1$ means no degrees of freedom are left to optimize the control input with respect to its constraints. Hence, the method of clipping is implemented after the EPSAC computation of the optimal input.

$^4$For example, $N_2 = N_1 + C$ where $C$ denotes the number of samples in the cost function.
Figure 6.5: Prediction procedure of the outlet temperature
Figure 6.6: Snapshots of the future outlet temperature profile at different sampling instants
Nevertheless, this whole prediction procedure can be simplified profoundly when taking a look at the physics of the system. Since only the manipulated outlet temperatures are of interest (starting from $N_1$), it is needless to go through the calculation of the entire temperature prediction of the outlet. When virtually applying a constant flow $q_{\text{base}}$, the temperatures resulting from this flow will be registered at the outlet exactly after $\frac{LS}{q_{\text{base}}}$ seconds! Physically, this makes good sense, since all water must be pumped out of the tube before the new manipulated temperatures can be measured. This is visualized in Figure 6.6. At time $t$, the outlet tube holds a certain temperature profile in which we are in fact not interested, because these temperatures can not be manipulated anymore. When virtually applying $q_{\text{base}}$, the water of the outlet tube is gradually removed to be replaced with water at temperatures which result from $q_{\text{base}}$. Since $q_{\text{base}}$ is constant (due to the choice $N_u = 1$), it will take exactly $\frac{LS}{q_{\text{base}}}$ seconds before all water is removed and the manipulated temperatures are registered. Moreover, typical effects of variable time delay as skipping samples at the outlet or holding the same temperature for more than one sampling instant (Figure 6.3) do not appear anymore! Exactly the same samples $x(t + k|t)$ as they were predicted by the dynamic models only, will be measured at the outlet. The reason is that $q_{\text{base}}$ remains constant, freezing the time delay and perfectly synchronizing the outlet temperatures with the ones entering the tube at every sampling instant$^5$.

---

$^5$The fact that the tube has got dynamics, does not change anything to this reasoning. If the dynamics of the tube are separated from the time delay it introduces, the temperatures entering the tube would then be the result of all process dynamics, hence the tube is represented as a time delay only.
Consequently, it is sufficient just to predict the temperatures resulting from the dynamics, without predicting the entire outlet temperature profile. It is important to stress however that even though this profile is not calculated as such, the outlet temperature is still the controlled variable! Going through the explicit procedure (as presented in Figure 6.3) would exactly lead to the same result, but would require much more effort and much more complexity to be overcome. In this way, the prediction procedure is thoroughly simplified. \( N_1 \) is no longer varying and obviously equal to one. Hence, \( N_2 \) is also constant. The variable time delay model is only used for noise computation. The non-linear EPSAC-structure as it is implemented for the heating tank system is presented in Figure 6.7.

6.1.3 Choice of \( T_s \)

Before any computer control can be implemented, the sampling time \( T_s \) must be chosen. Since the measured signals are fed through lowpass filters having a cutoff frequency of 1.1 Hz, aliasing will not occur if the sampling frequency is at least double, which is the Nyquist frequency in this case. However, the bandwidth of the system - linearized around the maximum flow - is approximately 0.004 Hz, which is far less than the proposed sampling frequency. In order not to overload the computer needlessly, a choice has to be made without the risk of mixing up high frequency disturbances with the signals of interest. That is why the sampling rate is chosen equal to 5 Hz, so that no aliasing can occur. The higher frequency signals are filtered out through averaging over a certain period. This period is the sampling time \( T_s \) used to control the system.

To determine \( T_s \), there is a commonly accepted rule of thumb proposing the sampling time to be approximately one tenth of the system time constant. In this case, the time constant is varying. A conservative choice for \( T_s \) would be the one resulting from the smallest time constant:

\[
\tau_{\text{min}} = \frac{V}{q_{\text{max}}} = \frac{1.13l}{1.8l/min} = 37.7\text{ sec}
\]

(6.8)

This means that \( T_s \) should be around 4 seconds. Still, there is an experimental part in the choice of \( T_s \) as well. If \( T_s \) is chosen too large, it would limit the freedom of the controller to rapidly adjust the input in order to get the temperature to the setpoint as fast as possible. Secondly, a large \( T_s \) leads to poor discretization of the non-linear model and consequently to poor estimations. On the other hand, taking \( T_s \) too small would probably make the controller too sensitive and increase the computer workload needlessly. After several experiments with different sampling times, it turned out that the best choice would be to take \( T_s \) equal to 4 seconds.

6.1.4 Influence of \( N_2 \)

The prediction horizon \( N_2 \) determines the number of predictions made starting from time \( t + 1 \). Based on these predictions, the optimal control strategy is calculated to minimize the cost function. Even though there is no general rule for selecting \( N_2 \) appropriately, its choice is indeed of crucial importance for the controller’s performance.
Figure 6.8: Effect of different $N_2$ on the controller performance

Figure 6.9: Process input for different $N_2$

Figure 6.8 compares the measured outlet temperature for different values of $N_2$, each time for the same experiment. Figure 6.9 shows the process inputs. The effect of small values of $N_2$ is clearly visible, because they lead to a large overshoot. It also appears that the response is steeper for these smaller $N_2$ values. This suggests that the controller loses stability as the prediction horizon is decreased. This makes good sense since less information about the process dynamics is provided to the controller. In fact, when $N_2$ is decreased, fewer samples are included in the cost function which makes the newly calculated input to be dependent on just some predictions. Hence, the error on these predictions gains in influence and might destabilize the controller.
The overshoot is more pronounced for the step-up than for the step-down setpoint change. This could be due to non-linearity effects, but in fact, the overshoot is exactly the same for step-up and step-down when $N_2$ exceeds 15 ($\approx 0.4^\circ C$ in both cases). Hence, the explanation must be found in the difference in time constants. From the calculation of the respective steady state inputs, these time constants are:

$$\tau_{up} = \frac{V}{q_{ss,up}} = \frac{1.13 l}{0.63 l/min} = 108 \text{ sec}$$

$$\tau_{down} = \frac{V}{q_{ss,down}} = \frac{1.13 l}{0.72 l/min} = 94 \text{ sec}$$

The time constant is smaller for the step-down, which means that a smaller $N_2$ already predicts an important part of the transient behavior and thus leads to a better performance. Apparently, a further increase of $N_2$ form 15 to 20 does not affect the controlled outlet temperature anymore. Hence, including additional predictions of the end part of the transient behavior in the cost function seems not to have much influence. That is because these predictions are already fairly close to the setpoint, leading to just small additional differences to be included in the cost function.

Eventually, $N_2$ has to be chosen in such a way that a significant part of the transient behaviour is predicted. This favours both stability and controller performance. The choice should be made with respect to the time constant of the system. For systems with variable time constant, it is reasonable to take the conservative choice, that is the one corresponding to the largest time constant. In this case, $N_2$ equal to 15 seems to be a good compromise. Since $T_s$ was chosen equal to 4 seconds, the transient behaviour is predicted for 60 seconds. This leads to a good performance and does not impose computational challenges on the computer.

### 6.1.5 Influence of $N_i$

As a further increase of $N_2$ does not lead to better controller performance and since the tank dynamics are non-linear, it is recommended to increase the number of iterations $N_i$, shifting from linear EPSAC to non-linear EPSAC control.

In EPSAC-control (for the special case of $N_u = 1$), $\delta u_{opt}(t|t)$ is calculated only once, out of the predictions resulting from $u_{base}(t + k|t) = u^*(t - 1|t - 1), k = 1..N_2$ and the vector of step coefficients $G$ (equation (6.5)), which in turn results from local linearization of the system around $u_{base}(t|t)$. Then, $\delta u(t|t)$ is added to $u_{base}(t|t)$ to obtain the new control input.

---

$u^*(t - 1|t - 1)$ denotes the previously derived optimal control input
The superposition is in fact not valid for non-linear systems. However, this can be cancelled if \( \delta u_{\text{opt}}(t|t) \) is gradually brought to zero. It can be expected that this will be the case if at the same sampling instant, \( \delta u_{\text{opt}}(t^1|t) \) is added to \( u_{\text{base}}(t^1|t) \) to result in the new \( u_{\text{base}}(t^2|t) \) for which the procedure can be done all over again. Hence, the influence of \( \delta u_{\text{opt}}(t^1|t) \) will gradually disappear so that after \( N_i \) iterations \( u_{\text{base}}(t^{N_i}|t) \) is fairly close to the optimal control input \( u(t|t) \).

This procedure was implemented for the heating tank system. An experiment was performed for a step-up and step-down setpoint change, first with \( N_i = 1 \) and secondly with \( N_i = 6 \). As it can be seen in Figure 6.10, the influence of \( \delta u_{\text{opt}} \) does disappear since the largest component of \( G \cdot \delta u_{\text{opt}} \) is in the second case almost zero. Despite the input \( u(t|t) \) for \( N_i = 6 \) being closer to the optimal control input, the influence on the controlled outlet temperature is hardly visible (Figure 6.11)\(^7\). This is not due to non-linear EPSAC control, but due to the fact that the system does not show much non-linearity in the region were the setpoint changes were made. Hence, the linear principles of superposition and stepcoefficient calculation are well applicable, and consequently, linear EPSAC shows already good results.

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\(^7\)The variable time delay is clearly visible when performing a setpoint change. At the first change, it is approximately 125 sec, while at the second change it has decreased to almost 100 sec.
CHAPTER 6. CONTROLLERS

Figure 6.11: Controlled outlet temperature for different $N_i$

Figure 6.12 shows the selected setpoints on the static characteristic of the tank. The curve is linearized around these setpoints, suggesting that local linearization around $u_{base}$ at every sampling instant is in fact a good approximation.

Figure 6.12: Static characteristic of the tank

Moreover, if we look at (a part of the) computed input of the EPSAC controller (Figure 6.13), the changes in $\delta u_{opt}$ are so small that the dynamic response of the real non-linear system will be very well approximated by the linearized system\(^8\).

To see whether or not severe non-linearity effects occur in the controlled temperature range, an experiment is performed in which the setpoint is gradually changed over the entire range,

\(^8\)Recalling the chosen control policy of $u_{base}(t + k|t) = u^*(t - 1|t - 1), k = 1..N_2$, the changes of input calculated by the EPSAC controller are in fact $\delta u_{opt}$. For non-linear EPSAC however, $u_{base}(t + k|t)$ is constantly adapted so that $\delta u_{opt}$ can not be read off the figure.
each time with the same step of 2.5°C. The same experiment is done for both linear and non-linear EPSAC control (Figure 6.14).

![Comparison of (N)EPSAC controlled outlet temperature over the entire range](image)

**Figure 6.14:** Setpoint changes over the entire temperature region

Obviously, both controllers give more or less the same result, indicating that non-linearity is just a feeble effect, surpassed by other effects the controllers have to compensate for. This is also the reason why the step responses differ, going from undershoot to overshoot. For example, there is a clear effect of the error on the outlet temperature predictions. Figure 6.15 shows the estimation of the NEPSAC-controlled outlet temperature in different parts of the range compared to its actual measurement. Clearly for lower temperatures, the slope of the measured output is steeper than the estimation suggests. The opposite occurs at higher
temperatures, where the effect is far more pronounced. The reason must be found in a change in tube dynamics when the flow changes. Referring to section 5.4, the time constant and the gain of the first order model of the outlet tube change with the flow (equation (5.20)):

\[ K_{\text{tube}} \sim \frac{1}{1 + \frac{\beta}{2\alpha}} \]

\[ \tau_{\text{tube}} \sim \gamma \left( \frac{1}{2\alpha} + \frac{1}{\beta} \right) \]

where \( \beta \) and \( \gamma \) are constants for a given insulation configuration and a specific insulation material. The parameter \( \alpha \) is however proportional to the flow:

\[ \alpha = \rho_w c_{p,w} q_{av} \]

\( q_{av} \) was the averaged flow over a time interval approximately equal to the time constant of the insulation. In this case, \( q_{av} \) can be seen as the averaged flow between two setpoint changes. Hence, if the controller operates in the high temperature part of the range, \( q_{av} \) and thus \( \alpha \) will be small. As a result, \( K_{\text{tube}} \) will decrease, indicating that more heat is lost. On the other hand, \( \tau_{\text{tube}} \) will increase. This effect is clearly visible on Figure 6.15.

![Figure 6.15: Estimation of the outlet temperature in different parts of the range](image)

Hence, the effect of the approximative tube model (with constant parameters) surpasses the feeble non-linearity effect. Since the non-linear EPSAC controller was specifically designed only to handle this non-linearity, it will in this case not lead to significantly better results. In Figure 6.16 an experiment with a large setpoint change of 5°C is shown. The process inputs are depicted in Figure 6.17. Referring to the static characteristic of Figure 6.12, this change is performed in the most non-linear zone of the system. There is a noticeable change in outlet temperature of both controllers, but the difference is so small it is not well suited for general conclusions.
6.1.6 Noise model

As expected, the errors in predictions caused by the approximative tube model, are also visible in the noise. Therefore, instead of making the model parameters adaptive to the operating range, it is useful to think of improving the noise model first [12], for two reasons. One is that accurate model parameters are not easily obtained. It requires a lot of effort to derive, test and implement adaptive parameters. The other one is that model errors can never be fully excluded in real life. There will always be external factors or combinations of factors which the model has not taken into account.
Moreover, for improving the noise model, it is sufficient to have information only about the frequency of the noise, without knowing which specific factors created the noise and to what extent. In real life, it can be that disturbances are detected always at the same typical frequencies, without exactly knowing where they come from or how their amplitude varies in time. On the contrary, for improvement of model parameters, this specific information is of importance.

Figure 6.18 shows the noise as it was extracted from the EPSAC-experiment presented in Figure 6.14. Since the parameters of the outlet tube change over the entire range, the noise will vary as well. That is why it is an interesting experiment to base the noise model improvement on. Spectral analysis of this noise (Figure 6.19) shows that the noise is mainly concentrated at very low frequencies, close to the DC-frequency. That is why the default noise model, filtering only the DC component, gives already good results. Though further improvement can be made if this low frequency noise is taken into account. To present an improved noise model, we take into account that the noise is not concentrated at one frequency in particular, but rather spread over a 0-0.02 Hz band. This is expressed by the parameter $a$, which is an additional tuning parameter of the filter:

$$\frac{C(q^{-1})}{D(q^{-1})} = \frac{1}{(1-q^{-1})(1 - a \cdot q^{-1})}$$  \hspace{1cm} (6.11)

For $a = 0$, the default noise model is found back, hence only the DC-frequency is passed and the steady state error is eliminated. For $a = 1$ the model becomes a double integrator, assuming a ramp noise pattern. In this case, an appropriate choice of $a$ is not straightforward, but it should at least be around 0.5. Based on the amplitude characteristic of the two parts of the model shown in Figure 6.20 and taking into account that the default noise model already gave good results, $a$ is chosen equal to 0.4.
After implementation, an experiment was performed in the upper part of the temperature range, where the tube model is expected to lose accuracy. In the first case, the EPSAC-controller was implemented with the default noise model. Secondly, exactly the same experiment was performed for an EPSAC-controller with an improved noise model. Figure 6.21 compares the measured outlet temperature in both cases (Figure 6.22 for the process inputs). Clearly, the overshoot caused by poor estimations of the outlet temperature is severely reduced. Moreover, due to natural disturbances, the steady state outlet temperature generally fluctuates in a band of $\pm 0.2^\circ C$ around the setpoint. This means that the overshoot of the improved controller is even in the range of the steady state band of the system! Hence, the controller performance is probably at its best!

![Bode diagram of the improved noise filter](image)

**Figure 6.20:** Bode diagram of the improved noise filter

![Controller performance with improved noise filter](image)

**Figure 6.21:** Controller performance with improved noise filter
6.2 Genuine PI-controller and Smith predictor

To give an idea how well the tuned EPSAC controller performs, both a genuine PI-controller and a PI-controller in the Smith predictor control-loop layout were implemented. The choice for a genuine PI-controller might be seen as ‘unfair’ because it does not make use of any predictions and is therefore not aware of neither variable time delay nor non-linearity. Though, in spite of all advances in process control, the PI-controller is still the most commonly encountered controller in the process industry. PI-controllers can address delays in the system dynamics, but serious practical limitations are reached when the delay becomes variable. This situation can be problematic when dealing with transportation delays, as in this case [1]. The Smith predictor on the other hand can be categorized as an advanced control method since it uses process models to estimate the output. It allows the internal PI-controller to control the process as if there was no time delay involved. If the models are correctly identified, this control loop can significantly enhance the performance of the PI-controller.

There are several tuning methods for PI-controllers used to control processes with time delay [1]:

- **Cohen and Coon tuning formula:** Cohen and Coon presented in [4] a method to determine adjustable parameters in order to obtain a desired degree of stability.

- **Frequency response method by Ziegler and Nichols:** this method gives good results when the dead time is small. When there is a large dead time, the parameters of the controller are severely de-tuned, resulting in a slow response [24]. Refinements were proposed by Hang, Åstrom and Ho [10].

- **Tavakoli-Fleming tuning rule:** based on dimensional analysis and numerical optimization techniques an optimal method is proposed for tuning PI-controllers for first
order systems with dead time [18].

Nevertheless, as these methods are mainly used when there is no model of the process on hand, it is more evident to use our knowledge of the model for the design of the PI-controller. As the tuning of the EPSAC-controller merely relies on process models, we choose to follow the same approach for tuning the PI-controller. The process models are used for off-line tuning with a CAD-tool based on the frequency response [6, 15].

First the tuning of the genuine PI-controller is discussed. Experiments are performed in different regions of the operating range in order to investigate its performance. Afterwards, the tuning of the Smith PI-controller is treated to compare results.

6.2.1 PI-controller

As the process is non-linear, an equilibrium point must be chosen in order to linearize the process before the frequency response method can be used. As an example, the equilibrium point of 1 l/min is chosen in this case. Recalling equation (5.28) discussed in section 5.5.1, the transfer function of the linearized process becomes:

\[
\frac{\Delta T_{\text{tank}}(s)}{\Delta q(s)} = \frac{(T_{\text{in}} - T^*)/q^*}{\frac{1}{q^*} s + 1} \quad \text{with} \quad q^* = 1 \frac{l}{\text{min}} = 0.01667 \frac{l}{\text{sec}}
\] (6.12)

Using the steady state characteristic (6.13) of the system:

\[
T^* = \frac{\dot{Q}_h}{\rho c_p q^*} + T_{\text{in}}
\] (6.13)

the transfer function of the linearized transfer function can be written as:

\[
\frac{\Delta T_{\text{tank}}(s)}{\Delta q(s)} = \frac{-\frac{\dot{Q}_h}{\rho c_p q^*}}{\frac{1}{q^*} s + 1} \cdot e^{-140s}
\] (6.14)

The transfer function of the tube is still given by equation (6.15):

\[
\frac{\Delta T_{\text{out}}(s)}{\Delta T_{\text{tank}}(s)} = \frac{K}{RC s + 1} = \frac{0.99}{29 s + 1}
\] (6.15)

The steady state delay corresponding to \(q^*\) amounts to 61 seconds. During control however, especially when exiting the linearization region, the delay can be significantly higher, even up to 2 minutes. To guarantee stability in a broader region than this linearization zone, the controller is designed for a worst case time delay of 140 seconds. With these values, the linear process model is now:

\[
\frac{\Delta T_{\text{out}}(s)}{\Delta q(s)} = \frac{-946}{68 s + 1} \cdot \frac{0.99}{29 s + 1} \cdot e^{-140s}
\] (6.16)

As tuning objectives, a closed loop settling time of 300 seconds is proposed (as the open loop settling time with time delay included is approximately 350 seconds) together with

\[\text{FRTool v1.3 developed at the control engineering department of Ghent University is used (Prof. dr. ir. R. De Keyser & ir. C. N. Vlasin, 2001).} \]
overshoot of 10% to have enough gain margin. Adjusting the gain and the integration time, it is impossible to attain the requirement for the closed loop settling time. Consequently, the border of 10% overshoot is crossed to lower the settling time as much as possible without losing too much stability. The tuning results in a gain $K_c$ of -0.0006 and an integration time $T_i$ of 124.3 seconds:

$$K_c \left(1 + \frac{1}{T_i s}\right) = -0.0006 \left(1 + \frac{1}{124.3s}\right)$$  \hspace{1cm} (6.17)

Figure 6.23: PI-design using the Nichols diagram

With this controller implemented, Figure 6.24 shows an experiment within the linear region where the controller was designed. Due to the extreme de-tuning (huge time delay), the controller reacts overcautious, resulting in a settling time of approximately 600 seconds (delay included). With a corresponding open loop settling time of approximately 350 seconds, it is clear that the PI-controller, even in its region of design, is very slow. It is not surprising that it performs much worse than the EPSAC controller. The process input is plotted in Figure 6.25.
Figure 6.24: PI-control within the design region

Figure 6.25: Process input of PI-control within the design region

Figure 6.26 shows an experiment out of the design region. The steady state flows of the setpoints are respectively 0.63 and 0.7 l/min (Figure 6.27). Obviously, the more the PI-controller exits its range, the larger the overshoot on the outlet temperature will be. After more than 40 minutes steady state is still not reached. Hence, the genuine PI-controller, tuned with the process models on hand, results in large overshoot and unacceptable long settling times. For out-of-region control, it might even become unstable.
6.2.2 PI-controller in the Smith predictor strategy

If the time delay model is accurately identified, the Smith predictor can be tuned so that the closed loop response is fast. The accuracy of the time delay model was proved for the EPSAC-control. With the Smith predictor relying on the same estimation strategy, we expect the performance to be a lot better than the one of the genuine PI-controller, which makes it more ‘fair’ to compare it with EPSAC-control. In the Smith predictor scheme (Figure 6.28) the controlled output without time delay is fed back to the PI-controller. This allows to tune the controller as if there was no time delay involved. Therefore, the controller does not need
severe de-tuning caused by the huge time delay.

\[
\frac{\Delta T_{\text{out}}(s)}{\Delta q(s)} = \frac{-946}{68s+1} \cdot \frac{0.99}{29s+1}
\]

Figure 6.28: Schematic representation of the Smith predictor approach

The transfer function of the process to tune the controller on is now:

\[
K_c \left(1 + \frac{1}{T_i s}\right) = -0.00166 \left(1 + \frac{1}{71.27s}\right)
\]

Taking the same tuning objective, a theoretical settling time of 300 seconds (without delay) can be obtained together with an overshoot of less than 10%. For the internal PI-controller, \(K_c\) is now -0.00166 and \(T_i 71.27\) seconds:

The Nichols chart is given in Figure 6.29.

Figure 6.29: PI-design based on the frequency response

Figure 6.30 shows an experiment of the PI-controller in the Smith predictor strategy, performed around the design range of 1 l/min (Figure 6.31).
With a settling time of 275 seconds (delay included) and an overshoot of 0.2 °C, its performance is comparable to the EPSAC-controller.

Figure 6.32 shows an out-of-region experiment with setpoint changes from 38 to 41 °C and back to 38 °C. The corresponding steady state flows are 0.63 and 0.72 l/min (Figure 6.33), thus clearly out of the linearization region of 1 l/min.
The overshoot is now about 1 °C for the step up, and 0.6 °C for the step down, the settling time (with delay) is about 420 seconds or 7 minutes. Obviously, the controller has lost much of its performance, while EPSAC (even without improved noise filter) is effective over the whole temperature range, from 25 to 45 °C.

To conclude, experiments have pointed out that the genuine PI-controller, with tuning merely based on a linearized process model, is not the best option for controlling this process with huge variable time delay. It needs severely de-tuning and might become unstable when exiting its region of design. Nevertheless, methods as presented in section 6.2.1 may result in better

Figure 6.32: PI-controller in Smith predictor out of its design region

Figure 6.33: Process input of PI-controller in Smith predictor out of its design region
performance. It would be interesting to investigate this more thoroughly. The advanced controller methods on the other hand perform much better as long as the process models are accurately identified. The Smith predictor with internal PI-controller performs well, at least within its region of design. In this case, there are also refinements possible, for example the inclusion of a feedback filter. Finally, the EPSAC-controller offers a lot of tuning knobs, of which some are adjusted to the process needs. The controller with improved noise filter pulls the outlet temperature within its natural range of $\pm 0.2 \, ^\circ\text{C}$ around the setpoint in a very short time. Moreover, it might only lose performance due to the varying tube model parameters but not due to its controlling strategy. Further improvement of the noise filter or increasing the control horizon $N_u$ can even lead to better performance. Increasing $N_u$ would at least allow to optimize the cost function with respect to the input constraints and therefore fully unfold the strengths of non-linear EPSAC.

In order to make a quick and interactive management of the different controllers and their parameters tuning, a Graphical User Interface was programmed. More information on the use of this GUI, the several programs, and the correct use of the plant itself can be found in the appendix.
Chapter 7
Conclusions

Having come to the end of this thesis, it is time to summarize a few realizations, conclusions and proposals for further research.

7.1 The setup

The setup has been built and is operating well. Due to a good choice of the several parameters involved, the requirements for the process itself have been fulfilled:

1. A unit change in the input produces a significant change in the output.
2. The closed loop time constant equals roughly one minute.
3. The time constant of the tank is more or less equal to the transport delay.

A steep static characteristic explains the output sensitivity to input changes, while the choice of a small tank volume and a high heat input, combined with a broad flow range results in a fast response time. The transport delay is of the same order of magnitude as the tank time constant, as the volumes of the tank and the tube are 1.13 and 1.02 liter respectively. A broad operational temperature range of approximately 12 °C is reached.

A constant water level is obtained in a cheap and simple, but yet effective way via the mechanical float switch. If the supply pressure of the tap water is sufficient, flows of up to 2 l/min can be applied without causing the water level to decrease. The choice of a float switch allows to use an open tank, which makes it easy to put other elements like the heater, the mixer and sensors inside the tank.

Using a peristaltic pump, a very accurate flow control is obtained, which is necessary for a precise temperature control. Besides its accuracy and fine resolution, it allows to make reliable calculations of the flow, which is very important in the calculation of the variable time delay. This is possible due to the linear relation between the pump speed and the flow. The pump speed itself can not be measured easily, but this problem is solved by controlling the DC-motor of the pump with a drive which has IxR feedforward compensation. Due to this feature an almost linear relation between the control voltage to the drive and the pump
CHAPTER 7. CONCLUSIONS

speed is achieved, and, consequently, a linear relation between the control voltage and the flow can be assumed with a high reliability. Another advantage of the IxR compensation is that changes in the setup (e.g. putting an outlet tube of different length) would not affect the linear relation between control voltage and flow, although the load would have been changed.

A suitable data acquisition board was chosen, insuring measurements with a sufficient resolution. The board is provided with enough in- and outputs for the several measurements and the flow control. More ports are available if one would consider further extensions.

Reliable Pt100 temperature sensors take care of the temperature registration. The circuitry to convert the signals into a useful voltage range between 0 and 10 V is soldered. Investigations and precautions have been taken thereby to insure measurements that are as accurate as possible. Noise on the signals is filtered out via a passive RC filter combined with a digital filter on the computer. Finally, a calibration of the signals has been carried out in order to achieve reliable results.

All the electrical and electronic parts are put in a sealed box, separated from the water circuit. Switches for the different devices (pump, heater, mixer, main switch) are provided. All cables and wires are placed in electrical wire grooves for protection and a neat arrangement. The water tank is made of plexiglass, which makes the process a bit more transparent and interactive. Flexible plastic hoses are used for the in- and outlet tube. The 9.5 meters long outlet tube is insulated, and curled up in a box. An easy attachment system to the water mains is provided.

7.2 Advanced Control

Regarding the EPSAC-based controller, first models of the several parts were derived and validated against experimental results. It turned out that the non-linear first order model of the tank is very accurate, while the tube dynamics are in reality very complex. A linear first order approximation was used for it, and proved to give acceptable results, although far from ideal. Last but not least a model for the variable time delay was deduced, which was refined to compensate for the sensor reaction time. It is important for the controller that this calculation of the time delay is very precise, as an error of only one or two samples causes differences between the estimation of the current model output and the real process output.

The difficulties caused by the varying time delay were discussed (prediction of future time delays, skipping samples or holding the same estimation over more samples, varying $N_1$), and an appropriate strategy (based on the Smith predictor scheme) to deal with it was proposed and implemented.

The influence and importance of parameters like the prediction horizon $N_2$ and the number of iterations $N_i$ has been examined. It turned out that $N_2 \cdot T_s$ should be a reasonable fraction of the (highest) time constant. $N_i$ seems not to have a very important influence, but this is due to the limited non-linearity within a limited temperature range. When $u_{base}$ of EPSAC was
compared to the one iterated by NEPSAC, it became clear that there was no much difference between the two, and hence, the influence of $N_i$ remained small.

A first improvement of the noise model compared to the default one was implemented, and proved already its effectiveness. With this, the closed loop time constant of approximately 1 minute is almost reached, while the overshoot remains very small.

As a reference, a genuine PI-controller and a PI-controller in the Smith predictor strategy were also implemented. It was shown that the genuine PI-controller, tuned with the process models on hand, needed severely de-tuning. Even then, it showed the tendency to become unstable when being operated out of the linearization region. The Smith predictor on the other hand can be classified as an advanced control method, which makes it a lot more suitable to control the heating tank system. It performed well for setpoint changes within its region of design, but showed large overshoot when exiting it.

Finally, a Graphical User Interface was programmed to make experimenting and switching between the different controllers easier and more accessible.

Although very promising results are obtained, there are still a lot of interesting options left for further research and improvement. The ones we have in mind are summarized briefly.

- Probably the most expectant topic to be further examined is the model of the tube. Maybe a better but still simple model can be derived. The first thing that can be tried concerning this problem is making the parameters of the transfer function adaptive to the flow and/or temperature. In this way it should be possible to obtain a more reliable tube model, resulting in a better control. It also turned out that a correct estimation of the time delay is extremely important for a good control. As the conversion factor between control voltage and flow might change slightly depending on the temperature of the pump motor (due to the not ideal IxR feedforward compensation, see section 3.1.2) it is possible that there is an error of $\pm 5$ seconds. Maybe a more steady motor temperature can be achieved by providing an efficient heat removal using heat sinks or a fan.

- Another promising feature is the noise model. A first improvement compared to the default noise model has been implemented, but it is certainly useful to investigate more complex models, resulting for example in a low pass filter with a very small cut-off frequency and a very steep slope.

- Something that is not yet investigated is the shape of the reference trajectory. In all experiments described in this thesis a step reference was used, while choosing a smoother transition could result in a control output with for example less overshoot.

- Another possibility lies in the control horizon $N_u$, which has been chosen equal to 1 up till now. Although this choice causes a profound simplification and has proved good results, there are some disadvantages: clipping is the only option as there are no other
degrees of freedom to calculate the optimal input with respect to input constraints. Especially when making large setpoint steps, a clipping situation is very likely to occur.

- An attempt can be made to use the (N)EPSAC-algorithm without the decoupling of the variable time delay from the dynamics. As explained complex problems and calculations can however be expected if the theory is strictly applied. But it is always possible that some simplifying assumptions (for instance not predicting the future time delay, but taking it equal to the current delay) could lead to good results.

Besides the EPSAC-method, also other controllers can be tried or improved. As the PI-controller in the Smith predictor scheme showed good results within its linearization region, it could be useful to split up the whole temperature range and to design for each separate range an appropriate PI-controller. Then the parameters of the PI-controller can be made adaptive depending on the current temperature (e.g. by interpolation between the parameters of the several designs).

The plant itself could also be extended with features which can make the heat input for instance manipulable (e.g. via a dimmer). In this way the plant could also be used as a test setup for problems with a constant time delay (e.g. a constant pump speed with a manipulated heat input). As an ultimate bonus, by manipulating both the pump and the heater, the setup could also be used as a MIMO\(^1\) system (with variable time delay), which would pose another very challenging research topic.

Starting with these suggestions, it is clear that quite a lot of further research can be done on this problem. A real setup is however available and working properly, while the difficulties with a variable time delay are mapped out. The first steps have been made and already resulted in a good control. Consequently, the most important conclusion that can be drawn from this thesis is that *EPSAC - if applied in a proper way - has no problem at all in dealing with real life processes with a variable time delay.*

---

\(^1\)Multiple Input Multiple Output
Appendix A

GUIDE

A.1 Introduction and Installation

This appendix serves as a manual on how to perform experiments on the heating tank system in a safe way. It also guides the user through the implementation of the different self-tuned controllers classified as:

- a genuine PID-controller: $K_p, T_i, T_d$
- a Smith predictor with internal PID-controller: $K_p, T_i, T_d$
- a non-linear EPSAC controller: $N_2, N_i, a$

The respective parameters form the heart of the controller and can be easily adapted in a Graphical User Interface (GUI). The GUI also serves to change the parameters of the dynamic models of the tank and the outlet tube and allows to redefine parameters which govern the execution of the experiment (e.g. input constraints, sampling time).

The GUI organizes all files involved in controlling the heating tank system. NI-DAQ from National Instruments and the Matlab Data Acquisition Toolbox exchange the control and measurement signals between the GUI and the process. These packages together with the GUI and the control files can be installed from the CD which accompanies this thesis. Make sure these packages are properly installed before installing the GUI. Information can be found in the readme.txt file. Once these packages are installed, select a directory and finish the installation by copying all GUI files to the directory. This includes all control files and the support files of the graphical user interfaces.

A.2 Control structure

The GUI control structure is shown in Figure A.1. This structure and its blocks are briefly discussed here. A detailed explanation is given in the following sections. STARTPROCESS allows to define general parameters that will be used during the experiment, for instance the parameters of the dynamic models. If these model parameters are changed,
it is recommended to execute an open loop experiment to see whether or not the adjustments are satisfactory. When a setpoint change is performed with \texttt{CONTROLPANEL}, the steady state flow is computed based on the static characteristic of the process. In that way, the predictions can be compared to the measured temperatures. If necessary, the model parameters (or other general parameters) can be adjusted after the experiment is stopped and the data is saved (this is not required).

\textbf{Figure A.1:} Structure of the GUI control

\texttt{CONTROLSELECT} offers the possibility to select a controller as soon as all general parameters are determined. Note that once the control loop is entered, the general parameters can no longer be adjusted. Each controller has its own GUI-file to determine the control parameters. When these are properly defined, the control starts after 80 seconds, which allows to make adjustments on the setpoint so that no severe control action is taken based on the initial setpoint ($38^\circ$C). Setpoint changes are performed with \texttt{CONTROLPANEL} and data can be saved after the experiment is stopped.

\section*{A.3 Interfaces}

When the installation is finished, the GUI is launched when \texttt{STARTPROCESS} is executed\textsuperscript{1}. Some general information is shown before the \texttt{proceed} pushbutton shows the entire user interface (Figure A.2). A help dialog appears to confirm if \texttt{CreateChannels.m} has succeeded to create the gateways to the data acquisition board.

\textsuperscript{1}Select the directory in which all GUI and control files are installed, then look for \texttt{STARTPROCESS.m} and execute it by either right mouse click and run or by dragging it to the command window.
From then on, the general parameters sampling time $T_s$, input constraints $q_{\text{min}}$ and $q_{\text{max}}$ and the initial input $q_{\text{init}}$ can be determined. Only the model parameters of the dynamic models can be adjusted. The delay model is protected from any changes because the process is very sensitive to a badly estimated time delay. Default parameters can be loaded if the user is not familiar with the process. The GUI does not accept invalid parameter values. As long as the parameters are not determined appropriately, no experiment can be executed. Figure A.3 and Figure A.4 are typical pop-up messages if the GUI finds strange parameter inputs.

After valid parameter values are entered, the choice is between an open loop experiment or a closed loop control experiment. Recall that, once the control part of the GUI is entered, no more parameter changes are allowed.

When proceeding by either selecting to perform an open loop test, or by shifting to process
control, the control program `MainProgram.m` is internally executed. This program links the GUI to all other control programs. Its functioning is crucial for the heating tank system control and consists of:

- copying the user-defined parameters.
- initializing internal parameters necessary for all underlying control programs.
- setting up the MATLAB data acquisition toolbox functions. These functions are the beating heart of the computer control. Therefore, they are important to understand when implementing additional controllers:
  
  - `AI.SampleRate=sRate` determines the sample rate. `MainProgram.m` sets `sRate` to 5Hz.
  
  - `AI.TriggerRepeat=inf` determines the number of triggers after the start command `start(AI)` is issued. Each trigger accounts for 1000 samples. Setting the trigger repeat function to infinite makes the data acquisition board to stop logging only when the stop command `stop(AI)` is explicitly issued.
  
  - `AI.SamplesAcquiredFcnCount=Ts.sRate` determines the number of samples after which a specific function, specified in the `AI.SamplesAcquiredFcn`-field is executed. Each time a number `Ts.sRate` of samples is acquired by the data acquisition board, the MATLAB toolbox executes `Control.m`, which in turn contains the three types of controllers and the open loop policy. Which of these controllers is actually applied to the process is determined by switching the variable `ControlSelect` in a switch-case structure.

As an example, we want to control the heating tank system with a non-linear EPSAC-controller with improved noise filter. `STARTPROCESS` is exited when clicking on the `ProcessControl` pushbutton. The selection panel of Figure A.5 appears. Once the controller type is selected, its respective parameters can be determined, which are in this case $N_2$, $N_1$ and $a$. We enter the default values by clicking on the `Default` pushbuttons. Again, the GUI takes care that no invalid parameters can be entered. With the default parameters loaded, process control is started. The interface is shown in Figure A.6.

Starting process control issues the `start(AI)` command which makes the data acquisition board to start logging. MATLAB data acquisition toolbox counts the number of samples logged upon start and executes `Control.m` if $T_s$ seconds are elapsed. `Control.m` executes in sequence:

---

2 AI refers to the analog input channel as it is created by `CreateChannels.m`. More information can be found in the manual of the MATLAB data acquisition toolbox.

3 When two start commands are given without any stop command in between, MATLAB processes an error. The only way to get it back running is to exit MATLAB and restart again.
APPENDIX A. GUIDE

Figure A.5: Interface of CONTROLSELECT

- **AcquireData.m** which reads the latest data from the temporary DAQ-board memory. It calls procedures ConvertIncTemp.m, ConvertOutTemp.m and ConvertVesselTemp.m to convert voltages into temperature. The conversion functions are determined by calibration.

- **Delay.m** which calculates the delay according to the formula presented in the thesis. It corrects for step-up and step-down setpoint changes because the PT100 sensor reacts differently in both cases.

- **StateEst.m** which estimates the outlet temperature both without and with delay. Therefore, it uses the discrete time index $N_d$ calculated by Delay.m.

- the switch-case structure for ControlSelect to determine which type of control (or open loop response) is selected. The respective control program PID.m, Smith.m or MBPC.m is executed with the parameters as determined in the GUI. The execution starts only after 80 seconds to prevent violent control actions due to the initial setpoint of 38°C. For the PI-controllers, this allows bumpless transfer. Violent control actions could strongly disturb the process and require too much time before it is back in steady state.

- **Show.m** which updates the real-time plot of temperatures, input control, variable time delay and noise.

During the experiment, the setpoint can be changed. Notice that these setpoint changes are only checked on their numerical value, **but not on their validity**! Use the real-time plot to make useful setpoint changes. CONTROLPANEL serves also to stop the experiment after which data can be saved. When clicking on the stop pushbutton, the interface of Figure A.7 appears. The choice is offered to apply the initial input $q_{init}$, set in STARTPROCESS, or to hold the latest
input as computed by the controller. This is particularly interesting if experiments want to be performed around a certain steady state temperature which does not correspond to $q_{\text{init}}$. The selected input will be applied while storing the data and during the next 80 seconds of a new control experiment. Hence, the equilibrium point is not lost.

Figure A.6: Interface of NEPSAC

Figure A.7: Interface of CTRLPANEL after the stop command is issued.

After the experiment, only the data relevant to the experiment is shown (Figure A.8) and can be saved in a *.mat-file. Mark the selection box for each data of interest and click on the
Store data pushbutton. Once the directory and the filename are chosen, the GUI confirms that the data has been saved and opens the SELECTCONTROLLER-interface again to perform a new experiment. If no saving is required, just hit the Proceed pushbutton.

![Interface of SAVEDATE after selection.](image)

If no other experiments are to be performed, exit the GUI only by clicking on the ExitProcess pushbutton, which is situated just beneath the explanation box of every user interface. This will stop the pump from turning, clear all variables and reset the internal structure to perform new experiments. Don’t forget to turn off the heater and the mixer manually!

### A.4 Precautions

When executing an experiment on the heating tank system, always make sure:

- **not to apply high flows during a long time.** This causes the pump motor to heat very rapidly, which in the extreme case could burn the coils. The default maximum flow that is implemented in the GUI insures safe operation. It is also wise to take a small pause of some ten minutes between long measurements (e.g. each hour) to allow the pump to cool down.

- **the pump is always turning when the heater is on.** If not, the system acts like an electric kettle. The temperature measurements and the siliconed tank are not designed for this purpose!

- all switches are turned off when stopping experiments, and the appliance is plugged out.
• not to forget to close the water valve before leaving, as the float switch might leak after some time. To avoid rusting of the mixer screw, the inlet tube should be disconnected from the water mains to lower the water level below the mixer screw. Afterwards, the inlet tube can be connected again to avoid air coming into the tubes.
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