Investigations on bubble behaviour at a dip tube for enhanced densitometry and level measurements

by

Sarah Vandekendelaere

Promotor: Prof. Dr. Ir. G. Janssens-Maenhout
Copromotor: Prof. Dr. Ir. W. Van Hove
Mentor: Z. Dzbikowicz

Scriptie ingediend tot het behalen van de academische graad van burgerlijk natuurkundig ingenieur

Academiejaar 2005 - 2006
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Toelating tot bruikleen

De auteur(s) geeft(geven) de toelating deze scriptie voor consultatie beschikbaar te stellen en delen van de scriptie te kopiëren voor persoonlijk gebruik. Elk ander gebruik valt onder de beperkingen van het auteursrecht, in het bijzonder met betrekking tot de verplichting de bron uitdrukkelijk te vermelden bij het aanhalen van resultaten uit deze scriptie.

Mei 2005,
Investigations on bubble behaviour at a dip tube
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Sarah Vandekendelaere

Abstract

In accountancy tanks of reprocessing facilities the pneumercator or dip tube technique is used for measuring a level, because the pneumercator delivers accurate measurements and the dip tubes are robust and easy in maintenance. The traditional pneumercator consists of three tubes of steel with a flat end and with different lengths and of pressure measurement cells recording the gas pressure at the dip tube’s tip. The pressure measurement can give very accurate determination of the level, if the health status of the dip tube and systematic errors are taken into account.

Blockage or plugging of the dip tubes usually occurs in tanks with highly concentrated solutions. The air bubbling causing up and down movements of the solution in the lowest part of the dip tube can cause drying out of the thin liquid films. Hence precipitation of salts of the concentrated solution occurs and solid particles deposit on the internal wall of the tube. The status of the dip tube’s tip is regularly checked by analysing the bubble behaviour during formation and the detachment frequency.

The pressure at the tip of the bubbling dip tube shows a regular but dynamic pressure signal for a given liquid level. The bubble dynamics, more in particular the periodical build-up of a bubble volume and therefore periodical displacement of water are commonly modelled with a systematic error of sinusoidal form. The bubble volume and the maximum distance of the bubble interface moves underneath the tip of the dip tube, show more a saw tooth behaviour.

The bubble formation behaviour and bubble volume function are investigated experimentally to establish a reliable bubble volume function in dependency of the
air flow supply. The impact of external parameters such as insertion depth of the dip tube, temperature of the liquid bath and liquid density and viscosity are studied with a sensitivity analysis.

**Keywords:**

Dip tube, level measurement, densitometry, bubble behaviour
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Chapter 1

Nuclear Safeguards

1.1 The Foundation of the International Atomic Energy Agency

Soon after the discovery of nuclear fission in the late thirties, it was calculated that with a chain reaction an enormous amount of energy could be produced. The potential of nuclear power plants has been directly experimented with success in 1941, with "Chicago Pile 1". Unfortunately in the early fourties, governed by the second world war, also non-peaceful applications were developed. In 1942, the Manhattan Project was set up to develop the first atomic bomb under the lead of Enrico Fermi. The tremendous power of this bomb was demonstrated at the end of the Second World War when the bombings of Hiroshima and Nagasaki took place. The monopoly of the USA as developer of nuclear weapons would soon end and proliferation of nuclear weapons would be a fact. An international program was set up to make sure that all the countries that would set up a nuclear program, would integrate a safeguards system.

The United States was willing to give up its monopoly, agreed with this plan and proposed the Baruch plan to put the promotion of nuclear energy under the responsibility of the United Nations. The main purpose of this plan was to ensure that the new knowledge would be spread, but only be used in civil programs and not in military ones. The plan was not accepted, since the Sovjet Union was developing its own nuclear weapons at that time. Their first successful test was performed in 1949.

In December 1953 President Eisenhower held his famous 'Atoms for Peace' speech before the general assembly of the United Nations. In this speech he promoted the achievement of Nuclear Disarmament and peaceful use of the nuclear technology. He succeeded in his goal, in 1956 the International Atomic Energy Agency (IAEA) was founded as an autonomous intergovernmental organization under the United Nations and in 1957 its statutes were approved. The goal of the organization would be 'to promote the use of nuclear technology for peaceful purposes and to ensure that no misuse of such
technology could be performed’. Articles II and III of the IAEA statute mention the controlling measures, also called Safeguards System.

The term Safeguards is not to be confused with Safety or Security. Safety is needed for the operation of civil nuclear installations, the transport of radioactive materials, the treatment and deposit of nuclear waste and the decommissioning of nuclear facilities, to protect the environment from any form of radioactive contamination. Security is needed as protection against the threats, sabotage and misuse of nuclear installations and their means. Safeguards is needed to survey the total quantity of Nuclear Material as inventory in a facility and to monitor Nuclear Material flows, to prevent or detect especially thefts or diversions of Nuclear Material.

1.2 The Birth of a Safeguards System

Despite the succesful foundation of the IAEA, there were quite some problems during the late fifties. The main problems were related to the actual implementation of the safeguards system and the political situation at the time. The implementation was realized in small steps, beginning with the definition of some basic rules for the exploitation of Research Reactors and later expanding to all kinds of reactors, reprocessing facilities and fuel fabrication plants. By 1958 the Cold War was raging and this postponed the start of the Agency to perform some of its important tasks. In the years that followed, the existing nuclear weapon arsenals were filled up. This development in history is now called the vertical proliferation.

During the 1960s there was also a horizontal proliferation, meaning that more and more countries started research and production:

![Figure 1.1: Horizontal proliferation](Image)

The threat for a nuclear war was ever growing and something had to be done fast.
The Cuban Missile Crisis, the appearance of 2 new Nuclear Weapons states (France and the United Kingdom), the proposals in NATO for a multilateral nuclear armed force and a handful of rumors about Germany and Italy also developing a bomb convinced that there was a need for a Nuclear Arms Control. In 1968 the Treaty on the Non Proliferation of Nuclear Weapons (NPT) was proposed and it entered into force in 1970.

1.3 The Non Proliferation Treaty and Classical Safeguards

The Non Proliferation Treaty (NPT) is a global treaty that is built around the following central ideas:

- The Non Nuclear Weapons States (NNWS) renounce nuclear weapons and therefore will not try to transfer, acquire or produce nuclear weapons.

- The Nuclear Weapons States (NWS) will not provide any NNWS with the technology to produce nuclear weapons or give any help to acquire those weapons.

- As a consequence of their willingness to put the fuel cycle under Safeguards, the NNWS will gain access to nuclear equipment, material and technology developed for peaceful purposes.

- All states will undertake negotiations towards nuclear disarmament.

A complete set of Safeguards has been negotiated for the NNWS. Two Safeguards Agreements exist: one for the single States (INFCIRC/153) and one for the regional state systems (e.g. EURATOM).

1.3.1 Classical Safeguards: INFCIRC/153

Document INFCIRC/153 is the basis for the Classical Safeguards and states:

The Agreement aims to provide the Agency’s right and obligation to ensure that safeguards will be applied, in accordance with the terms of the Agreement, on all sources or special fissionable materials in all peaceful nuclear activities within the territory of the State, under its jurisdiction or carried out under its control anywhere, for the exclusive purpose of verifying that such material is not diverted to nuclear weapons or other nuclear explosive devices.

The inspections of the nuclear material are designed so that they can achieve the three Safeguards Goals:
CHAPTER 1. NUCLEAR SAFEGUARDS

- **Significant Quantity:**
  The quantity of material that, if it was diverted, could be sufficient to manufacture a nuclear weapon. This is not to be confused with the critical mass, being the mass needed to start a nuclear reaction.

- **Timeliness Detection:**
  The conversion time is the time between the acquisition of the material and the finished nuclear device, and so the time within which diversion or theft of nuclear material needs to be detected for preventing from manufacturing a nuclear device.

- **Probability for False Alarm and for Non Detection:**
  To make the necessary measurements for the Material Accountancy System, Material Balance Areas (MBA’s) and Key Measurement Points are defined. The verification if the material is divided in three steps: counting of all the batches and items, a qualitative checking and a quantitative verification. The inspector compares the amount of material stated in the Inventory and compares it to the calculated amount. The Material Unaccounted For (MUF) is the difference in amount. If the MUF is positive, this can either mean that there has been a loss during the process or the material has been diverted. Another explanation is that the measurement errors are too large. The statistical error on the amounts (volumes or weights) can be modelled by a normal distribution around the expected average. The model designed by the IAEA contains 3 crucial parameters: standard deviation of 1%, detection probability of 95% and false alarm of 5%. Taking this into account, the maximum allowed MUF before alarm is $3.3\sigma$.

1.3.2 EURATOM Treaty

On 25 March 1957, the Euratom Treaty was signed by 6 countries of the European Union (Belgium, The Netherlands, Luxemburg, Germany, France and Italy) and went into force on 1 January 1958. By 1995, 9 other European Countries had signed the Treaty. All the countries that want to join the European Union are now obliged to ratify the Treaty.

Chapter VII of the Treaty deals with Safeguards and describes the three main principles: conformity, general control and territorial control.

- **Conformity:**
  Euratom inspectors should verify the conformity of the effective use of the nuclear material with the intended use.

- **General Control:**
  A general control should be performed on all the nuclear material located in whatever facility on that territory.
• **Territorial Control:**
  The inspectors are allowed to perform a control of the material in situ at the nuclear facility. Again the three safeguards goals are in use.

The member State should fully cooperate with the investigation to ensure that all the inspections are carried out as described in the Agreement.

### 1.4 The Crisis of Classical Safeguards and The Additional Protocol

#### 1.4.1 Crises in Iraq and the DPRKorea

In these countries, proliferation material was found during inspections in the early nineties. All the machines and the products that could be used to produce nuclear weapons were either taken away or left sealed on the property. In 1998 Iraq was not further prepared to cooperate and rejected the treaty. With the DPRK, an agreement was reached: in exchange for full cooperation, the promise not to proliferate anymore and under no circumstances reprocess anymore within the country, the US financed the complete replacement of Korea’s graphite moderated reactors into LWR’s and took steps to fully normalize the political and economical relations with Korea. Unfortunately in 2003 North Korea announced its withdrawal from the NPT effective as of 11 January 2003 but no agreed statement on the matter has been issued by the NPT States Parties, or by the NPT depositary States (Russia, UK and USA), or by the UN Security Council.

#### 1.4.2 The Additional Protocol

In response to the events in Iraq and the DPRK, the IAEA decided to add an additional Protocol, INFCIRC/540, to the Safeguards System. The purpose of this Additional Protocol (AP) is twofold: an improved information system, to have a better overview on a countries nuclear and nuclear-related activities (making use of satellite images and open source info) and an improved accessibility to suspected facilities, meaning that the inspectors can visit all the installations in the country, also the non-declared ones, and that the inspections can take place on short-time notice (24h). Signing the AP is not mandatory, but most of the countries that signed the NPT are willing to sign the AP. Most countries signed it, but haven’t ratified it yet.
1.5 The Conceptual Framework for Integrated Safeguards

In 2002, a framework for ‘Integrated Safeguards’ was presented. Its goal is to optimally prevent any further proliferation. The ‘Facility Level Approach’ focuses on:

- timeliness verification for irradiated fuel
- unannounced inspections
- increased cooperation with the State System of Accountancy and Control
- the use of surveillance for physical protection and scaling.

In reality, the actual revolution lies in the ‘State Level Approach’ and the integrated safeguards for the State’s nuclear facilities. This system takes into account the State’s nuclear fuel cycle with the specific design of all the facilities and the development plans. This approach will lead to a ‘tailor made’ system for each state.

1.6 Other Treaties

1. The FMCT or the Fissile Material Cut-off Treaty.
   After the Cold War, the US and the former USSR realized that there was too much nuclear material in the military arsenal, so they reconsidered the Baruch Plan. In 1993 a consensus was reached in the United Nations’ general assembly to adopt a resolution that stated a ‘non-discriminatory multilateral and internationally and effectively verifiable treaty banning the production of fissile material for nuclear weapons or other explosive devices’. By signing this treaty, the NWS agreed to stop producing. Unfortunately, the plan is not in force yet because some countries stalled the decision process.

2. The CTBT or the Comprehensive Test Ban Treaty.
   The Partial Test Ban Treaty was signed in 1963 and prohibited any nuclear tests in the atmosphere, in space or under water, and in 1996 the CTBT was signed by all Nuclear Weapons States that prohibits all nuclear tests.

3. The Tlatelolco Treaty.
   With this treaty, some countries in South America and the Caribbean declared themselves as ‘Nuclear Weapon Free Zone’. This means that they would not host nuclear weapons from states that are not bound by the Treaty. The additional protocols also stated that other states owning property in the Zone would not test their weapons there and that they would not threat with or effectively use nuclear weapon against one of the States of the Treaty.
4. The Rarotonga Treaty

In 1985 this treaty was signed that bans the manufacture, the acquisition or receipt of any nuclear explosives. It also prohibits the countries from stationing nuclear material for other countries.

1.7 Nuclear Material

Under the NPT the following materials are under control:

- non-fissile nuclear material:
  Uranium, containing the mixture of isotopes occurring in nature; uranium depleted in the isotope 235; Thorium; any of the foregoing in the form of metal, alloy, chemical compound or concentrate; any other material containing one or more of the foregoing in such concentrations as the Board of Governors shall from time to time determine.

- fissile nuclear material:
  Plutonium-239; U-233; uranium enriched in the isotopes 233 or 235; any material containing one or more of the foregoing; and such other fissionable material as the Board of Governors shall form time to time determine.

Not included in the NPT is the ore (the source material).
Chapter 2
Introduction to the dip tube technique

2.1 General information

The dip tube system or pneumercator is a precise robust instrument commonly used in chemical and pharmaceutical process industry, and particularly in the reprocessing industry, to determine accurately the volume of a vessel inventory by means of pressure measurement. Via the hydrostatic head the level and with the calibration curve of the vessel the volume of the vessel inventory is derived. The Dip tube system consists of
three stainless steel tubes of outside diameter 10mm and inside diameter 6mm, an air flow supply and pressure transducers. Through each of the tubes a constant air flow is sent and the pressure in each of the tubes is measured. For a small enough flow rate, the measured air pressure equals the hydrostatic head of the liquid column plus the pressure of the void in the tank, which is measured separately.

The longest tube is the level tube and is inserted into the liquid over an unknown insertion depth $L$. The intermediate tube is the density tube, which has its tip at a well-known distance $S$ above the tip of the level tube. The third tube is the reference tube, that should never touch the liquid.\(^1\)

The level in the tank is calculated using the difference in measured pressures between the different tubes and the formula for the hydrostatic head:

$$P = \rho gh$$

(2.1)

By taking the difference in pressure between the level tube and the density tube, the average of the density of the liquid between the tips of these two tubes is measured. Using this density and the difference in pressure between the level tube and the reference tube, the insertion depth of the level tube can be determined. This is not the level of the liquid in the tank, since the tip of the level tube does not touch the bottom of the tank: a certain height is needed to let the bubble grow. The liquid volume contained in this part of the tank is called the heel. (see also part on calibrations)

The pneumercator system is easy to use and to maintain and an accuracy of 0.5 mm on the calculated tank level can be achieved, taking into account the health of the tip of the dip tubes. Due to a precipitation of the salts in the highly concentrated solutions in the tank, the tip of the dip tube can become clogged: the liquid enters the dip tube and the air flow dries out the thin liquid film leaving a layer of salt on the inside of the dip tube. This is partly prohibited by using air humidifiers, but can never be totally avoided. This clogging of the dip tubes tip causes a change in the bubble behaviour and an increase of the frequency.

Since the pressure that is used to calculate the density and the level of the liquid in the tank is a timely average of the dynamic pressure signal, any change in this signal compared to the signal under calibration conditions will lead to an error in the calculations. An investigation of the impact of external parameters on the bubble formation behaviour and the bubble volume and height must therefore be carried out. In this work the effect of a change in temperature of the liquid bath, density and viscosity of the liquid, insertion depth of the dip tube and flow rate will be investigated.

\(^1\) A measurement in the reference tube is necessary since the tank will not be kept under atmospheric pressure. For security reasons there will always be an underpressure in the tank.
2.2 Calculation of the level

The assumption of a low air flow rate is made, meaning that the measured pressure equals the sum of the hydrostatic head of the liquid column and the pressure of the void of the tank:

\[
\begin{align*}
\begin{cases}
  P_L - P_R = \rho g h \\
  P_L - P_D = \rho g S
\end{cases}
\Rightarrow
\begin{cases}
  \rho = (P_L - P_D)/(\rho g) \\
  h = (P_L - P_R)/(\rho g)
\end{cases}
\end{align*}
\]

(2.2)

Correction factors are introduced to take some effects into account:

1. **Temperature**

   To take the changes in temperature into account, a thermal expansion coefficient \( \alpha \) is introduced:

   - **Height**
     \[
     H_m = \frac{P_L - (P_R + \text{BiasL})}{\rho g}
     \]
     (2.3)

     with
     
     - \( H_m \) = measured height of the liquid
     - \( P_L \) = Pressure measured on the Level Tube
     - \( P_R \) = Pressure measured on the Reference Tube
     - \( \text{BiasL} \) = Offset between Level and Reference Tube

   \[
   H_r = \frac{H_m}{1 + \alpha[T_m - T_r]}
   \]
   (2.4)

   with
   
   - \( H_r \) = height of the liquid at the reference temperature
   - \( T_m \) = measurement-temperature
   - \( T_r \) = reference temperature
   - \( \alpha \) = Thermal expansion coefficient

   - **Volume**
     \[
     V_r = \frac{V_m}{1 + 3\alpha[T_m - T_r]}
     \]
     (2.5)

     with
     
     - \( V_r \) = Volume measured at Reference Temperature
     - \( V_m \) = Volume at temperature of measurement

---

\(^2\) The bias is defined as the difference in pressure measured in the two tubes when the tubes are not inserted in the liquid and the flow rates are equal in both tubes.
• **Probe separation**

\[
S_r = \frac{S_m}{1 + \alpha(T_m - T_r)}
\]  
(2.6)

with

- \(S_m\) = Probe separation at measurement temperature
- \(S_r\) = Probe separation at reference temperature

• **Liquid density**

Indirectly this affects the calculation of the density of the liquid:

\[
\rho = \frac{(P_L - (P_D + BiasD))}{\rho.S_m}
\]  
(2.7)

with

- \(P_D\) = Pressure measured at the Density Tube
- \(BiasD\) = Offset between Level and Density transducers

• **Water density**

The direct effect of the temperature on the density of the water:

\[
\rho_{H_2O} = 998.47654 + 0.27997T^1 - 2.1435.10^{-6}T^2 + 4.3709.10^{-4}T^3
\]
\[-5.44028.10^{-6}T^4 + 2.72562.10^{-8}T^5\]  
(2.8)

2. **Buoyancy:**

The buoyancy effect is important during the weighing of the increment during the calibration. When two samples with the same mass but different volume are weighed, they will only result in the same reading if the density is the standard density (20°C, 1013.25 mbar and 50% relative humidity). When the air density increases resp. decreases, the sample with the largest volume will appear lighter resp. heavier.

\[
W_r = W_m + (V_m - V_{std})Q_{air}
\]  
(2.9)

1. with

- \(W_r\) = Liquid weight corrected for buoyancy
- \(W_m\) = Measured weight
- \(V_m\) = Displaced volume of weighed liquid
- \(V_{std}\) = Displaced volume of standard weights

---

3The bias is defined as the difference in pressure measured in the two tubes when the tubes are not inserted in the liquid and the flow rates are equal in both tubes.
and

\( Q_{air} = \text{Air density} \)

The direct influence of the temperature on the density of air:

\[
\rho_{air} = \frac{0.4645\text{Bar} - (0.085594 AT^2 - 1.8504 AT + 34.470)\cdot RH}{(2.73016 + AT)\cdot 10^6}
\]  \( (2.10) \)

with

Bar = Barometric pressure in mm Hg

RH = Relative Humidity in %

AT = Air Temperature in °C
Chapter 3

Tank calibrations

3.1 Inventory measurement in bulk handling facilities

Bulk handling facilities are confronted with challenges to the performances of weighing and volume measurement of their inventory. These measurements are the basis for nuclear material accountancy and verification and are utilised in the parts of the nuclear fuel cycle which have a very high strategic importance and where the measurement uncertainty has to be minimized. Therefore the TAME laboratory at the JRC, Ispra, is developing systems to increase the precision with which the volume and weight can be measured and followed during the whole cycle within the facility. Training courses for inspectors of the IAEA are organised in the laboratory. The course material for these courses was used as a reference for this chapter.

3.1.1 Nuclear Material Accountancy

According to the rules of international safeguards, a separate Nuclear Material Accountancy (NMA) is to be kept for Uranium and Plutonium. Therefore these materials have each a specific Material Balance (MB) in a well-defined Material Balance Area (MBA). A MB contains 5 columns and as many lines as there are different materials. For each material the data sheet contains the following data:

1. identification of the material

2. the beginning inventory (BI) or the amount of this material present in the MBA at the beginning of the period

3. the receipts (R) or the amount of this material which enters the MBA during the period
4. the shipments (S) or the amount of this material which leaves the MBA during the period

5. the end inventory (EI) or the amount of this material that is present in the MBA at the end of the period

Keeping in mind that the precision with which these amounts are measured is not infinite, the balance will not be zero. The imballance is defined by the IAEA as Material Unaccounted For (MUF) but is also called sometimes Inventory Difference (ID). This MUF can be calculated as following:

\[
MUF = \sum_i BI_i + \sum_i R_i - \sum_i S_i - \sum_i EI_i
\]  

(3.1)

**Definition:** The Material Unaccounted For (MUF) defined in a Material Balance Area the imbalance between the Beginning Inventory and Receipts at one side and the Shipments and End Inventory at the other side.

The MUF is a consequence of the measurement errors and is therefore a statistical variable. To make sure that the imbalance comes only from these statistical errors, only a MUF that is smaller than twice its standard deviation $2 \sigma_{MUF}$ is accepted.

### 3.1.2 Quantity determination at bulk handling facilities

In a bulk handling facility both weighing and volume determination are used as measurement systems. By coupling well-determined volumes with level indications, a quantity measure is given by the level reading. It is not the operation of the measurement system that poses problems, but its accuracy. The accuracy of a single quantity measurement (including all errors on calibration, mixing, sampling and analysis) in a reprocessing plant must be 0.3 % or better, otherwise significant quantities of nuclear material can be unaccounted for.

**Example:** 10 tanks of 500l containing a solution of 200g/l. If the systematic error is 1%, this corresponds to 10kg of Pu

To make the link between level and volume, a very accurate calibration is carried out which allows to look up for a given level the corresponding volume. Once this calibration curve is available, the content of the tank is monitored by recording the density and the level.

### 3.2 Description of the used tanks and equipment

A tank can have two different functions from the safeguards point of view: it can be used to measure the transferred quantities or the inventories. In the first case, only the
full and empty level are of real interest. In the second case the whole tank must be well-calibrated.

3.2.1 Inventory tanks

Before the different types of tanks are discussed, some definitions are given.

Definition: The inventory Key Measurement Points (KMP) are defined in terms of the principal stage of the technological process as: stores for nuclear material received and shipped, process line for nuclear material conversion, process line for recovering solid and liquid wastes,... The choice of the KMP allows for a facilitated internal accountancy of the nuclear material.

Definition: The Input and Outpunt Accountancy Tanks (IAT and OAT) are inventory tanks on the important KMP at the beginning respectively the at the end of the reprocessing line, which are used for complete physical inventory taking and verification of each nuclear material processed.

The existing reprocessing plants have a design based upon safety. Therefore the shape of the tanks is determined by safety requirements, beside some aspects of available space. Especially to avoid criticality, a shape with small volume over surface ratio is needed. The more concentrated the solution (output tanks) the more restrictive this requirement. Slabs or annular tanks in which neutron absorbing materials can be introduced are commonly found.

In the areas where high concentrations are not expected, the conventional tanks are cylindrical. The lower part is non-linear, the rather large middle part is linear and the upper part is non-linear again. Since the ratio of the volume over the height of the solution is rather large, an accurate level measurement is required to have an accurate volume determination.

The output accountancy tanks exist in various geometrical shapes, all designed to meet the safety requirements to avoid criticality because of the highly concentrated solutions of Plutonium or Enriched Uranium. The most used types are:

- the slab tank: maximum width of 10 cm, very suitable for fissile material accountancy, reinforcement on the inside to avoid deformations (again important for criticality reasons) but expensive to manufacture.

- annular or ring tanks: similar characteristics to the slab tank, cross section is a ring with gap width of about 10 cm.

- harp tanks: mostly used in the United Kingdom, legs have typically a diameter of 6 inches (152.4 mm). There are the D-tank, the E-tank and the Monteju-tank.
D-tank has a very non-linear profile and the E-tank is almost linear, since its legs are inclined in such a way that the horizontal cross section remains fairly constant and the profile approaches that of a cylindrical tank.

3.2.2 Recommended equipment for accountancy tanks

All the accountancy tanks are equipped with a pneumercator or dip tube system. This system measures density and level of the liquid in the tank. Inserting more than one density tube allows to compare the density at different heights and so indicates the heterogeneity of the liquid solution. If necessary, the signal can be given to mix the solution.

The diameter of the tubes must be at least 8 mm, to avoid clogging and to minimize the pressure build up in the tube. The clogging mainly occurs in highly concentrated solutions and can be avoided using air humidifiers. Like that, the air bubbling causing the up and down movements of the solution in the lowest part of the tube are not causing so much drying out of the thin liquid films. Hence solid deposits on the internal wall of the dip tube are anticipated.

The connection of the dip tube with the air supply is situated just above the top of the tank, minimizing the pressure build-up in the instrument lines. For safety reasons though, this is not always possible: an unintentional venting of the instrument line at a pressure below the tank pressure could induce an air-lift, resulting in a transfer of radioactive material into the working place. Therefore the air inlet is situated at a higher position, which causes some pressure build-up, dependent on the diameter, length, wall roughness, shape and bends of the measurement line. This build-up can be reduced by reducing the number of bends in the line, and to avoid differences in build-up, all the connection lines from the air supply to the dip tubes should run in parallel and have equal
Figure 3.2: A vertical cylindrical tank with upper non-linear part in the form of a cone

lengths.

Although the tanks are designed to be very stiff, equipment is needed to measure the deformations that would still occur. Also the fixation of the tank should be checked regularly to avoid a tilting of the tank in case of an earthquake.

Ideally for safeguards purposes both a density and a concentration measurement system should be installed at the same sampling point.

### 3.3 Specification of the liquids used

Calibrations are carried out using water, since its properties are so well-known. The solutions that need to be measured have very different properties. They are solutions of Plutonium and Enriched Uranium that typically have a density of about 1.4 kg/l. They can contain up to several hundreds of grams of the nuclear material per liter. The difference in liquid properties will have an effect on the accuracy of the dip tube system, this is discussed later in this work.
3.4 Mass/Volume methodology

A calibration can be done in two modes: continuously or discrete, and in two ways: incremental or decremental. One of the following scales can be used: weight, volume or concentration. To avoid temperature effects during the calibration, the primary tank calibrations are carried out in continuous mode and with the weight as a scale. This is called the Classical Gravimetric Calibration.

3.4.1 Gravimetric Calibration Method

During a calibration, weighed increments are added and the corresponding water level is measured. The weight is used as a scale because of its invariance against temperature variations. In practice, a flask with water is weighed before and after each increment to the tank, giving the weight of the added water: $W_{f_i} - W_{e_i}$. By measuring the temperature
of the water in the tank after each increment, the density of the water can be calculated using the formula of Jones and Harris (1992):

\[ \rho_{th,i} = 999.84847 + 6.337563 \times 10^{-2}.T - 8.523829 \times 10^{-3}.T^2 \\
+ 6.943248 \times 10^{-5}.T^3 - 3.821216 \times 10^{-7}.T^4 \]  

(3.2)

Knowing this density and adding the increments, the total volume after increment \( i \) is:

\[ V_i = \sum_{j=1}^{i} \left( W_fj - W_ej \right) / \rho_{th,i} \]

(3.3)

The level tube records at each increment the level \( L_i \) with the calculated density\(^1\). The result of the calibration is the curve of the corresponding couples \((L_i, V_i)\) that represent the profile of the tank.

### 3.4.2 Calibration plan

Before a new tank can be used, an extended calibration must be carried out. When an inspection takes place later, this whole procedure is not repeated, but a simple calibration verification is performed. The steps taken during a calibration will be discussed in the following paragraph.

**Preparation**

Before the real calibration procedure can start, the Engineering Flow Sheet (EFS) is studied. In this document, the function of the tank is explained and all the possibilities for receiving or transferring solutions are given. Also other components for for example temperature, density and level instrumentation, warning alarms and mixing and sampling devices are indicated. It is the task of the operator and inspector to investigate how these components can influence the calibrations or measurements.

The Detailed Construction Drawings (DCD) allow the inspector to determine the expected calibration regions in the tank and to calculate also the heel and the theoretical calibration equation for each region. With these estimations, the influence of the presence of internal components in the calibrations can be identified. The physical points of interest are highlighted.

Comparing the EFS and the DCD lets recognize lines not drawn on the EFS and omissions in the DCD.

\(^1\)The formula for the density is used as long as the density tube does not touch the liquid.
In the next preparation step, the changes in the tank’s design are checked out. Especially the extension of instrument lines is relevant, since they often represent a source of errors. All the information about the tank from its initial design up to its current status is checked.

**In-field calibration preparation**

- Before the start of the calibration, all the necessary instruments must be available and in a good condition. The instruments that are available on the site can be used, but the inspector can also decide to bring a second set of instruments for comparison. The operator should provide the inspector of the necessary information, including the material of construction of the standard weights and a copy of the certificate of the national authorities concerning the calibration of the standard weights.

- A very small effect that does bias the results slightly is the air buoyancy effect. This is the effect of replacing a volume of air contained in an object on a scale by a volume of standard weights or calibration input. To estimate the scale of this effect, the ambient temperature, humidity and local pressure close to the scale must be recorded.

- In the next step, the instruments the operator uses during a calibration verification are checked. If the inspector decides so, extra measurement instruments are added to the system and tested together with the operator’s systems. During these tests, the zero points are checked (possible biases) and all the procedures necessary to ensure that the tank is in best condition to carry out a calibration are identified. Once all the instruments are adjusted, they are sealed to avoid a change in setting later-on during the calibration and the measurements. The weighing scale must pass a test in which at least 10 points between 0 and maximum must be determined with an accuracy of 0.005%.

- After this, the leak test is performed. This is done by capping the tubes and inserting freon gas, or by performing an X-ray. This test is very important in the calibration procedure, since any leak could make the calibrations useless. Leaks at joints can be detected by a steady decrease of pressure. Applying soap and looking where the bubbles appear is the simplest way to see where the leak is.

- The density of the calibration fluid must be known very precisely. If the inspector finds it necessary, a sample of the liquid can be analysed in an external laboratory, but in any case the liquid density is measured on the site with a high precision densitometer.
• All the lines and valves should be as much as possible in the condition and position as needed for the routine measurements as will be performed later. All the valves on possible transfer routes should be closed and if possible sealed, to avoid accidental transfers out of or into the tank.

• In the case of a first calibration, the feed line of the calibration liquid should be as direct as possible. Even wetting this feed line before the first increments can be important to avoid errors in the weight of the increment that enters the tank.

• The determination of the heel is the last step before the actual calibration of the tank. The heel of a tank is the total content below the level dip-tube and comprises the volume of the liquid from the bottom of the tank to the lowest point of the tube, which dips the most deeply into the tank. Obviously the heel indicates the minimum amount of liquid, responding to the lowest level that can be measured. Mostly this calibration of the heel is only done during the very first calibration of the tank. An estimate of the volume of the heel is made using the DCD. Small increments are added to the tank. When the first bubble is detected in the pressure signal of the level dip tube, the sum of the increments is the total volume of the heel. The pressure signal will rise abruptly and will indicate about 5mm, or 50 Pa.

**In-field calibration procedure**

• The non-linear part
  Usually not much attention is paid to this part of the tank, but it can not be totally ignored since it contains a lot of well localized internal equipment that later could be used as reference points for later recalibrations. Especially the height at which the non-linear part goes over into the linear part is important. With this height the length of the dip tubes can be checked (e.g. shortening of the tubes by corrosion).

• The linear part
  In the linear part, the increments can be larger. About 10 points would be enough, but the prover to deliver the increments is usually not large enough so that more than 10 points are taken. Some points of interest in the linear region are the position of the tip of the dip tube and the overflow line. The latter will give an indication of the maximum volume that the tank can contain keeping in mind that some routine operations such as mixing must take place without an overflowing of the tank. The linear part is characterised by a linear relationship between the pressure and the level. Also the slope, the ratio of the volume increment over the level increment,

\[ \frac{\text{volume increment}}{\text{level increment}} \]

2Little drops of the calibration liquid can stick to the inside wall of the feed line, so that less liquid enters the tank than initially intended.
is constant\textsuperscript{3}. Special care is to be given to temperature variations. If the tank with calibration liquid is empty and liquid from a different location is added to the tank, the temperature and thus the density can be different. For water, formula (3.2) can be used. For other solutions, the density can be calculated with the standard density and $\rho_0$ and the thermal expansion coefficient $\beta$ by the formula

$$\rho_T = \rho_0 + \beta (T - T_0)$$  \hspace{1cm} (3.4)

With this density and the measured pressure difference between the level and density dip tubes the probe separation $S_R$ can be calculated as:

$$S_R = \frac{p_L - p_D}{g \cdot \rho}$$  \hspace{1cm} (3.5)

Comparing this value with the distance between the tubes as stated in the DCD allows to certify the status of the dip-tubes.

- Testing after calibration
  
  At the end of the calibration, the valves are checked again to see if no changes occurred during the operations. Opening a valve will cause liquid to flow from the tank into a line. The change in level reading allows now to determine the volumes of each line. The leak test can now be repeated under a higher pressure and the zero points of the manometers are checked again.

\textsuperscript{3}In stead of using the level in the tank, the slope is used. This allows to detect inserted objects in the tank. Example: if a box with a certain volume replaces some diverted solution in the tank, the level will remain unchanged, but the slope in that region will decrease.
Chapter 4

Theory: Forces acting on the bubble

The maximum bubble volume before detachment is the result of a balance of forces acting on the bubble. These forces will not be discussed in detail, the purpose is to get a feeling of what kind of forces there are present in the system.

Before discussing these forces, some assumptions are made to simplify the system:

- The bubble is formed quasi-statically.
  In practice this means that the air flow is very low, and the currents in the liquid surrounding the bubble caused by the previous detachment can be neglected.

- The dip tube is positioned perfectly vertical, the edges of the recipient are far away and the tip of the tube is perfectly flat.
  This implies that there are no forces acting on the bubble which would cause a preference to detach at a certain side.

- The bubbles are perfectly spherical when detached and perfect spherical segments when still attached to the tube.

The bubble will detach from the tube when it is energetically favourable. To determine at what volume the threshold lays, the different forces on the bubble are determined. In figure 4.1 2 situations are shown with the same bubble volume. The energy in the 2 situations will be compared and the situation with the lowest energy will be the most favourable.

*Situation 1:*

The total energy in the first situation is the sum of:

- energy of the Archimedes force

- interfacial energy in the bubble liquid/gas interface

- surface tension energy at the ring of air-metal liquid contact
CHAPTER 4. THEORY: FORCES ACTING ON THE BUBBLE

Figure 4.1: 2 situations with the same bubble volume: bubble attached to the tube and bubble detached.

\[ E_1 = \rho_{\text{liq}} g V_0 (H + z_0) + \sigma S_1 + E_{\text{air-metal-liquid}} \]  

Situation 2:
The total energy in the second situation is the sum of:

- energy of the Archimedes force
- interfacial energy in the bubble liquid/gas interface
- surface tension energy at the ring of air-metal-liquid contact
- surface tension energy at the air-liquid contact at the dip tube’s tip.

\[ E_2 = \rho_{\text{liq}} g V_0 H + \sigma S_2 + E_{\text{air-metal-liquid}} + \sigma \frac{\pi d^2}{4} \]  

In another assumption, the energy due to the air-metal-liquid contact is only dependent of the inner diameter of the dip tube’s tip, and so the \( E_{\text{air-metal-liquid}} \) is equal in both equations. Subtracting 4.2 from 4.1 gives:

\[ F[V_0(r)] = E_1 - E_2 \]

\[ = \rho_{\text{liq}} g V_0 z_0 + \sigma \left( S_1 - S_2 - \frac{\pi d^2}{4} \right) \]  

All the terms on the left hand side can be written as a function of \( r \) or \( V_0 \). Predictions about the trends are made for volume and bubble height. The results of
these calculations and the trends to be suspected are found in Figures 4.2 and 4.3. The following trends are found:

- Increasing volume and bubble height for decreasing density
- Increasing volume and bubble height for increasing surface tension

Figure 4.2: Prediction of the maximum bubble volume as a function of density and surface tension
Figure 4.3: Prediction of the bubble height as a function of density and surface tension
Chapter 5

Fluid-dynamic simulations of bubble formation at the tip of a dip tube

5.1 Introduction on the applied CFD code COMSOL

COMSOL Multiphysics is a modeling package for the simulation of any process that can be described with partial differential equations. The models are solved using the finite element (FEM) method in an acceptable time. Models can be created in 3D, 2D, 1D and in axial symmetry. For the modelling of the pneumercator, a 2D axial symmetric model was created and for the tilted tube configuration a 2D model was created. The models were solved with the fluid dynamics module, using the Navier-Stokes equation set in the transient analysis.

5.2 Steady formation of an axi-symmetric bubble

To understand the physics of the bubble formation and the impact parameters, fluid-dynamic simulations have been carried out. The discontinuous point of bubble pinch off as well as instabilities for detaching of the bubble were excluded to simplify the simulations. In a first step half of the experimental setup with straight tube was modelled in 2D with a hybrid mesh, shown in figure 5.1. The mesh has been redefined to guarantee that it has no impact on the simulation results. The calculation time for this 2D model with 32260 mesh elements was 188 s. The boundary conditions and the parameter settings, including the properties of the liquid, are given in table 5.1 and table 5.2. The subdomain settings can be found in table 5.4.

| Number of degrees of freedom | 146193 |
| Number of boundary elements | 611    |
| Number of elements           | 32260  |
| Minimum element quality      | 0.5733 |

(5.1)
Figure 5.1: Axi-symmetric mesh

<table>
<thead>
<tr>
<th>Boundary</th>
<th>1,3</th>
<th>2,7-8,11-12</th>
<th>4-5</th>
</tr>
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<tbody>
<tr>
<td>Type</td>
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<td>No slip</td>
<td>Inflow/outflow velocity</td>
</tr>
<tr>
<td>z-velocity (v0) (m/s)</td>
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<td>0</td>
<td>-1</td>
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<tr>
<td>Pressure (p0) (Pa)</td>
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<td>0</td>
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(5.2)

<table>
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<td>Pressure (p0) (Pa)</td>
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(5.3)

<table>
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<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>1000</td>
<td>1.3</td>
</tr>
</tbody>
</table>

(5.4)

| Dynamic viscosity (Pa.s) | 10⁻⁵ | 10⁻⁶ |

The shape of the bubble can be seen clearly in Figure (5.2). A detaching bubble can not be achieved in the simulation program. During the simulations, parameters such as liquid density, inflow velocity and pressure at the dip of the tube (corresponds to the change of insertion depth of the dip tube) were changed, but without result. The maximum velocity and the profile as seen in figure 5.2 do not change when changing the parameters.

The observation was made that for infinite time the bubble always stays nicely under
the tube. To see what happens if the bubble is forced to grow from underneath the tube, a second model was set up containing a tilted dip tube.

5.3 Bubble formation at a tilted tube

In order to model the bubble growth forced from underneath the tube towards the side, a second model was setup with a tilted tube. By tilting the tube the instability problem is avoided. This time the complete dip tube is modelled with a full 2D hybrid mesh, as shown in figure 5.3, that showed to be sufficiently fine with the extra refined part around the tip of the dip tube. The calculation time for this mesh with 13060 elements was 16.023 s. The boundary conditions for a stationary solution and parameter settings are given in tables 5.5 and 5.6. The density and dynamic viscosity of the liquid are respectively 1000 kg/m$^3$ and $10^{-5}$ Pa.s. The results are presented in figure 5.4. The bubble shape observed in the results with this model seems to be more stretched out in vertical direction, because it simulates a strong vertical flow into the water. No impact of changes in the parameters (density, flow velocity, pressure at the dip tube) on the size and shape of the bubble could be simulated.
5.4 Behaviour of the liquid entering the dip tube

In order to understand the air-liquid interfacial periodic changes, a last model for the water going into the dip tube was set up with a moving mesh. A model was produced using a combination of the Navier-Stokes module and the moving mesh application mode, based on the reference model of the sloshing tank. A report generated by COMSOL containing the details of the model can be found in the appendix. As in the model with the tilted tube, the mesh was refined around the tip of the tube until no further
impact on the simulation results was seen. The calculation time for this mesh with 2260 elements was 223.5 s. Figure (5.5) shows screenshots of the movie produced by COMSOL Multiphysics. The moving boundary underneath the tube is used to simulate the movement of the liquid surrounding the tube after the detachment of the previous bubble caused by the turbulence. The oscillating movement of the liquid entering the tube is clearly demonstrated with this simulation.
Figure 5.5: Screenshots of the movie generated by COMSOL, showing the movement of the liquid in the tube
Chapter 6

Experimental Setup

6.1 Introduction

In order to investigate the behaviour of the bubble at the dip tube’s tip, an experiment was set up that allowed to change external parameters such as temperature of the liquid bath, position of the dip tube in the liquid bath, flow rate and liquid properties such as density and viscosity. To follow the shape and volume of the bubble during growth and detachment, 2 cameras were used.

6.2 The recipient

The choice of glass was determined by the requirements to make accurate pictures of the bubbles at the tip of the tube dipping in different solutions of different densities.

- Glass is recommended because the reflections during the transmission of light of all wavelengths through the glass can be neglected.

- A flat-sided recipient was chosen in order to avoid deformations due to the use of a curved surface. In case of a cylindrical recipient deflections and deformation of the image is to be anticipated and corrections to be considered. Moreover correction factors are specific for each setup and need to be recalculated when the camera is moved.

- A relatively small recipient of 20l was utilised in order to be able to investigate the bubble formation in different kinds of solutions, with densities varying from 1.0 kg/l to 1.3 kg/l.
6.3 The dip tubes

As already discussed into detail, the measuring device consists of 3 dip tubes of different length. The longest is the level tube, the second is the density tube and the shortest is the reference tube. They are soldered together and are positioned over the aquarium using a plastic support that allows to move the tubes up and down.

All three tubes are identical in shape and diameter. They have a flat end with inner diameter measured to be 6 +0.1 mm and outer diameter 10 +/- 0.1 mm. The material used is stainless steel.

6.4 The air flow regulators

The device used to regulate the air flow through the tubes consists of three Bronkhorst Hi-Tec El-Flow Mass Flow Controller/Meters (type SN M2201795A F-201C-RGA-22V) united in a box. The technical specifications can be found on the following website of the manufacturer \(^1\) or in the appendix.

The length of the connecting tubes was kept as short as possible, order of 1m, in order to avoid establishing a compressible air reservoir, damping and so disturbing the dynamic bubble behaviour. The constant pressure drop due to friction could be neglected because bending the pipes has been avoided and the diameter to length ratio was sufficiently large.

A compressor supplied dry air with flow rates varying from 0.5 l/h to 14 l/h. A possible offset between the air flow set via the computer program and the actually supplied air flow was anticipated but not measured. In real situations the air flow supply is rather

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\(^1\)http://www.bronkhorst.com/files/downloads/brochures/folder-el-flow.pdf/
Figure 6.2: On the left: the pressure acquisition device, on the right: the dip tubes

not precisely known, because it does not affect the volume determination as long as it is kept constant and through all dip tubes equal.

6.5 The pressure acquisition

The pressure was measured with a pressure acquisition unit of type MPX50, housing a pressure transducer of vibrating cylinder type. This type of pressure transducer is known as the most accurate. The calibration and technical characteristics of the MPX have been downloaded from the web\(^2\).

The DPM was connected to a data acquisition unit that was included in a single pressure measurement device, that was connected directly to the computer. The data were acquired in the form of an excel-datasheet with a datarate of 10 kHz. The data acquisition unit allowed settings in the range from 5 kHz to 40 kHz.

The calibration of the pressure measurement device has been done by connecting a pump and a second pressure measurement device connected to a computer\(^3\). The pressure measured with this second device was then plotted against the voltage measured by first device. The following calibration relation was derived:

\[
\text{Pressure (kPa)} = \text{Device output (V)} \times 10.063 - 18.219 \quad (6.1)
\]
CHAPTER 6. EXPERIMENTAL SETUP

6.6 The camera

The cameras that are used are from the type Basler A600f series. The datasheet and the manual can be downloaded from the manufacturer’s website\(^4\). The A602f that is used here has a maximum frame rate of 100 images/s, but the actual maximum frame rate depends on the shutter speed. When the shutter speed is too low, some images will be skipped and less than 100 frames per second will be actually taken, which will lead to an uncorrect synchronisation with the pressure signal. Fortunately this can be seen directly since when the shutter time is too large, the number of pulses given by the clock and the number of images stored by the program don’t match. When this happens, the problem can be solved simply by lowering the shutter time. Also some frames can be lost during the acquisition, because of programs running in the background. Windows can generate

\(^{2}\)http://www.datasheetarchive.com

\(^{3}\)The same program was used as for the pressure measurement during calibrations.

\(^{4}\)http://www.baslerweb.com
interrupts that cause the loss of a few images. The only thing that can be done to avoid this kind of losses as much as possible is to run only the necessary program (for saving the images) and to disconnect from all networks. After such a loss, the acquisition should be restarted.

During the experiment two cameras were used, placed under an angle of 90°. This is done to calculate the volume of some shapes and with these data calculate the error on the calculated volume and diameter of these shapes.

Some problems using 2 cameras were experienced:

- Data acquisition at a very high rate should be possible, so a fast computer is needed.
- The program that processes the images demands a certain contrast in the image and no reflection spots should appear near the edge of the bubble. This means that the position of the cameras and the light sources is very important.
- To use the formulas with the cameras under 90 degrees, it is necessary to have them exactly under 90 degrees, otherwise another uncertainty is introduced in the experimental results. For an estimation on that error, see the paragraph on the use of the camera.
- The chances of losing an image during the acquisition become larger. Both cameras must acquire all the images to have a correct synchronisation later on.

---

5In about 10% of the measurements this kind of loss was encountered.
6Before starting the acquisitions, some days had to be spent on getting to know the cameras and the program. After that it was clear that using indirect lighting was the right option to avoid reflection spots near the edge of the round bubble. Therefore pannels with white paper were installed next to the recipient. A second light source was added to compensate for the loss of light and to optimise the lighting for both cameras.
6.7 The temperature measurement device and heater

A simple thermometer and a heat source were inserted in the recipient. This equipment was not really high-standard and the range of the heater was small, from 20°C to 34°C. The errors on the temperature are therefore estimated to be 10%\textsuperscript{7}.

6.8 The clock for synchronization

The images are acquired at a rate of 100 Hz and the pressure at 10 kHz. This means that the signals could not be synchronized without a trigger and a clock was needed. The trigger consists of a repetition of a peak followed by a flat signal, which is on a time line equidistant. During the peak with flat signal 100 pressure data are acquired whereas

\textsuperscript{7}A temperature difference of 2°C could not be avoided during the time needed to perform the experiments (about 1 hour per temperature). The estimate is therefore taken to be 2°C/25°C+uncertainty on the thermometer=10%
only one picture is taken. The acquisition is launched on the rising flank of the peak, for the pressure measurement only taking into account the first rising of the flank and for the images taking into account every rising flank. This allowed to match the separate images with the pressure signal and make a correct synchronisation in time.

Since the clock and the pressure are measured and recorded at the same time and stored in one Excel-file, no further synchronisation had to be carried out for those signals. Only a conversion of the number of the pressure data rate to the image rate had to be performed, coupling the number of pressure data to the according image number. Since the clock signal is very stable this calibration only had to be carried out for one signal, and the calibration formula could be used for all the data sets. The following steps were taken during this calibration:

1. Deleting of all the pressure data recorded before the first trigger signal

2. Making a new time scale fitting the rising flank of each trigger signal to the according image number, using the row number in the excel-sheet.

3. The following formula transforms the row number of the rising flanks to the image
number.

\[
\text{Image number} = \text{row number of the pressure data} \times 0.0096 + 0.9841 \quad (6.2)
\]

6.9 Programs

As already mentioned, a few programs were used for the acquisition and the processing of the data:

- regulation of the flow
- acquisition of the images
- acquisition of the pressure
- processing of the images in the Bubble-program

Only the last program is worth going into detail in, since it poses some demands for the setup of the experiment.

The Bubble program.

The acquired images (JPEG-files) were stored in a folder together with a reference image containing only the tube in the exact same position as during the acquisition of the images. The Bubble program is used to treat these images, taking the following steps (see Figure (6.10)):

1. Selection of the reference frame: This frame only contains the image of the tip of the dip tube.

2. Selection of the diameter of the dip tube and calibration: outer diameter 10mm, and selection of the area where the bubble starts to grow.

3. Captured frame is selected

4. Inversion of the image and subtraction of the reference frame. The only white in the picture is now the inside of the bubble.

5. Finding the edges around and in the bubble. It is important in this step that the edges are sharp, meaning that there are no spots due to reflection near the edges. If this were the case, the calculated volume would be too small and the shape of the bubble in the processed image would not be nice and round.

6. Removing the black spots inside the bubble again due to reflection.
7. The reference image is subtracted again to make sure that the edge of the bubble is exactly where it should be and that it is not displaced because of the smoothing effects applied.


For the calculation of the volume with orthogonal cross section image, there is assumed that the bubble is axially symmetric, so the following formula can be used:

\[
V = \pi \int_0^h f(z)^2 \, dz \approx \pi \sum_{\text{each line}} f(z)^2 \Delta z
\]  

(6.3)

with \(\Delta z\) the length according to 1 pixel and \(f(z)\) the number of pixels on the line multiplied by \(\Delta z\) and divided by 2. \(\Delta z\) is determined in step 2) as follows: the number of white pixels is counted and the length corresponding to that number is given to be the
outer diameter of the dip tube, 10mm. A simple division gives $\Delta z$, the number of mm according to 1 pixel.

The error on the volume is calculated using $\Delta z$ as the error on $f(z)$, so that gives:

$$\text{Error}[	ext{Volume}] = 2 \sum_{\text{each line}} f(z).\Delta z.\text{Error}[f(z)]$$

\hspace{1cm} (6.4)
Chapter 7

Using the cameras

7.1 Introduction

The cameras were used to follow the shape and the volume of the bubble during the entire cycle. The accuracy with which this volume is calculated needs to be determined to make a conclusion about the usability of the images. In the first step, perfect spherical shapes are captured. From this the error on the diameter can be derived and this error will be the error on the diameter and height of the bubble. In the second step, a mathematical calculation of the error on the volume of a perfect ellipsoidal shape will be determined. If the ellipsoid is seen by the cameras under the good angle, the volume could be calculated with zero error, but when the cameras see the ellipsoid under a wrong angle, meaning that not the maximum and minimum axes are seen, an error will be introduced in the volume calculation. The maximum error will be used as the error on the bubble volume. This error will be dependent on the ratio of the diameters of the ellipse and the viewing angle. In a third and last step, the volume is calculated of some irregular shapes. The error on these shapes will be an indication of the error on the volume calculated during the detachment of the strongly deformed bubble.

7.2 Verification of the volume of perfect spherical shapes

To measure the volume of a bubble, images are taken with a camera and the bubble program then calculates the volume. For details on how this is done, see the section on the Bubble program. In this section, measured volume and the calculated volume will be compared. First, the diameter of the metal balls were accurately measured and the volumes were calculated. Then an image was taken of the entire ball hanging on a magnetic stick inserted in the dip tube. As for the calculation of the bubble volume, the diameter of the dip tube is taken as a reference. The results are compiled in the following
CHAPTER 7. USING THE CAMERAS

Figure 7.1: Calibration image and processed image of ball 1.

Figure 7.2: Calibration image and processed image of ball 2.

<table>
<thead>
<tr>
<th></th>
<th>measured volume (mm$^3$)</th>
<th>calc.volume (mm$^3$)</th>
<th>volume error</th>
</tr>
</thead>
<tbody>
<tr>
<td>ball 1</td>
<td>575.322</td>
<td>580</td>
<td>($\frac{580-575.322}{575.322}$) = 0.813%</td>
</tr>
<tr>
<td>ball 2</td>
<td>348.490</td>
<td>349</td>
<td>($\frac{349-348.49}{348.49}$) = 0.146%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>measured diameter (mm)</th>
<th>calculated height (mm)</th>
<th>calculated diameter (mm)</th>
<th>error on diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>ball 1</td>
<td>10.319</td>
<td>10.36</td>
<td>10.39</td>
<td>($\frac{10.39-10.319}{10.319}$) = 0.688%</td>
</tr>
<tr>
<td>ball 2</td>
<td>8.731</td>
<td>8.78</td>
<td>8.76</td>
<td>($\frac{8.76-8.731}{8.731}$) = 0.332%</td>
</tr>
</tbody>
</table>

Conclusions:

1. The volume of a perfect spherical shape is very accurate, the error is smaller than 1%.

2. Both the camera and the bubble program are well calibrated in horizontal and vertical direction. The error that is made is small enough to be neglected.
7.3 Error calculation using 2 cameras

When the shape of the bubble is not perfectly spherical, but ellipsoidal, then the volume could be calculated with the precision of a sphere, in case the cameras are viewing the bubble under the right angle. In this section, an estimation will be made on the error on the volume for different ratios of the diameters of the ellipse and different viewing angles.

The formula for an ellipse is:

\[
\begin{align*}
    x &= a \cos \theta \\
    y &= b \sin \theta 
\end{align*}
\]  

(7.1)

The formula for a line going through the centre of the ellipse:

\[
y = \alpha x
\]  

(7.2)

Formula for the line perpendicular to the first one:

\[
y = -\frac{1}{\alpha} x
\]  

(7.3)

![Figure 7.3: Ellipse with diameters a and b and 2 perpendicular lines through the centre, under an angle alpha with the main axes](image)

The images of the 2 cameras give 2 diameters of the bubble in perpendicular views. The simple formula \(\pi a b\) for the surface of 1 cross section can only be used when the largest and smallest diameter of the ellipse are known. Since the angle under which the ellipse is seen is not known, an angle-dependence has to be introduced to learn the maximum error.

To calculate the different surfaces, the distance between the centre and the point on the surface has to be known. To calculate this, the interception of the ellipse and the 2 perpendicular lines is calculated:
Interception 1:

\[
\begin{align*}
\left\{\begin{array}{l}
y = \alpha x \\
x = a \cos \theta \\
y = b \sin \theta
\end{array}\right.
\Rightarrow \left\{\begin{array}{l}
\theta = \arctan \left( \frac{\alpha a}{b} \right) \\
x = a \cos \left( \arctan \left( \frac{\alpha a}{b} \right) \right) = \frac{a}{\sqrt{1 + \left( \frac{\alpha a}{b} \right)^2}} \\
y = b \sin \left( \arctan \left( \frac{\alpha a}{b} \right) \right) = \frac{\alpha a}{\sqrt{1 + \left( \frac{\alpha a}{b} \right)^2}}
\end{array}\right.
\Rightarrow \sqrt{x^2 + y^2} = a \cdot b \cdot \sqrt{\frac{1 + \alpha^2}{b^2 + a^2 \alpha^2}} = r_1
\end{align*}
\]

Interception 2:

\[
\begin{align*}
\left\{\begin{array}{l}
y = \frac{1}{\alpha} x \\
x = a \cos \theta \\
y = b \sin \theta
\end{array}\right.
\Rightarrow \left\{\begin{array}{l}
\theta = \arctan \left( -\frac{a}{\alpha b} \right) \\
x = a \cos \left( \arctan \left( -\frac{a}{\alpha b} \right) \right) = \frac{a}{\sqrt{1 + \left( \frac{a}{\alpha b} \right)^2}} \\
y = b \sin \left( \arctan \left( -\frac{a}{\alpha b} \right) \right) = \frac{a}{\alpha \sqrt{1 + \left( \frac{a}{\alpha b} \right)^2}}
\end{array}\right.
\Rightarrow \sqrt{x^2 + y^2} = a \cdot b \cdot \sqrt{\frac{1 + \alpha^2}{\alpha^2 b^2 + a^2}} = r_2
\end{align*}
\]

With these radii known, the surfaces of the different cross sections can now be calculated. Three methods are compared.

Method 1:

Using the formula \( \pi \cdot r_1 \cdot r_2 \) :

\[
\pi \cdot r_1 \cdot r_2 = \frac{\pi \cdot a^2 b^2 (1 + \alpha^2)}{\sqrt{b^2 + a^2 \alpha^2} \cdot (a^2 b^2 + a^2)} = \frac{\pi \cdot a \cdot b \cdot (1 + \alpha^2)}{\sqrt{1 + \varepsilon^2 a^2} \cdot (\frac{a^2}{\varepsilon^2} + 1)}
\]

with \( \varepsilon = \frac{a}{b} \). The error on the 'real' surface \( \pi \cdot a \cdot b \) is here:

\[
\frac{(1 + \alpha^2)}{\sqrt{1 + \varepsilon^2 a^2} \cdot (\frac{a^2}{\varepsilon^2} + 1)}.
\]

Method 2:
Figure 7.4: Errors for different angle from 0 to $\pi/2$ for different diameter ratios $\varepsilon$. The thicker lines correspond to the error made by using the geometrical average, the thin lines correspond to the algebraic average. The green lines correspond to $\varepsilon = 1.1$, the grey lines to $\varepsilon = 1.2$ and the black lines to $\varepsilon = 1.25$.

Calculating the volume like there would be only 1 camera and taking the geometrical average of the volumes. This method gives the same results as the first method, since $\pi r_1 r_2 = \sqrt{(\pi r_1^2) \cdot (\pi r_2^2)}$.

**Method 3:**
Calculating the volume like there would be only one camera and taking the algebraic average of the volumes:

$$\frac{(\pi r_1^2) + (\pi r_2^2)}{2} = \pi a b \frac{(1 + \alpha^2)}{2} \frac{(\varepsilon + \frac{1}{\varepsilon})}{(1 + \varepsilon^2 \alpha^2) \cdot (\frac{\alpha^2}{\varepsilon} + 1)}$$

(7.8)

The errors are now plotted as a function of $\alpha$ for different values of $\theta = \arctan(\varepsilon)$:

In Figure (7.4) the errors for the different methods are plotted against the viewing angle. The integrated error over all angles is the same for both methods, but the algebraic average gives the smallest error in the larger range of angles. The following conclusion can be drawn:

**Conclusion:**
The algebraic average of the volumes calculated separately by the two cameras gives the smallest error on the actual volume over the largest range of angles.

**Conclusion:**
The error on the algebraic average of ellipsoidal volumes increases with increasing diameter ratio $a/b = \varepsilon$.

The maximum error occurs at an angle of $\pi/4$ and 0 degrees and is 2.5% for $\varepsilon = 1.25$. 


These conclusions are valid for perfect ellipsoidal shapes, no conclusions about non-ellipsoidal shapes can be drawn from these calculations.

### 7.4 Consequences of the bubble not being elliptic

Since the bubble is not axisymmetric and probably even not exactly elliptic, there is another volume error to take into account. Here, a more experimental rather than theoretical investigation was made. The volumes of a few non-symmetric pieces were determined by measuring the volume and by calculating the volume using the images made by the cameras.

#### 7.4.1 The pieces

The pieces are an acorn, a kindersurprise-egg and 2 Inox shapes fabricated in the workshop. The $\varepsilon$ for the ellipsoidal shape is 2.

![Figure 7.5: 1: kindersurprise-egg, 2: acorn, 3: ellipsoid, 4: bullet](image)

#### 7.4.2 Measuring the volume

To measure the volume, the Archimedes principle was used. By measuring the mass of a liquid in which the piece is floating, supported by a standard and then subtracting the

![Figure 7.6: Archimedes weighing method](image)
volume of the liquid and the recipient, the Archimedes force can be measured:

\[ F_{Archimedes} = \rho_{H2O} \cdot g \cdot V_{object} \]  
\[ m_{total} \cdot g = m_{(H2O+recipient)} \cdot g + \rho_{H2O} \cdot g \cdot V_{object} \]

\[ \Rightarrow V_{object} = \frac{m_{total} - m_{(H2O+recipient)}}{\rho_{H2O}} \]

With \( \rho_{H2O} = 0.997 \text{ g/cm}^3 (25^\circ \text{C}) \).

The results of the measurements are compiled in the following table:

<table>
<thead>
<tr>
<th>mass</th>
<th>mass measured volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>without insertion (g)</td>
<td>with insertion (g)</td>
</tr>
<tr>
<td>Inox bullet</td>
<td>106.613</td>
</tr>
<tr>
<td>Inox ellipsoid</td>
<td>106.628</td>
</tr>
<tr>
<td>Eikel (Acorn)</td>
<td>106.461</td>
</tr>
<tr>
<td>Kindersurprise-egg</td>
<td>105.581</td>
</tr>
</tbody>
</table>

7.4.3 Calculating the volume

In this section, the volumes are calculated using the images taken by the cameras. As in shown above, the algebraic average gives the smallest error when working with ellipsoidal shapes. This conclusion was made based on the range of angles the algebraic average gives a smaller error than the geometric average. In the case of the capturing of these irregular shapes, the angle between the main axes of the shape and the cameras was taken as close as possible to zero\(^1\). In this case the geometrical average would give the smallest error. To compare the errors, both averages are calculated. The respective errors will be calculated in the paragraph below.

The results are compiled in the following table:

<table>
<thead>
<tr>
<th>camera 1 (cm(^3))</th>
<th>camera 2 (cm(^3))</th>
<th>geometric avg (cm(^3))</th>
<th>algebraic avg (cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inox bullet hor</td>
<td>26.472</td>
<td>5.011</td>
<td>11.517</td>
</tr>
<tr>
<td>Inox ellipsoid hor</td>
<td>16.426</td>
<td>5.015</td>
<td>9.076</td>
</tr>
<tr>
<td>Acorn hor</td>
<td>19.272</td>
<td>1.962</td>
<td>6.149</td>
</tr>
<tr>
<td>vert</td>
<td>5.923</td>
<td>7.534</td>
<td>6.680</td>
</tr>
<tr>
<td>Kindersurprise-egg hor</td>
<td>27.300</td>
<td>47.355</td>
<td>35.955</td>
</tr>
<tr>
<td>vert</td>
<td>31.323</td>
<td>35.233</td>
<td>33.220</td>
</tr>
</tbody>
</table>

\(^1\) The shapes were positioned in such a way that the maximum and minimum diameter were seen by the cameras.
The volume was also determined measuring the weight. The density of the inox the
shapes are made of is 8.03 g/cm³. The volume of the bullet determined in that way is
\[
\frac{81}{8.03} = 10.087
\]
and for the ellipsoidal shape:
\[
\frac{64}{8.03} = 7.970.
\]

### 7.4.4 Conclusions

The volume determined using the Archimedes method and the volume calculated using
the density of the inox pieces is as good as equal. Therefore it can be concluded that the
volumes determined using the Archimedes method are accurate enough to make a decision
on the error made using the camera. In the following table the errors are calculated for
the different pieces:

<table>
<thead>
<tr>
<th>Piece</th>
<th>Archimedes (cm³)</th>
<th>Camera (algebraic) (cm³)</th>
<th>Error (algebraic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Innoss bullet</td>
<td>10.340</td>
<td>15.7415</td>
<td>\frac{15.7415−10.340}{10.34} = 52.24%</td>
</tr>
<tr>
<td>Innoss ellipsoid</td>
<td>7.998</td>
<td>10.72</td>
<td>\frac{10.72−7.998}{7.998} = 34.03%</td>
</tr>
<tr>
<td>Acorn</td>
<td>4.439</td>
<td>10.617</td>
<td>\frac{10.617−4.439}{4.439} = 139.17%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.7885</td>
<td>\frac{6.7885−4.439}{4.439} = 52.93%</td>
</tr>
<tr>
<td>Kindersurprise-egg</td>
<td>30.586</td>
<td>37.3275</td>
<td>\frac{37.3275−30.586}{30.586} = 22.04%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>33.278</td>
<td>\frac{33.278−30.586}{30.586} = 8.8%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Piece</th>
<th>Archimedes (cm³)</th>
<th>Camera (geometric) (cm³)</th>
<th>Error (geometric)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Innoss bullet</td>
<td>10.340</td>
<td>11.517</td>
<td>\frac{11.517−10.340}{10.34} = 11.4%</td>
</tr>
<tr>
<td>Innoss ellipsoid</td>
<td>7.998</td>
<td>9.076</td>
<td>\frac{9.076−7.998}{7.998} = 13.5%</td>
</tr>
<tr>
<td>Acorn</td>
<td>4.439</td>
<td>6.149</td>
<td>\frac{6.149−4.439}{4.439} = 38.5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.680</td>
<td>\frac{6.680−4.439}{4.439} = 50.5%</td>
</tr>
<tr>
<td>Kindersurprise-egg</td>
<td>30.586</td>
<td>35.955</td>
<td>\frac{35.955−30.586}{30.586} = 17.6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>33.220</td>
<td>\frac{33.220−30.586}{30.586} = 8.6%</td>
</tr>
</tbody>
</table>

It can be clearly seen that the errors are in any case larger than 10% for the more
irregular forms.

When looking at the bubble, the volume will be well calculated as long as the bubble
is small. From the moment that the bubble will grow laterally to one side, its shape will
deviate strongly from the spherical or ellipsoidal shape and the error will be too large to
make any conclusions about the volume.
CHAPTER 7. USING THE CAMERAS

7.5 Conclusion

The error on the volume is negligible when the pieces are perfectly spherical. The error on the height is negligible, leading to the assumption that the error on the diameter and the height of the captured bubbles will be negligible.

When the shape is perfectly ellipsoidal with a ratio of the diameters less than 1.25, the error is smaller than 2.5%. The diameters of the bubble have been compared in different situations. The maximum ratio has been estimated to be about 1.2, excluding the part of the bubble cycle where the bubble is detaching and therefore heavily deformed. The error in that part of the bubble cycle can be taken conservatively 2.5%. During the detachment, the error should be assumed to be larger than 10%.
Chapter 8

Visualisation of a bubble cycle

8.1 Physical interpretation of the bubble formation

In Figure (8.1) the signals are shown during one bubble cycle, synchronized with images of the bubble. Different regions can be defined:

1. Detachment of the previous bubble. This part is characterised by a dramatic drop in both the pressure and the volume.

2. Large oscillations in the pressure signal caused by the force balance between the force due to the pressure of the accumulated air in the tube and the bubble and the pressure of the surrounding liquid.

3. At the moment that enough air has flown into the tube, the built up pressure is high enough to push the liquid out of the tube and the new bubble becomes visible. The oscillations disappear and the pressure rises to reach its maximum value.

4. The pressure stays constant at its maximum value and the volume and bubble height increase linearly.

5. When the volume has reached a certain value, the bubble expands mainly laterally and the bubble height stays constant (figure 8.3).

6. Detachment of the bubble, the cycle is repeated.
Figure 8.1: Entire bubble cycle, showing the different regions and images of the bubble at that point of the cycle.

8.2 Model for the systematic dynamic error in the volume determination

Although in the literature a sinusoidal function is given to simulate the bubble volume fluctuations, it is clearly demonstrated with these experiments that a sigma-function is more appropriate.

8.3 Model for the systematic dynamic error in the hydrostatic head

The depth the bubble peaks out of the dip tube or bubble height also has a specific pattern. The following regions can be seen in the signal:

1. The bubble height is zero as long as the bubble is being formed inside the tube
2. Almost linear increase
3. Slower increase of the bubble height
Figure 8.2: A linear increase followed by a sharp drop in the volume can be seen, corresponding to a sigma function

4. Flattening of the curve at the maximum bubble height: the bubble height stays constant

5. Dramatic drop of the bubble height at the detachment of the bubble

Figure 8.3: Different regions in the bubble height-signal:

<table>
<thead>
<tr>
<th>region</th>
<th>image number range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20-29</td>
</tr>
<tr>
<td>2</td>
<td>29-60</td>
</tr>
<tr>
<td>3</td>
<td>60-138</td>
</tr>
<tr>
<td>4</td>
<td>138-148</td>
</tr>
<tr>
<td>5</td>
<td>148-151</td>
</tr>
</tbody>
</table>
Chapter 9

Results

9.1 Variation of the bubble volume

The maximum volume just before detachment can not be read from the graphs for the reason that the shape of the bubble is so deformed that the error on the calculated volume is too large. Therefore, the volume just before the detachment of the bubble from the tube is derived from the number of images in the cycle. For a certain flow rate, the number of cycles is measured. With this information, the total time for the cycle can be calculated and together with the flow rate, the maximal volume of the bubble can be calculated with the following formula:

\[ \text{Vol}_{\text{max}} = \frac{\text{Flow rate (l/h)} \cdot 10^6 \times \frac{mm^3}{s}}{3600} \times \frac{\text{number of images in 1 cycle}}{\text{images per second}} (s) \] (9.1)

This volume was calculated for the different solutions at different temperatures and insertion depths, and was plotted against the flow rate (l/h). The results for water can be seen in Figures (9.1) to (9.4):

The plots for salt water and sugar water show the same trends and have the same values.

For flow rates below 2l/h, the volume calculated with formula (9.1) increases for decreasing flow rates at constant insertion depth and increases for increasing insertion depth at constant flow rate. When looking at the volume-plots, this trend can not be seen (see Figure (9.5)). The volume just before detachment stays constant for a certain liquid at a certain temperature and for varying insertion depth. The observation is made that the slope of the volume and the cycle length change with the insertion depth: for larger insertion depths, the length of the cycle increases and the slope of the volume signal decreases. This observation can be explained as follows: when the bubble is formed at a deeper point in the liquid, the pressure in the surrounding liquid will be higher and the air will be compressed more. To reach the same volume at detachment, the time between
the formation of the new bubble and its detachment is larger. This larger number of images in the cycle leads to a larger maximum volume according to formula 9.1, but in reality the volume just before detachment stays constant for flow rates below 2 l/h.

For flow rates higher than 2 l/h, the volume increases linearly with flow rate. This trend is also found in the volume plots (see Figure (9.6)). Looking at the formula 9.1, this can be explained as follows: for the higher flow rates, the downward pressure on the bubble surface caused by the air flow pushes the bubble downward and keeps it from leaning totally to one side. This way, the bubble stays attached to the tube for a relatively longer time and the volume before detachment increases. ¹ This conclusion is also compatible with the observations of the increasing bubble height with increasing flow rates.

### 9.2 Variation of the bubble height

The bubble height is the maximum distance the bubble peaks out of the tube, see figure 9.7. The height that is found in the plots in this section is the geometrical averages of the heights seen by the 2 cameras.

---

¹If the flow rate doubles, and the volume stays constant, then the number of images in one cycle must half. If the flow rate doubles and the volume increases, the number of images in one cycle less than halves. This means that there is relatively more time for one cycle for increasing flow rate.
CHAPTER 9. RESULTS

9.2.1 Dependency of bubble height on flow rate for a dip tube at different positions in different liquids

Plots were produced showing the bubble height as a function of flow rate, for different liquids.

Figures 9.8, 9.9 and 9.10 show the bubble height as a function of flow rate for the different liquids at different insertion depths. The different liquids water, salt water and sugar water have densities at the temperature of 29°C that vary from 1 g/cm³ for water to 1.151 g/cm³ for salt water to 1.295 g/cm³ for sugar water, and kinematic viscosities that vary from 0.8393 \(10^{-6}\) m²/s for water to 1.28 \(10^{-6}\) m²/s for salt water and 36.1722 \(10^{-6}\) m²/s for sugar water.\(^2\)

An almost linear increase of the bubble height with increasing flow rate can be observed for the case of the three different liquids. The insertion depth, which was varied over only a small range, from 2 to 12 cm, did not shown an effect on the slope of the linear increase of the bubble height with increasing flow rate.

The following conclusions can be drawn:

**Conclusion:**
The bubble height is smaller for liquids with larger density and viscosity. For acqueous solutions the bubble height decreases with increasing salt or sugar concentration.

\(^2\)ref. http://www.engineeringtoolbox.com

\(^3\)Measurements for the kinematic viscosity of salt water and sugar water performed in Politecnico di Milano, for results see appendix
different from the difference in bubble height between salt water and sugar water. The difference between the water and salt water is mainly due to a significant difference in density. The difference between salt water and sugar water is mainly due to significant changes in both density and viscosity.

Conclusion:

No effect of the change in insertion depth of the dip tube into the water bath could be observed with the small variation range of 10 cm.

9.2.2 Temperature effects on the bubble height increase

The effect of the temperature was investigated on the linear increase of bubble height with increasing flow rate. The same setup was used with the same variation in insertion depth and with the same aqueous solutions. The temperature range varied from 20°C to 38°C and the results are given in the figures 9.11, 9.12 and 9.13 for the different aqueous solutions at a chosen representative insertion depth. No clear trend (increase or decrease) could be observed with increasing temperature.

Conclusion:

No trend in the bubble height change with increasing temperature could be observed.

Despite the dependence of the density, the viscosity and the surface tension of the liquid on the temperature, the temperature variation range of 18°C did not show a clear effect on the bubble height change. For the maximum variation in temperature from 20°C to 38°C the density of water changes from respectively 0.998 $\frac{g}{cm^3}$ to 0.992 $\frac{g}{cm^3}$, the kinematic viscosity changes from $1.0463 \times 10^{-6} \frac{m^2}{s}$ to $0.6971 \times 10^{-6} \frac{m^2}{s}$ and the surface tension
Figure 9.4: Volume variation as a function of flow rate for water at 33.6°C

varies from $72.75 \times 10^{-3} \text{Nm}^{-2}$ to $69.86 \times 10^{-3} \text{Nm}^{-2}$. The liquid property changes are all too small to observe the impact of a 18°C temperature change.

9.3 Pressure histograms

9.3.1 Observations in the histograms

When the histograms of the pressure signals for a certain liquid at a chosen temperature and insertion depth and different flow rates are compared, more peaks are observed at higher flow rates. The origin of these multiple peaks could be a difference in bubble shapes. The higher the flow rate, the more deformed the bubble is before the detachment, and the more differences in the detaching bubble could be expected: detachment volume, cycle length, bubble height... resulting in a variety of pressure signals for one flow rate, giving multiple peaks in the pressure histogram of a number of cycles. Another explanation for the increasing number of peaks in the pressure histogram with increasing flow rates is the presence of the oscillations at the beginning of the signal. At higher flow rates the pressure signal of one cycle exists of only the oscillation part and the region with constant pressure is non-existent. This implies more pressures will be well-represented in the histogram.

That the latter explanation is correct, can be illustrated in the following way:

The pressure signal of different bubble cycles for water at 29 °C, insertion depth 6 cm and flow rate 14 l/h are shown in figure 9.14.
CHAPTER 9. RESULTS

Figure 9.5: Volume signals for water at 29°C, insertion depths 2 cm and 8 cm and flow rates 0.5 l/h and 0.8 l/h. The maximum volume is constant.

A short cycle and a long cycle are observed\(^4\), but the pressure signals are for the largest part the same for the different cycles. The pressure histogram of a long cycle and a short cycle are compared in figure 9.16. The accumulated histograms are plotted in Figure (9.17) No significant differences that could lead to extra peaks can be observed. The following conclusion can be made:

**Conclusion:**

*For high flow rates the pressure signal consists only of oscillations, no constant region is observed.*

Figure 9.14 is an illustration of this fact. The same pattern in the pressure signal is found for all liquids used in the experiment at all temperatures and insertion depths.

**Conclusion:**

*At high flow rates 2 different bubble cycles are observed: a short one and a longer one.*

**Conclusion:**

*The multiple peaks in the pressure histograms stem from the oscillations in the pressure signal and are not consequence of the difference in length of the cycles at high flow rates*

The peaks in the histogram correspond to the local maxima and minima of the oscillations: there are two minima situated between 0.62 kPa and 0.63 kPa and the last small peak is at a value between 0.63 kPa and 0.64 kPa. In the histogram the highest peak

\(^4\) The total difference in length between the two cycles is in the order of 0.001 s while the average cycle length is in the order of 0.1 s.
Figure 9.6: Volume signals for water at 22.5°C, insertion depth 2 cm and 12 cm and flow rates 2 l/h and 4 l/h. An increasing volume with increasing flow rate can be observed.

is situated in the bin with values between 0.62 kPa and 0.63 kPa. The second peak is situated around 0.65 kPa, where the fourth maximum is situated and there is a bump in the first oscillation.
Figure 9.7: Definition of the bubble height $h$

Figure 9.8: Bubble height as a function of flow rate at 29°C and insertion depth 4 cm for different liquids
CHAPTER 9. RESULTS

Figure 9.9: Bubble height as a function of flow rate at 29°C and insertion depth 6 cm for different liquids

Figure 9.10: Bubble height as a function of flow rate at 29°C and insertion depth 8 cm for different liquids
CHAPTER 9. RESULTS

Figure 9.11: Bubble height as a function of flow rate for insertion depth of 2 cm in water at different temperatures

Figure 9.12: Bubble height as a function of flow rate for insertion depth of 2 cm in salt water at different temperatures
Figure 9.13: Bubble height as a function of flow rate for insertion depth of 8 cm in sugar water at different temperatures

Figure 9.14: Pressure signals for 4 cycles for water at 29C, insertion depth 6 cm
Figure 9.15: Pressure histogram for water at 29°C, insertion depth 6 cm and a low flow rate of 1 l/h

Figure 9.16: Histograms for a long and a short cycle
Figure 9.17: Accumulated histograms for the short and the long cycle
Chapter 10

Conclusions

The quality of the experimental data (accuracy on the volume 2.5%) allowed to provide data from which the following conclusions could be drawn:

- One bubble cycle can be recognized in the pressure signal with the subsequent regions:

  1. detachment of the previous bubble characterized by the sharp drop in pressure, volume and bubble height.

  2. oscillations in the pressure signal, the bubble is not visible due to the entrance of the liquid in the tube after the previous detachment and the bubble height and the volume are zero.

  3. oscillations disappear and the pressure rises to its maximum value. The volume and the bubble height increase linearly.

  4. the pressure and the bubble height stay constant at their maximum value while the volume increases linearly. The bubble expands laterally.

- The volume of the bubble stays constant for flow rates below 2 l/h, and grows almost proportionally for increasing flow rates up to 14 l/h. The formula

\[
\text{Vol}_{\text{max}} = \frac{\text{Flow rate (l/h)} \cdot 10^6}{3600} \left( \frac{\text{mm}^3}{s} \right) \times \frac{\text{number of images in 1 cycle}}{\text{images per second}}(s)
\]  

(10.1)

should be used with care, since air is compressible. The effect of this compression is seen at flow rates below 2 l/h since there the volume calculated by the cameras does not equal the volume calculated with the formula above.

- The effect of the different parameters on the bubble height was investigated. The following conclusions were drawn:
1. The bubble height increases almost linearly with increasing flow rate.

2. The bubble height decreases with increasing density and viscosity of the liquid. The higher the salt or sugar concentration in an aqueous solution, the smaller the bubble height.

3. The temperature change during the experiments was not large enough to see any effect on the bubble height, despite the dependency of the density, viscosity and surface tension of the liquid on the temperature.

4. The change in insertion depth was not large enough to see an effect on the bubble height

- The histograms show an increasing number of peaks for increasing flow rate. The origin of these multiple peaks is the increasing importance of the oscillations in the pressure signal. The peaks occur at the maxima and minima of these oscillations. The small difference in the length of the bubble cycle does not have a significant effect on the histogram.
Appendix A

Basic equations of fluid dynamics

A.1 Bernoulli equation in potential form

The Navier-Stokes equation for incompressible flow:

\[
\frac{d\rho}{dt} = \rho \mathbf{g} - \nabla P + \mu \nabla^2 \mathbf{v} \tag{A.1}
\]

is derived from the conservation laws taking the following assumptions:

1. incompressible flow
2. constant viscosity
3. laminar flow

From this equation, the Euler equation is derived, assuming the liquid to be inviscid: \( \mu = 0 \):

Euler Equation:

\[
\frac{d\rho}{dt} = \rho \mathbf{g} - \nabla P \tag{A.2}
\]

Integrating the Euler equation along a streamline gives the Bernoulli equation. To perform this integration, new coordinates are postulated:

- \( s \rightarrow \) coordinate along the streamline
- \( n \rightarrow \) coordinate perpendicular to the streamline

In this coordinate system, the different terms in the equation become:
APPENDIX A. BASIC EQUATIONS OF FLUID DYNAMICS

\[ \frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \frac{\partial \mathbf{v}}{\partial s} + n' \frac{\partial \mathbf{v}}{\partial n} \]

with \( s' = v \) and \( n' = 0 \)

\[ \frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \frac{\partial \mathbf{v}}{\partial s} \]

\[ \nabla P = \frac{\partial P}{\partial s} \mathbf{e}_s + \frac{\partial P}{\partial n} \mathbf{e}_n \]

And now the component of the equation along the streamline becomes (taking dot-product with \( \mathbf{e}_s \)):

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} \cdot \mathbf{e}_s + \mathbf{v} \cdot \frac{\partial \mathbf{v}}{\partial s} \cdot \mathbf{e}_s \right) = \rho \mathbf{g} \cdot \mathbf{e}_s \quad \frac{\partial P}{\partial s} \mathbf{e}_s \cdot \mathbf{e}_s - \frac{\partial P}{\partial n} \mathbf{e}_n \cdot \mathbf{e}_s \quad \text{(A.3)} \]

\[ = \rho \mathbf{g} \cdot \mathbf{e}_s - \frac{\partial P}{\partial s} \quad \text{(A.4)} \]

\[ \quad \Leftrightarrow \left( \frac{\partial \mathbf{v}}{\partial t} \cdot \mathbf{e}_s + \mathbf{v} \cdot \frac{\partial \mathbf{v}}{\partial s} \right) ds = \rho \mathbf{g} \cdot \mathbf{e}_s ds - \frac{\partial P}{\partial s} ds \quad \text{(A.5)} \]

\[ \quad \Leftrightarrow \left( \frac{\partial \mathbf{v}}{\partial t} \cdot \mathbf{e}_s + \frac{1}{2} \frac{\partial v^2}{\partial s} \right) ds = -\rho g dz - \frac{\partial P}{\partial s} ds \quad \text{(A.6)} \]

For a steady flow, \( \frac{\partial \mathbf{v}}{\partial t} = 0 \), so integrating the above equation for \( s \): \( 0 \to \infty \) means:

\( v \to 0 \), \( p \to p_\infty \), \( z \to z_\infty \):

Bernoulli equation:

\[ (p - p_\infty) + \rho g (z - z_\infty) + \frac{v^2}{2} = 0 \quad \text{(A.7)} \]

Or in standard form:

\[ p + \rho g z + \frac{v^2}{2} = \text{cst} \quad \text{(A.8)} \]

The following assumptions were made to obtain this equation:

- permanent flow
- incompressible fluid
- inviscous fluid
- equation can only be applied along a streamline
APPENDIX A.  BASIC EQUATIONS OF FLUID DYNAMICS

For an irrotational flow, $\nabla \times \mathbf{v} = 0$ is true, so the velocity vector can be replaced by a potential function, defined as:

$$\mathbf{v} = -\nabla \phi$$  \hspace{1cm} (A.9)

With the minus sign so that the potential decreases when the velocity is positive.

When this potential is substituted in the Bernoulli equation, taking back into account the term for unsteady flow, the equation becomes:

$$\frac{p - p_\infty}{\rho} + g.(z - z_\infty) - \frac{\partial}{\partial t} \phi + \frac{1}{2} (\nabla \phi)^2 = 0$$  \hspace{1cm} (A.10)

A.2 Derivation of the dimensionless Bernoulli-equation

Making the equation dimensionless allows to solve the equation independently of initial conditions and geometry.

The dimensionless parameters are:

- $p^* \frac{\sigma}{d} = p - p_\infty$
- $v^*_N.\mathbf{u} = v_N$
- $\frac{v^* d}{2a} = t$
- $z = z^* \frac{d}{2} + z_\infty - z^*_\infty = z_\infty + z^* \frac{d}{2} - \frac{\sigma}{\Delta \rho.g.S_B}$
- $S_B = d.S'_B = \frac{z.D^2_{u}(\rho_L - \rho_C)}{\sigma}$
- $\nabla = \frac{\nabla^*}{d/2}$
- $\phi = \phi^*.\mathbf{u}.\frac{d}{2}$

With all these definitions, the equation can be rewritten:

$$\frac{p - p_\infty}{\rho} + g.(z - z_\infty) - \frac{\partial}{\partial t^*} \phi + \frac{1}{2} (\nabla^* \phi)^2 = 0$$  \hspace{1cm} (A.11)

$$p^* \frac{\sigma}{\rho d} + g.\left(z^* \frac{d}{2} - \frac{\sigma}{\Delta \rho.g.S_B}\right) - 2u^* \frac{\partial}{\partial t^*} \left(\frac{\nabla^*}{d/2} \left(\phi^*.\mathbf{u}.\frac{d}{2}\right)\right) + \frac{1}{2} \left(\frac{\nabla^*}{d/2} \left(\phi^*.\mathbf{u}.\frac{d}{2}\right)\right)^2 = 0$$  \hspace{1cm} (A.12)

$$p^* \frac{\sigma}{\rho d} + z^* \frac{g.d}{2} - \frac{\sigma}{\Delta \rho.S_B} - u^2 \frac{\partial}{\partial t^*} (\nabla^* \phi^*) + \frac{1}{2} u^2 (\nabla^* \phi^*)^2$$  \hspace{1cm} (A.13)

$$p^* + z^* \frac{g.d \rho \sigma}{2} - \frac{\sigma}{\Delta \rho.S_B} \rho \sigma \rho \sigma + \rho \sigma u^2 \left[ - \frac{\partial}{\partial t^*} (\nabla^* \phi^*) + \frac{1}{2} (\nabla^* \phi^*)^2 \right]$$  \hspace{1cm} (A.14)
\[ p^* + z^*. E\ddot{\phi} + We \left[ -\frac{\partial}{\partial t^*} (\nabla^* \phi^*) + \frac{1}{2} (\nabla^* \phi^*)^2 \right] = \frac{1}{S_B^*} \]  

(A.14)

With dimensionless numbers:

\[ E\ddot{\phi} = \frac{g\rho d^2}{2\sigma} \]  

(A.15)

\[ We = \frac{\rho du^2}{\sigma} \]

and

\[ S_B^* = \frac{S_B}{d} \]  

(A.16)

The \( S_B \) that occurs in the equation is the Sauter diameter. It is a statistical diameter calculated as follows: if there is a volume \( V_G \) of air and a volume \( V_L \) of water, then the total volume in the tank is \( (V_G + V_L) \). The void is calculated as \( \varepsilon = \frac{V_G}{V_G + V_L} \). The volumes of all the bubbles are summed up. The Sauter diameter is now the diameter the bubbles should have, if they were all equal in volume and \( \varepsilon \) should still have the same value.

If the viscosity is to be taken into account, an extra term is added coming from the Boussinesq equation:

\[ \text{Extra term for viscosity} = 2.Ca.\frac{\partial^2 \phi^*}{\partial n^*^2} \]  

(A.17)

\( Ca = \frac{\mu u}{\sigma} \) is the capillarity number and the second derivative is proportional to the change of velocity perpendicular to the streamline.

### A.3 The Rayleigh-Plesset equation

To derive the Rayleigh-Plesset equation, the Bernoulli equation in potential form is used, applied to the bubble-system. \( u \) is the velocity of the bubble radius, \( R_B \) is the bubble radius and \( r \) is the distance from the centre of the bubble.

\[ \pi(R_B, r, t) = \frac{R_B^2}{r^2} \frac{dR_B}{dt} \]  

(A.18)

\[ \phi = -\frac{R_B^2}{r} \frac{dR_B}{dt} = \frac{11}{3} \frac{dR_B^3}{dt} \]  

(A.19)

It is clear that the potential contains the change of the volume over time. This velocity potential will be substituted into the dimensionless Bernoulli equation in the following form:
\[-\frac{\partial \Phi}{\partial t} + \frac{1}{2} (\nabla \Phi)^2 + \frac{p}{\rho} + \bar{g} \cdot \bar{x} = c_k \] (A.20)

The potential is substituted in the case of \( r = R_B \):

\[
\frac{3}{2} \left( \frac{\partial R_B}{\partial t} \right)^2 + R_B \frac{\partial^2 R_B}{\partial t^2} + \frac{p_{RB}}{\rho} + \bar{g} \cdot \bar{x} = c_k
\] (A.21)

and in the case of \( r = \infty \):

\[
\frac{p_{\infty}}{\rho} + \bar{g} \cdot \bar{x} = c_k
\] (A.22)

The combination of equations A.21 and A.22 yields the Rayleigh-Plesset equation for the radius as a function of time of a bubble in a surrounding liquid:

\[
\frac{3}{2} \left( \frac{\partial R_B}{\partial t} \right)^2 + R_B \frac{\partial^2 R_B}{\partial t^2} = \frac{p_{\infty} - p_{RB}}{\rho}
\]
Appendix B

Viscosity measurements

The viscosity of the salt water and the sugar solution were measured with an Ubbelohde-Viscosimeter ISO 3105, DIN 51 562, Part 1, BS 188, NFT 60-100 Ref. No 530 was used. For salt water, a capillary number 0c was used, for sugar water Ic. The respective constants K are 0.003 and 0.03 and the kinematic viscosities are calculated with formula B.1:

\[ \nu = K (t - \theta) \]  

(B.1)

In which \( \theta \) is the Hagenbach correction on the time needed for the liquid to pass between the two marks on the capillar. The results of the measurements are shown in plots B.1 and B.2.
Figure B.1: Kinematic viscosity of salt solution with density 1.15 g/cm³ as a function of temperature
Figure B.2: Kinematic viscosity of sugar solution with density 1.29 g/cm³ as a function of temperature

\[ y = -2.1962x + 99.92 \]

\[ R^2 = 0.9914 \]
Appendix C

COMSOL report straight tube model
1. Table of Contents

- Title - COMSOL Model Report
- Table of Contents
- Model Properties
- Geometry
- Geom1
- Materials/Coefficients Library
- Solver Settings
- Postprocessing
- Variables

2. Model Properties

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Application modes and modules used in this model:
3. Geometry

Number of geometries: 1

3.1. Geom1

3.1.1. Point mode
3.1.2. Boundary mode

3.1.3. Subdomain mode
4. Geom1

Space dimensions: Axial symmetry (2D)
Independent variables: r, phi, z

4.1. Mesh

4.1.1. Mesh Parameters

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Subdomain: 2

Maximum element size: 0
Element growth rate: 1, 3-4
APPENDIX C. COMSOL REPORT STRAIGHT TUBE MODEL

Boundary
- Maximum element size
- Element growth rate
- Mesh curvature factor
- Mesh curvature cut off

Point
- Maximum element size
- Element growth rate

4.1.2. Mesh Statistics
- Number of degrees of freedom: 146193
- Number of boundary elements: 611
- Number of elements: 32260
- Minimum element quality: 0.5733

4.2. Application Mode: Incompressible Navier-Stokes (ns)
Application mode type: Incompressible Navier-Stokes
Application mode name: ns

4.2.1. Application Mode Properties

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Weak constraints
Reference frame
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4.2.2. Variables
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4.2.3. Boundary Settings

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4.2.4. Subdomain Settings

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| Integration order | 4 4 2 | 4 4 2 |
| (gorder) | Constraint order | 2 2 1 | 2 2 1 |
| (corder) | Density (rho) | kg/m^3 | 1000 | 1.3 |
| Dynamic viscosity | Pa·s | 10^(-5) | 10^(-6) |

5. Materials/Coefficients Library

5.1. Water, liquid

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6
Dynamic viscosity (eta) \( \eta(T) \)
Thermal conductivity (k) \( k(T) \)
Kinematic viscosity (nu0) \( \nu_0(T) \)
Density (rho) \( \rho(T) \)

6. Solver Settings

Solve using a script: off
Analysis type Transient
Auto select solver On
Solver Stationary linear
Solution form Automatic
Symmetric Off
Adaption Off

6.1. Direct (UMFPACK)

Solver type: Linear system solver

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pivot threshold</td>
<td>0.1</td>
</tr>
<tr>
<td>Memory allocation factor</td>
<td>0.7</td>
</tr>
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</table>

6.2. Advanced

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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint handling method</td>
<td>Elimination</td>
</tr>
<tr>
<td>Null-space function</td>
<td>Automatic</td>
</tr>
<tr>
<td>Assembly block size</td>
<td>5000</td>
</tr>
<tr>
<td>Use Hermitian transpose of constraint matrix</td>
<td>Off</td>
</tr>
<tr>
<td>Use complex functions with real input</td>
<td>Off</td>
</tr>
<tr>
<td>Type of scaling</td>
<td>Automatic</td>
</tr>
<tr>
<td>Manual scaling</td>
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</tr>
<tr>
<td>Row equilibration</td>
<td>On</td>
</tr>
<tr>
<td>Manual control of reassembly</td>
<td>Off</td>
</tr>
<tr>
<td>Load constant</td>
<td>On</td>
</tr>
<tr>
<td>Constraint constant</td>
<td>On</td>
</tr>
<tr>
<td>Mass constant</td>
<td>On</td>
</tr>
<tr>
<td>Damping (mass) constant</td>
<td>On</td>
</tr>
<tr>
<td>Jacobian constant</td>
<td>On</td>
</tr>
<tr>
<td>Constraint Jacobian constant</td>
<td>On</td>
</tr>
</tbody>
</table>
Appendix D

COMSOL report tilted tube model
APPENDIX D. COMSOL REPORT TILTED TUBE MODEL

COMSOL Model Report

1. Table of Contents
- Title - COMSOL Model Report
- Table of Contents
- Model Properties
- Geometry
- Geom1
- Solver Settings
- Postprocessing
- Variables

2. Model Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<td>Company</td>
<td></td>
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<tr>
<td>Department</td>
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Application modes and modules used in this model:
- Geom1 (2D)
- Incompressible Navier-Stokes

3. Geometry

Number of geometries: 1

3.1. Geom1

3.1.1. Point mode
3.1.2. Boundary mode

3.1.3. Subdomain mode
4. Geom1

Space dimensions: 2D
Independent variables: x, y, z

4.1. Mesh

4.1.1. Mesh Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum element size</td>
<td></td>
</tr>
<tr>
<td>Maximum element size scaling factor</td>
<td>1</td>
</tr>
<tr>
<td>Element growth rate</td>
<td>1.3</td>
</tr>
<tr>
<td>Mesh curvature factor</td>
<td>0.3</td>
</tr>
<tr>
<td>Mesh curvature cut off</td>
<td>0.001</td>
</tr>
<tr>
<td>Resolution of narrow regions</td>
<td>1</td>
</tr>
<tr>
<td>Resolution of geometry</td>
<td>10</td>
</tr>
<tr>
<td>x-direction scale factor</td>
<td>1.0</td>
</tr>
<tr>
<td>y-direction scale factor</td>
<td>1.0</td>
</tr>
<tr>
<td>Mesh geometry to level</td>
<td>Subdomain</td>
</tr>
</tbody>
</table>

Subdomain

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum element size</td>
<td></td>
</tr>
<tr>
<td>Element growth rate</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX D. COMSOL REPORT TILTED TUBE MODEL

4.1.2. Mesh Statistics

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of degrees of freedom</td>
<td>59043</td>
</tr>
<tr>
<td>Number of boundary elements</td>
<td>108</td>
</tr>
<tr>
<td>Number of elements</td>
<td>13060</td>
</tr>
<tr>
<td>Minimum element quality</td>
<td>0.6098</td>
</tr>
</tbody>
</table>

4.2. Application Mode: Incompressible Navier-Stokes (ns)

Application mode type: Incompressible Navier-Stokes
Application mode name: ns

4.2.1. Application Mode Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default element type</td>
<td>Lagrange - P_2 P_1</td>
</tr>
<tr>
<td>Analysis type</td>
<td>Stationary</td>
</tr>
<tr>
<td>Stress tensor</td>
<td>Total</td>
</tr>
</tbody>
</table>

5
Frame Reference frame
Weak constraints Off

4.2.2. Variables
Dependent variables: u, v, p
Shape functions: shlag(2,'u'), shlag(2,'v'), shlag(1,'p')
Interior boundaries not active

4.2.3. Point Settings
Point  1-8
style Pa {0,{0,0,255}}

4.2.4. Boundary Settings
Boundary 1-2, 4, 6, 8 3, 7
Type No slip Outflow/Pressure
y-velocity (v0) m/s 0 0
Pressure (p0) Pa 0 100000

4.2.5. Subdomain Settings
Subdomain 1
Integration order 4 4 2
(gporder)
Constraint order 2 2 1
(cporder)
Dynamic viscosity (eta) Pa·s 10^(-5)

5. Solver Settings
Solve using a script: off
Analysis type Stationary
Auto select solver On
Solver Stationary linear
Solution form Automatic
Symmetric Off
Adaption Off
5.1. Direct (UMFPACK)

Solver type: Linear system solver

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pivot threshold</td>
<td>0.1</td>
</tr>
<tr>
<td>Memory allocation factor</td>
<td>0.7</td>
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</table>

5.2. Advanced

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint handling method</td>
<td>Elimination</td>
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<tr>
<td>Null-space function</td>
<td>Automatic</td>
</tr>
<tr>
<td>Assembly block size</td>
<td>5000</td>
</tr>
<tr>
<td>Use Hermitian transpose of constraint matrix</td>
<td>Off</td>
</tr>
<tr>
<td>Use complex functions with real input</td>
<td>Off</td>
</tr>
<tr>
<td>Type of scaling</td>
<td>Automatic</td>
</tr>
<tr>
<td>Manual scaling</td>
<td></td>
</tr>
<tr>
<td>Row equilibration</td>
<td>On</td>
</tr>
<tr>
<td>Manual control of reassembly</td>
<td>Off</td>
</tr>
<tr>
<td>Load constant</td>
<td>On</td>
</tr>
<tr>
<td>Constraint constant</td>
<td>On</td>
</tr>
<tr>
<td>Mass constant</td>
<td>On</td>
</tr>
<tr>
<td>Damping (mass) constant</td>
<td>On</td>
</tr>
<tr>
<td>Jacobian constant</td>
<td>On</td>
</tr>
<tr>
<td>Constraint Jacobian constant</td>
<td>On</td>
</tr>
</tbody>
</table>

6. Postprocessing
7. Variables

7.1. Boundary

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_x_ns</td>
<td>Viscous force per area, x component</td>
<td>2 * nx_ns * eta_ns * ux+ny_ns * eta_ns * (uy+vx)</td>
</tr>
<tr>
<td>T_x_ns</td>
<td>Total force per area, x component</td>
<td>nx_ns * eta_ns * (uy+vx)-nx_ns * p+2 * nx_ns * eta_ns * ux</td>
</tr>
<tr>
<td>K_y_ns</td>
<td>Viscous force per area, y component</td>
<td>ny_ns * eta_ns * (vx+uy)+2 * ny_ns * eta_ns * vy</td>
</tr>
<tr>
<td>T_y_ns</td>
<td>Total force per area, y component</td>
<td>2 * ny_ns * eta_ns * vy-ny_ns * p+nx_ns * eta_ns * (vx+uy)</td>
</tr>
</tbody>
</table>

7.2. Subdomain

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>U_ns</td>
<td>Velocity field</td>
<td>sqrt(u^2+v^2)</td>
</tr>
<tr>
<td>V_ns</td>
<td>Vorticity</td>
<td>vx-uy</td>
</tr>
<tr>
<td>cellRe_ns</td>
<td>Cell Reynolds number</td>
<td>rho_ns * U_ns * h/eta_ns</td>
</tr>
<tr>
<td>res_u_ns</td>
<td>Equation residual for u</td>
<td>rho_ns * (u * ux+v * uy)+px-F_x_ns-eta_ns * (uxx+uxx+uyy+vxy)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Expression</td>
</tr>
<tr>
<td>------------</td>
<td>--------------------------------------------------</td>
<td>----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>res_tst_u_ns</td>
<td>Variational equation residual for u</td>
<td>nojac(rho_ns) * (nojac(u) * ux+nojac(v) * uy)+px-nojac(eta_ns) * (uxx+uxy+uyy+vyy)</td>
</tr>
<tr>
<td>res_sc_u_ns</td>
<td>Shock capturing residual for u</td>
<td>rho_ns * (u * ux+v * uy)+px-F_x ns</td>
</tr>
<tr>
<td>res_v_ns</td>
<td>Equation residual for v</td>
<td>rho_ns * (u * vx+v * vy)+py-F_y ns-eta_ns * (vx+uyx+vy+y)</td>
</tr>
<tr>
<td>res_tst_v_ns</td>
<td>Variational equation residual for v</td>
<td>nojac(rho_ns) * (nojac(u) * vx+nojac(v) * vy)+py-nojac(eta_ns) * (vx+uyx+vy+y)</td>
</tr>
<tr>
<td>res_sc_v_ns</td>
<td>Shock capturing residual for v</td>
<td>rho_ns * (u * vx+v * vy)+py-F_y ns</td>
</tr>
<tr>
<td>beta_x_ns</td>
<td>Convective field, x component</td>
<td>rho_ns * u</td>
</tr>
<tr>
<td>beta_y_ns</td>
<td>Convective field, y component</td>
<td>rho_ns * v</td>
</tr>
<tr>
<td>Dm_ns</td>
<td>Mean diffusion coefficient</td>
<td>eta_ns</td>
</tr>
<tr>
<td>da_ns</td>
<td>Total time scale factor</td>
<td>rho_ns</td>
</tr>
</tbody>
</table>
Appendix E

COMSOL report moving mesh model
1. Table of Contents

- Title - Sloshing Tank
- Table of Contents
- Model Properties
- Constants
- Geometry
- Geom 1
- Solver Settings
- Postprocessing
- Variables

2. Model Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
<tr>
<td>Model name</td>
<td>Sloshing Tank</td>
</tr>
<tr>
<td>Author</td>
<td>COMSOL</td>
</tr>
<tr>
<td>Company</td>
<td>COMSOL</td>
</tr>
<tr>
<td>Department</td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>Copyright (c) 1994-2005 by COMSOL AB</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://www.comsol.com">www.comsol.com</a></td>
</tr>
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File name: C:\Documents and Settings\svdekkend\Desktop\Femlab laatste versies\18 mei.mph

Application modes and modules used in this model:
- Geom1 (2D)
  - Moving Mesh (ALE)
  - Incompressible Navier-Stokes

2.1. Model description

Sloshing Tank
This model of sloshing in a rocking tank demonstrates COMSOL Multiphysics’ ability to simulate dynamic free surface flow with the help of a moving mesh.

3. Constants

<table>
<thead>
<tr>
<th>Name</th>
<th>Expression</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>rho</td>
<td>1.25</td>
<td>1.25</td>
<td>Glycerol density (kg/m^3)</td>
</tr>
<tr>
<td>nu</td>
<td>1.49</td>
<td>1.49</td>
<td>Glycerol viscosity (Pa·s)</td>
</tr>
<tr>
<td>phi_m</td>
<td>0</td>
<td>0</td>
<td>Maximum angle of inclination (rad)</td>
</tr>
<tr>
<td>ax</td>
<td>1</td>
<td>1</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>freq</td>
<td>9.81</td>
<td>9.81</td>
<td>Acceleration due to gravity (m/s^2)</td>
</tr>
</tbody>
</table>

4. Geometry

Number of geometries: 1

4.1. Geom1
4.1.1. Point mode

4.1.2. Boundary mode
4.1.3. Subdomain mode

5. Geom1

Space dimensions: 2D
Independent variables: X, Y, Z

5.1. Scalar Expressions
APPENDIX E. COMSOL REPORT MOVING MESH MODEL

5.2. Mesh

5.2.1. Mesh Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum element size</td>
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</tr>
<tr>
<td>Maximum element size scaling factor</td>
<td>1</td>
</tr>
<tr>
<td>Element growth rate</td>
<td>1.3</td>
</tr>
<tr>
<td>Mesh curvature factor</td>
<td>0.3</td>
</tr>
<tr>
<td>Mesh curvature cut off</td>
<td>0.001</td>
</tr>
<tr>
<td>Resolution of narrow regions</td>
<td>1</td>
</tr>
<tr>
<td>Resolution of geometry</td>
<td>10</td>
</tr>
<tr>
<td>x-direction scale factor</td>
<td>1.0</td>
</tr>
<tr>
<td>y-direction scale factor</td>
<td>1.0</td>
</tr>
<tr>
<td>Mesh geometry to level</td>
<td>Subdomain</td>
</tr>
</tbody>
</table>

5.2.2. Mesh Statistics

- Number of degrees of freedom: 19333
- Number of boundary elements: 118
- Number of elements: 2260
- Minimum element quality: 0.6314
5.3. Application Mode: Moving Mesh (ALE) (ale)

Application mode type: Moving Mesh (ALE)
Application mode name: ale

5.3.1. Application Mode Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Default element type</td>
<td>Lagrange - Quadratic</td>
</tr>
<tr>
<td>Smoothing method</td>
<td>Winslow</td>
</tr>
<tr>
<td>Analysis type</td>
<td>Transient</td>
</tr>
<tr>
<td>Defines frame</td>
<td>Frame (ale)</td>
</tr>
<tr>
<td>Motion relative to</td>
<td>Reference frame</td>
</tr>
<tr>
<td>Weak constraints</td>
<td>Non-ideal</td>
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</table>

5.3.2. Variables

Dependent variables: dx, dy
Shape functions: shlag(2,'lm3'), shlag(2,'lm4'), shlag(2,'dx'), shlag(2,'dy')
Interior boundaries not active

5.3.3. Boundary Settings

<table>
<thead>
<tr>
<th>Boundary Type</th>
<th>Mesh displacement</th>
<th>Mesh velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>constrcoord</td>
<td>global</td>
<td>local</td>
</tr>
<tr>
<td>Mesh velocity</td>
<td>m/s</td>
<td>{'u'nx+'v'ny',0}</td>
</tr>
<tr>
<td></td>
<td>1, 9</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Mesh</td>
<td>Mesh velocity</td>
</tr>
</tbody>
</table>
APPENDIX E. COMSOL REPORT MOVING MESH MODEL

(veldeform)
defflag
veldeflag
weakstr
Boundary 3, 8
Type Mesh
constrcoord global
Mesh velocity (0;0)
(veldeform)
defflag {1;0}
veldeflag {0;0}
weakstr 0

5.3.4. Subdomain Settings
Subdomain 1 2
Shape functions
(shape)
shlag(2,'lm3') shlag(2,'lm3')
shlag(2,'lm4') shlag(2,'lm4')
shlag(2,'dx') shlag(2,'dy') shlag(2,'dy')
type free none
Subdomain initial value 1 2
x-displacement m X X
(dx)
y-displacement m Y Y
(dy)

5.4. Application Mode: Incompressible Navier-Stokes (ns)
Application mode type: Incompressible Navier-Stokes
Application mode name: ns

5.4.1. Application Mode Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default element type</td>
<td>Lagrange - P_2 P_1</td>
</tr>
<tr>
<td>Analysis type</td>
<td>Transient</td>
</tr>
<tr>
<td>Stress tensor</td>
<td>Total</td>
</tr>
<tr>
<td>Frame</td>
<td>Frame (ale)</td>
</tr>
<tr>
<td>Weak constraints</td>
<td>Off</td>
</tr>
</tbody>
</table>
5.4.2. Variables
Dependent variables: u, v, p
Shape functions: shlag(2,'u'), shlag(2,'v'), shlag(1,'p')
Interior boundaries active

5.4.3. Boundary Settings

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Type</th>
<th>y-velocity (v0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, 4-5, 7</td>
<td>Neutral</td>
<td>0 m/s</td>
</tr>
<tr>
<td>1, 9</td>
<td>Slip/Symmetry</td>
<td>0</td>
</tr>
<tr>
<td>3, 8</td>
<td>Inflow/Outflow velocity</td>
<td>0.5<em>sin(2</em>pi<em>freq</em>t)</td>
</tr>
<tr>
<td>6</td>
<td>No slip</td>
<td>0</td>
</tr>
</tbody>
</table>

5.4.4. Subdomain Settings

<table>
<thead>
<tr>
<th>Subdomain</th>
<th>Integration order (gorder)</th>
<th>Constraint order (cporder)</th>
<th>Density (rho)</th>
<th>Dynamic viscosity (eta)</th>
<th>Volume force, x-dir. (F_x)</th>
<th>Volume force, y-dir. (F_y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 4 2</td>
<td>2 2 1</td>
<td>rho</td>
<td>nu</td>
<td>grav_x'*rho</td>
<td>grav_y'*rho</td>
</tr>
<tr>
<td>2</td>
<td>4 4 2</td>
<td>2 2 1</td>
<td>1.29</td>
<td>nu</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Solver Settings

Solve using a script: off
Analysis type: Transient
Auto select solver: On
Solver: Time dependent
Solution form: Automatic
Symmetric: Off
Adaption: Off

6.1. Direct (UMFPACK)
Solver type: Linear system solver

Parameter      Value
Pivot threshold    0.1
Memory allocation factor  0.7

6.2. Time Stepping

Parameter      Value
Times            0:0.1:2
Relative tolerance    0.001
Absolute tolerance        0.0010
Times to store in output  Specified times
Time steps taken by solver   Free
Manual tuning of step size    Off
Initial time step            0.0010
Maximum time step             1.0
Maximum BDF order              5
Singular mass matrix      Maybe
Consistent initialization of DAE systems  Backward Euler
Error estimation strategy     Exclude algebraic
Allow complex numbers       Off

6.3. Advanced

Parameter      Value
Constraint handling method    Elimination
Null-space function      Automatic
Assembly block size       5000
Use Hermitian transpose of constraint matrix    Off
Use complex functions with real input    Off
Type of scaling         Automatic
Manual scaling          
Row equilibration      On
Manual control of reassembly    Off
Load constant          On
Constraint constant   On
Mass constant          On
Damping (mass) constant  On
Jacobian constant       On
Constraint Jacobian constant  On
Bibliography


[12] Ubbelohde-Viscosimeter manual
Onderzoek van bellengedrag voor verbeterde niveau- en dichtheidsmetingen met een pneumercator

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Abstract—In dit artikel worden de resultaten naar het onderzoek van het bellengedrag van een pneumercator toegelicht. De nadruk wordt gelegd op het volume en de hoogte van de bellen net voor het loslaten van de tube.

Keywords—Bellengedrag, dichtheidsmeting, niveaumeting, Nuclear Safeguards

I. Introduction

De pneumercator techniek wordt gebruikt om de dichtheid en het niveau van de vloeistof in inventaris tanks in heropwerkingsfabrieken te meten. Volgens het niet-proliferatie verdrag (NPT) moet dit met een zekere nauwkeurigheid gebeuren. Het pneumercator systeem dat hiervoor gebruikt wordt is robuust en gemakkelijk in gebruik en onderhoud. De metingen gebeuren met een goede nauwkeurigheid wanneer de systematische fouten en de staat van de tubes in rekening gebracht worden. Een afzetting van de zouten uit de vloeistof in de tank veroorzaakt een verstopping van de tube en dit leidt tot afwijkingen in het gemeten druksignal en een verhoging van de bellenvrequentie.

In dit werk worden de vorming van de bellen aan de tube en hun volume experimenteel bestudeerd. De impact van externe parameters zoals de temperatuur van de vloeistof, de positie van de tubes in de vloeistof, de snelheid van de luchtstroom en de eigenschappen van de vloeistof wordt onderzocht met een sensitiviteitsanalyse.

II. Inleiding tot de pneumercator techniek

De pneumercator techniek maakt gebruik van de druk gemeten in drie tubes. Twee van deze tubes, waarvan het lengteverschil S nauwkeurig gekend is, worden ondergedompeld in de vloeistof en de derde tube raakt het vloeistofoppervlak nooit. Door een luchtstroom door de tubes te sturen worden bellen geblazen in de vloeistof. Door een gemiddelde over de tijd (10 s) te nemen van het dynamische druksignal in elk van de tubes en deze waarden te vullen in formule (1) kunnen de dichtheid en het niveau van de vloeistof bepaald worden.

$$p = \rho gh$$  

III. Experiment

Het experiment maakt gebruik van een reservoir voor de vloeistof, vloeistoffen met verschillende dichtheids- en viscositeits-eigenschappen, 2 camera’s, een pneumercator set, een drukmeter en een apparaat om de luchtstroom te regelen. De camera’s worden gebruikt om de groeiende bellen te filmen aan een snelheid van 100 beelden per seconde. De druk wordt meten met een snelheid van 10 kHz en wordt gesynchroniseerd met de beelden gebruik makend van een klok. Voor het experiment wordt de tip van de langste tube 2 tot 12 cm onder het vloeistofoppervlak geplaatst bij een temperatuur variërend van 20 tot 38 °C. De dichtheid van de vloeistof varieert van 0.5 l/h tot 14 l/h. De eigenschappen van de vloeistoffen zijn samengevat in tabel (2):

<table>
<thead>
<tr>
<th>Vloeistof</th>
<th>Dichtheid (g cm⁻³)</th>
<th>Viscositeit (10⁻⁶ m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>0.998 - 0.992</td>
<td>1.0463 - 0.6971</td>
</tr>
<tr>
<td>salt water</td>
<td>1.15</td>
<td>1.5919-0.9763</td>
</tr>
<tr>
<td>sugar water</td>
<td>1.29</td>
<td>55.9560-16.3884</td>
</tr>
</tbody>
</table>

IV. Resultaten

A. Visualisatie van één cyclus

De signalen gemeten tijdens de vorming en het loslaten van één bel aan de tube worden weergegeven in figuur (2). Er kunnen duidelijk verschillende regio’s onderscheiden worden:

1. Loslaten van de vorige bel, gepaard gaande met een scherpe daling van volume en bellenhoogte
2. Grote oscillaties in het druksignal bij het begin van de groei van de nieuwe bel
3. Uitsterven van de oscillaties, bel wordt zichtbaar en volume en bellenhoogte beginnen toe te nemen
4. Stijgen van de druk tot de constante maximale waarde waarbij volume en bellenhoogte beginnen toe te nemen
5. Laterale toename van het volume, stagnatie van de toename van de bellenhoogte
6. Bel laat los van de tube, herhaling van regio 1

B. Model voor systematische dynamische fouten

Het volume signal heeft duidelijk de vorm van een zaagtandfunctie en niet van een sinusoidale functie, zoals in de literatuur beschreven wordt. Het bellengedrag wordt ook eerder benaderd door een zaagtandfunctie.

C. Variatie van het volume

Aangezien de berekening van het volume met de camera’s niet voldoende nauwkeurig is in de regio waar de bel de tube loslaat omwille van de onregelmatige vorm van de bel, wordt het volume bepaald met formule
E. Observaties in de druk histogrammen

Wanneer de histogrammen voor een aantal bellencyclen bekeken worden, dan neemt men met stijgende luchtsnelheid een stijging van het aantal pieken waar. Deze toename duidt niet op een toename van het aantal types bellen, maar is een gevolg van de beperking van het drusignaal tot oscillaties. Door het ontbreken van een regio met constante druk in het drusignaal voor hoge luchtsnelheden verschijnen er pieken in het histogram in de lokale maxima en minima van de oscillaties. De drusignaal voor de bellencycle bij hoge luchtsnelheden zijn op een klein verschil in de cycluslengte na identiek voor alle bellen.

V. Conclusies

In de signalen voor één cyclus kunnen verschillende regio’s onderscheiden worden. Het volume signaal en de bellenhoogte kunnen gemodelleerd worden met een zaagtandfunctie. Het belvolume is constant voor lage luchtsnelheden en stijgt lineair voor snelheden vanaf 2 l/h. De hoogte van de bel stijgt met dalende dichtheid en viscositeit van de vloeistof en met stijgende luchtsnelheid. De kleine variaties van positie en temperatuur vertonen geen effect op de bellenhoogte. Door de samendrukbaarheid van de lucht in de bel kan formule (3) niet gebruikt worden om het maximale volume te berekenen bij lage luchtsnelheden. De stijging van het aantal pieken met stijgende luchtsnelheid in de drukhistogrammen is afkomstig van de oscillaties in het drusignaal en duidt niet op een stijging van het aantal soorten bellen. Er worden een korte en een lange cyclus waargenomen maar de histogrammen van deze twee cycli zijn niet te onderscheiden.

REFERENCES
