PUBLIC CAPITAL, ECONOMIC GROWTH
AND THE LABOUR MARKET

Dissertation

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By

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Public Capital, Economic Growth
and the Labour Market

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Gerdie Everaert
Gent, November 20, 2000
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ABSTRACT: This chapter provides a general motivation for analysing the impact of public capital on macroeconomic performance (section 1). It briefly outlines the hypothesised impact of public capital on productivity and economic growth, labour market performance and the result of fiscal consolidation programs (section 2) as well as the corresponding research objectives related to these hypotheses (section 3). Further, it presents the general outline of this dissertation together with a summary of its main results (section 4). An analysis of the policy implications of the obtained results (section 5) and a non-exhaustive outline of directions for future research (section 6) close this chapter.

KEYWORDS: Public capital, economic growth, labour market performance, fiscal consolidation.

1. ORIENTATION

Over the last two decades macroeconomic policy in Europe has been inspired by the belief that price stability and sound public finances are the cornerstones of a macroeconomic environment supporting sustainable economic growth and low unemployment. In the 1990s, this belief was formalised by the signing of the Maastricht Treaty and the Stability and Growth Pact, making low inflation, balanced government budgets and a strong currency the prime objectives of European macroeconomic policy.

Regained macroeconomic stability. Taking stock at the outset of the 21st century, most European countries have indeed succeeded in attaining the postulated objectives. First, most governments have managed to bring down their budget deficit in line with the conditions for entering the Monetary Union. Some countries are even expected to register a surplus in 2000. Second, average inflation in Euroland has declined from 11.7% in the period 1974-82 to 2.1% in the period 1995-99.

The argument that the created stable macroeconomic environment will (automatically) set Europe on a sustainable higher economic growth path with low unemployment remains to be proven, though. Admittedly, unemployment rates in Euroland have declined considerably in recent years while economic growth will be no less than 3.75% in 2000. However, 9% of the European labour force is still stuck in unemployment while only 60% of the population at
working age is currently employed. This remains very weak, both from a historical and a cross-sectional (i.e. compared to e.g. the US) perspective. Further, the excellent growth record in 2000 cannot be taken as evidence that the economy has been set on a sustainable higher growth path.

**Double macroeconomic challenge.** Despite the regained macroeconomic stability, most European countries still face a double macroeconomic challenge. First, there is the enduring need to reduce the high government debt ratios. Both social considerations – e.g. safeguarding the Welfare State – and the prescriptions of the Stability and Growth Pact underpin this need. Governments should be aware that the budgetary room created by the current upswing in economic activity might disappear quickly as the economy slips into a new recession. Second, there is the challenge to reap the fruits of the achieved macroeconomic stability in terms of a further reduction in unemployment rates and an increase in employment rates.

Both challenges are clearly highly interrelated. On the one hand, labour market performance affects the government’s financial balances through the amount of unemployment benefits, social contributions and direct tax receipts. On the other hand, the state of the government’s financial balances affects labour market performance through the level of taxes and transfers and through the composition of public spending, i.e. consumption versus investment.

**Adequate policy mix.** Partly due to this mutual dependence, the success of the undertaking relies heavily on an adequate policy mix. In this respect, we do not challenge the view that a stable macroeconomic environment is a prerequisite for high economic growth and low unemployment. In itself, it is not a sufficient condition, though. If the road to – for instance - sound public finances is paved with policy actions that are unfavourable to the labour market, one can hardly argue that employment will benefit from the created stable environment.

This dissertation is mainly concerned with one particular aspect of this policy mix, i.e. public investment. Our apriori hypothesis is that public capital supports both labour market performance and the long-run supply potential of the economy. Being supportive for macroeconomic performance, public investment is also expected to cause positive feedbacks on the government’s financial balances.
Crumbling public infrastructures. Table 1 compares the level of public investment, as a percentage of GDP, observed in the 1970s with the level attained in the 1990s for a number of OECD countries. Given the expected positive contribution of public capital to both macroeconomic performance and public finances, it is surprising to see that over the last two decades public capital outlays have been drastically reduced in most OECD countries. In Belgium, for instance, the rate of public investment dropped from an average of 4.84% of GDP in the 1970s to a poor 1.77% in the 1990s. This implies a drop to 37% of the level reached in the 1970s. A similar dilution of public investment shares is observed in Denmark, Ireland and the UK. Somewhat less pronounced but still very significant reductions have taken place in Austria, Canada, Germany, the Netherlands, Norway and Sweden. Among the countries reported in table 1, only Finland, Spain and Portugal are able to present an increase.

Table 1  Evolution in public investment (in % of GDP).

<table>
<thead>
<tr>
<th>Country</th>
<th>inv70</th>
<th>inv90</th>
<th>inv90/inv70</th>
<th>inv70</th>
<th>inv90</th>
<th>inv90/inv70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>5.03%</td>
<td>2.78%</td>
<td>0.55</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td>4.84%</td>
<td>1.77%</td>
<td>0.37</td>
<td>Japan</td>
<td>9.18%</td>
<td>7.95%</td>
</tr>
<tr>
<td>Canada</td>
<td>3.58%</td>
<td>2.45%</td>
<td>0.68</td>
<td>Netherlands</td>
<td>3.84%</td>
<td>2.56%</td>
</tr>
<tr>
<td>Denmark</td>
<td>4.17%</td>
<td>1.86%</td>
<td>0.45</td>
<td>Norway</td>
<td>4.62%</td>
<td>3.46%</td>
</tr>
<tr>
<td>Finland</td>
<td>2.13%</td>
<td>3.08%</td>
<td>1.45</td>
<td>Portugal</td>
<td>2.53%</td>
<td>3.84%</td>
</tr>
<tr>
<td>France</td>
<td>3.55%</td>
<td>3.31%</td>
<td>0.93</td>
<td>Spain</td>
<td>2.42%</td>
<td>3.75%</td>
</tr>
<tr>
<td>Germany</td>
<td>3.79%</td>
<td>2.30%</td>
<td>0.61</td>
<td>Sweden</td>
<td>4.60%</td>
<td>2.73%</td>
</tr>
<tr>
<td>Ireland</td>
<td>4.85%</td>
<td>2.37%</td>
<td>0.49</td>
<td>Switzerland</td>
<td>4.55%</td>
<td>3.49%</td>
</tr>
<tr>
<td>Italy</td>
<td>3.10%</td>
<td>2.61%</td>
<td>0.84</td>
<td>UK</td>
<td>5.09%</td>
<td>2.08%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>US</td>
<td>3.70%</td>
<td>3.32%</td>
</tr>
</tbody>
</table>

Note: inv70 (inv90) measures the average level of nominal public investment in 1970-1979 (1990-1999) in % of nominal GDP.
Source: OECD, Statistical Compendium, 2000/1.

In itself, the observation that public investment relative to GDP has declined is no evidence that public capital is currently undersupplied. This decline may be due to the simple fact that the demand for publicly provided goods has declined. (e.g. smaller school-aged population, fewer registered cars). A closer inspection of the exact timing of the reduction in public investment suggests a different story, though. In Belgium for instance, almost 90% of the global decline in public investment reported in table 1 has occurred during the period 1982-87, a period characterised by strong fiscal consolidation. Also in other countries, strong reductions in public investment appear to be situated in periods of contractionary fiscal policy.1 Consistent with this heuristic finding, de Haan, Sturm and Sikken (1996) find evidence that fiscal stringency is a key element in the explanation of the sharp decline in

1 See Everaert (1999) for more details.
public capital investment in a lot of OECD-countries. Similarly, Alesina and Perotti (1995) argue that during periods of tight fiscal policy, cuts in government expenditures primarily involve cuts in public investment. This is due to “the political reality that it is easier to cut back or postpone investment spending than it is to cut current expenditures” (Oxley and Martin, 1991), for investments are a less rigid component of public outlays.

As contractionary fiscal policy seems to be – to a large extent – responsible for the drastic reduction of public investment in a large number of OECD countries, some authors have criticised the composition of fiscal consolidation programs. The necessity of restoring public investment at higher rates to support economic growth and employment has for instance been stressed by the European Commission (1993) and in a policy initiative paper written by a group of economists at the initiative of Drèze and Malinvaud (1994). In the US, new infrastructures played a key role in Clinton’s economic plans. In Belgium, higher expenditures on public investment – with special attention to enhancing mobility - were an important aspect of the 1999 negotiations preceding the formation of the Flemish and the Federal Government. However, the resistance in some countries against the proposed European investment programs proves that their expected positive contribution to macroeconomic performance is not evident and needs a stronger scientific background.

2. PUBLIC CAPITAL HYPOTHESIS

A large literature has studied the impact of public capital on the long-run growth potential of the economy. Critics generally point at the well-known crowding-out effects of public capital formation. To the extent that the government must borrow to finance its investments, increases in real interest rates and displacement of private investments cannot be excluded. Empirically, however, a lot of authors conclude that the net effect of public capital formation is positive. Estimating an aggregate production function, Aschauer (1989a, 1989b, 1989c) for instance found positive and highly significant effects of infrastructure on productivity growth and private sector investment in the US and the G7.

Productivity and growth. In response to Aschauer’s seminal work, a huge literature has studied the productivity and growth effects of public investment. Initially, similar results were obtained. More recently, however, Aschauer’s work has come under serious attack. Most authors still accept the underlying intuition but are highly critical concerning the concrete
methodological development of the idea, both on theoretical and methodological grounds. Major issues that have been raised concern the non-stationarity of the data, the direction of causality, possible misspecification of the production function (i.e. the Cobb-Douglas specification implies the substitution elasticities between factors of production to be equal to one) and the endogeneity of other factors of production (i.e. private labour and capital). Trying to come up with more justifiable approaches, the literature has proceeded along several different lines (see e.g. Sturm et al., 1997, for a broad survey). As is well known by now, the results of this literature are quite controversial. Some of these studies confirm Aschauer’s ‘public capital hypothesis’, others reject it.

**Labour market.** Much less attention has been paid to the labour market implications of public investment. Figure 1 is consistent with our hypothesis that there exists a positive long-run relationship between public investment and labour market performance. This figure includes cross-section data for 21 OECD countries. The upper part of figure 1 relates the average employment rate (i.e. employment as a percentage of the population at working age) in the 1980s to the average level of public investment (in percent of GDP) in the 1960s and the employment rate in the 1990s to the level of public investment in the 1970s. This yields 40 observations (two for each country, except for Australia and Switzerland where due to data limitations we only have one). Although there appears to be quite some dispersal, one might conclude that there is a positive correlation. Countries where public investment was high during the 1960s and 1970s appear to do better in terms of employment during the 1980s and 1990s. The lower part of figure 1 relates the unemployment rate in the 1980s and the 1990s to public investment in the 1960s and 1970s. Consistent with the previous results, a negative relationship emerges.

**Fiscal consolidation.** In recent years a growing literature has tried to explain the success or failure of fiscal consolidation programs. In this explanation, the composition of the consolidation program usually plays a crucial role. One of the hypotheses states that in order to be successful, i.e. lead to a permanent reduction in government debt and deficit ratios, fiscal consolidation must not rely on cuts in government investment. As mentioned above these cuts may erode the macroeconomic basis of a country, implying lower tax receipts and higher social expenditures. Evidence supporting this hypothesis has been provided by – among others - Alesina and Perotti (1995) and McDermott and Wescott (1996). Also own our research has contributed to this conclusion (Heylen and Everaert, 2000).
3. RESEARCH OBJECTIVES

This dissertation intends to study the effects of public investment more closely. Our contribution to the debate can be situated at four different levels:

(i) A number of methodological problems have not yet been adequately tackled in the literature studying the effects of public capital on economic growth. A first potentially
serious problem concerning the estimation of production and cost functions is that the underlying rate of technological progress is not directly measurable. Some authors simply ignore technological growth. Others have proceeded by including a linear time trend as a proxy. To the extent that technological growth is a stochastic process, this approach is doomed to yield biased estimates. Second, due to slow capital accumulation, the capital stock series usually included in empirical analyses often exhibit I(2)-components, implying that the conditions underlying the standard cointegrated VAR methodology are not satisfied. Third, most studies stick to a single equation framework, making their results vulnerable to simultaneous equation bias. Taking these problems as a starting point, our first goal is to strengthen the methodological framework of Aschauer’s work.

(ii) The effect of public capital on labour market performance has received almost no attention. To investigate whether the correlation shown in figure 1 is - at least partially - due to a causal relation running from public capital to employment or whether it is a mere coincidence, driven by the evolution of other variables, is our second objective.

(iii) Most studies that try to explain the success or failure of fiscal consolidation programs have taken a cross-country perspective (see e.g. Alesina and Perotti, 1995; McDermott and Wescott, 1996; Heylen and Everaert, 2000). An interesting contribution would be to check whether the finding that successful fiscal consolidations should not rely on cutting public investment survives using time series data.

(iv) Concerning the possible relation between public capital and employment it is important – especially from policy considerations – to understand the source of the high unemployment persistence observed in many OECD countries. Two alternative hypotheses are (full) hysteresis and convergence to an increased natural rate. Full hysteresis (meaning that all changes in the unemployment rate are permanent) implies that independent of the causes of the high unemployment rate, each policy measure, e.g. a boost in public investment, that succeeds in reducing unemployment has permanent effects. If the increase in the unemployment rate is due to adjustment to an increased natural rate, however, effective policy measures require dealing with the causes of the increase in the natural rate directly, among which possibly the reduction in public investment.
4. OUTLINE AND MAIN RESULTS

The remainder of this study consists of four chapters. The first two chapters contribute to the debate on growth and productivity effects of public investment by explicitly dealing with some of the problems currently encountered in the literature. The third chapter extends the analysis to the labour market. Each of these chapters uses time series data for the Belgian economy. The last chapter investigates the source of unemployment persistence in the OECD. It employs time series data for 21 countries.

Productivity and growth. Conceptually, chapter 1 sticks to the production function approach initially proposed by Aschauer. Methodologically, however, it differs in at least three crucial respects. (i) Given the stochastic non-stationary behaviour of the variables traditionally included in the analysis, we check whether the production function estimates constitute a stable equilibrium relation using the residual-based Engle-Granger cointegration methodology. (ii) As an alternative to fitting a deterministic trend we construct a proxy for the stock of knowledge by accumulating data on granted patents. (iii) We check the direction of causality from the estimates of an error-correction model. In contrast to standard Granger causality tests, this approach allows for causality resulting from the estimated long-run equilibrium. The estimates reveal a significant positive cointegrating relationship between public capital and private sector productivity, with causality running from public capital to multifactor productivity. The estimated output elasticity of capital lies around 0.29. In order to check the robustness of our results, we change the set-up of the analysis in two different ways. First, as a lot of critical assumptions underpin the construction of capital stock data (see appendix A for a brief note on this subject), we construct new public capital stock series by implementing some small adjustments in the original assumptions. Second, we use the Philips-Hansen fully modified least squares estimation procedure to correct for possible simultaneous equation bias in the production function. The results are found to be fairly insensitive to both alternative specifications.

Using the cointegrated VAR methodology, the underlying idea in the second chapter is to determine the rate of technological growth as a common stochastic trend in output, private capital and public capital based on the balanced growth restrictions derived from a simple neoclassical growth model. Due to slow capital accumulation, capital stock series often exhibit a near I(2)-trend, especially in the small samples usually available to the applied
researcher. These I(2)-trends introduce significant inertia in the system, implying slow adjustment of output towards its long-run steady state. Moreover, the standard I(1) cointegrated VAR methodology is no longer valid in the case of I(2)-trends. Therefore, capital stock series enter the analysis in first differences. Inspired by the neoclassical growth model, the balanced growth path of output is now being determined by private and public investments, with private and public capital stock growth rates capturing the medium-term slow adjustment towards this steady-state. Using Belgian data for the period 1953-96, the analysis supports Aschauer’s hypothesis that the decline in public capital investment has lowered the balanced growth path of real output. In contrast to Aschauer’s results, the output elasticity of public capital is found to be only a fraction 0.4 of the output elasticity of private capital. Assuming that the output elasticity of private capital is about 0.33, these estimates imply an output elasticity of public capital of about 0.14.

The third chapter investigates the output and labour market effects of public capital formation in Belgium within a broader structural model, explaining - among other variables - private output, private employment and unemployment, private capital formation, wage bargaining and price setting. From the supply side of the economy, which is modelled using a translog cost function, we can derive the contribution of public capital to private sector production capacity. The estimates of this cost function show that - for given output and wages - services from public capital significantly reduce private sector total cost. An increase in the public capital stock with 1 Euro reduces long-run private sector cost with 0.24 Euro. The output elasticity of public capital implied by these estimates equals 0.31.

The main conclusion concerning our first research objective is that public capital has a significant positive impact on productivity and economic growth. Although there is some variation in the size of the output elasticity of public capital, this finding is robust over the three alternative econometric techniques used.

Labour market. Our model in chapter 3 also allows us to analyse the impact of public capital on labour market performance. In this respect, at least three different channels of influence of public capital on private employment can be distinguished: (i) direct complementary or substitution effects for given output and wages, (ii) indirect effects on real wages, due to changes in labour productivity and/or the unemployment rate, (iii) indirect effects caused by changes in aggregate demand.
The estimates of the translog cost function allow quantification of the impact of public capital on private sector inputs for given output and wages, i.e. direct complementary or substitution effects. The results suggest that public capital and labour are substitutes. Public capital and private capital are found to be complements. This means that public capital is labour saving in the production function, i.e. a given level of output will be produced with less labour if the government extends the public capital stock.

This substitutive relationship does not necessarily imply that labour demand actually decreases with increasing public investment. Apart from the direct substitution effect in the production function, public capital also affects the demand side of the economy and wage bargaining between employers and unions. In addition to higher public investment spending, positive demand side effects might stem from higher private investment spending and higher private consumption due to an increase in the household disposable income. The impact of public capital on real wages is uncertain. On the one hand, the increase in labour productivity exerts upward pressure on bargained wages. On the other hand, the substitution of labour for private and public capital should dampen union’s wage claims.

To find out whether the negative relationship between public capital and employment is altered once demand and wage effects are taken into account, the model is simulated under an alternative public investment policy. The simulations reveal a considerable rise in aggregate demand in response to an increase in public investment. However, as labour demand is found to be largely insensitive to changes in aggregate demand, the benefits in terms of employment are very moderate, accounting for only a small decrease in the unemployment rate. One should be very cautious in interpreting this result, though. Although the idea that in the long run employment and output are independent is consistent with apriori expectations, one would expect to find a (considerable) positive impact in the short run. The observed very small impact of output on labour market performance is most probably due to an overestimation in the dynamic cost function of the speed of adjustment of factor shares toward their long-run equilibrium.

Further, due to a strong rise in labour productivity, also real wages increase in response to an increase in public investment. As the employment share in total cost was found to be independent of public capital, the productivity benefits resulting from an increase in the public capital stock can - for given costs - only be incorporated in real wages at the expense of
lower employment, though. Given the estimated low responsiveness of real wages to unemployment, the increase in unemployment has offset only part of the rise in real wages. Therefore, the increase in real wages has induced a further rise in the unemployment rate.

The combined effect of the three channels through which public capital affects private employment is clearly negative. Simulating the model under the assumption that the strong decline in Belgian public investment over the period 1982-89 did not occur\(^2\), we observe a 3.8%-points increase in the 1996 unemployment rate compared to the benchmark simulation. Direct substitution of labour for public capital contributes 3.3%-points to this overall increase while the rise in real wages adds another 0.8%-points.

*The answer to our second research objective is that public investments reduce private sector employment and raise private sector unemployment.*

**Fiscal consolidation.** Finally, our model in chapter 3 allows for a time series test of the well-known hypothesis that in order to be successful, fiscal consolidation should not rely on cutting public investment. From the above mentioned simulation we learn that the government deficit widens considerably during the first years of the increase in public investment, reaching a maximum increase in the deficit equal to 2.9% of GDP. As the positive growth effects of a higher public capital stock slowly increase tax receipts, the increase in the budget deficit shrinks to 0.8% of GDP in 1996. Although this is still a significant burden on the government’s financial balances, extrapolation of the evolution of the government budget suggests a (growing) decrease, relative to the benchmark simulation, in the deficit from about 1998 onwards.

*Concerning our third research objective, the conclusion is that a structural reduction in government debt and deficit ratios cannot be accomplished by cutting public investments.*

**Persistence in unemployment rates.** The fourth chapter tests whether the high persistence in unemployment rates observed in most European countries is due to (full) hysteresis against the alternative hypothesis that it is caused by adjustment toward an increased natural rate. A direct methodology for assessing the nature of unemployment persistence is analysing the time series properties of the unemployment rate using standard univariate unit root tests.
Usually, such tests cannot reject the presence of a unit root in the unemployment rate, pointing to full hysteresis. Implicitly, the alternative hypothesis in these tests states that the unemployment rate reverts to a constant natural rate, though. We investigate whether the results change if we allow for a variable natural rate under the alternative hypothesis by modelling infrequent level shifts, identified using an outlier detection algorithm. Once such level-shifts are allowed for, univariate unit root tests strongly reject the null of a unit root in almost all OECD-countries.

In terms of the alternative persistence perspectives, this result is in favour of the hypothesis that the rise in OECD unemployment rates is due to adjustment to an increased natural rate. Nevertheless, it would be hard to advocate that the identified level-shifts in the actual unemployment rate coincide with shifts in the natural rate. More likely, infrequent large shocks are caused by extreme adverse shocks to the unemployment rate – whatever their cause - implying jumps in the direction of a natural rate, which might have increased years before the upshot in actual unemployment.

Ljungqvist and Sargent (1998) have developed a general equilibrium search model, which is able to account for this feature. In this model workers accumulate skills on the job and lose skills during unemployment. Its main implication is that even with unfavourable labour market characteristics – implying an upward shift in the natural rate - low unemployment is sustainable as long as the economy is not subject to any major adverse shocks. The intuition behind this result is that the availability of a lot of ‘good jobs’ counteracts the adverse effects of generous unemployment benefits. When the economy is hit by a severe adverse shock, however, generous benefits erode the ability of the labour market to adjust. Ljungqvist and Sargent (1998) argue that in this interpretation, the smooth performance of the European economies in the 1950s and 1960s concealed an inherent instability. Their model implies that the gradual build-up of the welfare state – implying unfavourable labour market characteristics like higher direct taxes on labour and generous unemployment benefits – was a virtual ‘time bomb’ waiting to explode. The oil shocks of the 1970s caused the “explosion”.

Concerning our fourth research objective, the results show that the observed high persistence in European unemployment rates is due to adjustment towards an increased natural rate.

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The 1996 hypothetical public capital stock would be 28.5% higher than the capital stock observed in reality.
5. POLICY IMPLICATIONS

In this section we present the main policy implications of the results outlined in section 4. As the results concerning the impact of public capital are derived using Belgian data, the related policy implications in principle apply to Belgium only. For other countries, they can serve as a first indication, though. Before proceeding, it should also be stressed that the presented material must not be considered to contain final results concerning the research questions put forward in section 3. Improvements and extensions open plenty of room for future research. Section 6 therefore contains a non-exhaustive research agenda.

**Beneficial supply and demand effects.** The results outlined in section 4 suggest that when the route to macroeconomic stability relies on cutting government investment, economic growth and productivity of private sector factors of production are structurally reduced. Taken at face value, the alleged beneficial supply and demand effects of public capital suggest that the Belgian government should raise public capital spending. We believe that this is the main policy implication of our research. Despite its beneficial effects, we do not wish to advocate an *unconditional* strong increase in government investment, though. Two arguments underpin the need for some reticence. First, our results show that, ceteris paribus, public capital reduces private sector employment. Second, increases in government investment should be balanced against the enduring need to reduce the high government debt ratio. Each of these two arguments is discussed in turn.

**Adverse labour market effects.** The empirical analysis in chapter three indicates that public capital reduces private sector employment. This result implies that the positive correlation between public investment and the employment rate observed in figure 1 is produced by the evolution in a third factor, affecting public investment and employment simultaneously, rather than resulting from a direct causal relation running from public capital to employment. One possible candidate is the massive build-up of government debt since the 1970s, forcing the government either to raise taxes or to cut spending, often investment spending (see e.g. de Haan et al., 1996). The potential negative labour market consequences of higher taxes are evident (see e.g. recent research by Daveri and Tabellini, 2000). Higher taxes can raise labour cost, either directly (employer taxes) or indirectly when workers claim higher wages to safeguard their net income (employee taxes, indirect taxes). To the extent that the increase in
taxes coincides with cuts in public investment spending, a positive correlation between public investment and employment may emerge.

Despite the negative relation between public capital and employment observed in the data, a number of critical comments are called for. (i) As noted above, the impact on employment of beneficial demand effects from higher public investment is probably underestimated in our research. Therefore, we believe that - at least in the short-run – the relation between public capital and employment is less unfavourable than our results suggest. (ii) The empirical analysis relies on a broad public capital stock concept. The observed negative relation does not imply that all investment projects bring about substitution of labour. Investments in training facilities for the structural unemployed, sheltered workshops, mobility of low-class workers, etc. are for instance expected to yield beneficial effects in terms of employment if labour supply is successfully reallocated from sectors and/or regions with high unemployment to sectors and/or regions with high labour demand. Further, infrastructure investment aimed at structural conversion of weak regions might also yield an important contribution to employment in these regions. (iii) Part of the rise in unemployment in the aftermath of higher public investment is due to upward pressure on real wages prompted by an increase in labour productivity. If the government wants to translate higher investment spending into higher employment, it should see to it that the beneficial productivity effects of higher public investments are not skimmed off by higher real wages. In this respect, investments aimed at enhancing effective labour supply (supra) are an obvious choice as they can be expected to exert downward pressure on real wages. Further, supplementary measures to moderate real wage claims might be in place.

**Structural reduction of government debt.** The second argument for not unconditionally increasing government investment results from the enduring need to reduce the high government debt ratio in Belgium. Our empirical analysis confirms the hypothesis that an increase in government investment implies no permanent worsening of the government budget. As it takes quite some time, however, for the indirect positive output effect of an increase in investment spending to feedback into the government budget, a significant short-run increase in the deficit - raising government debt and interest payments - arises. This short-run negative impact implies that - given the still very high public debt ratio in Belgium – the government should be cautious if it wants to expand public investment spending. In the light of the arduous consolidation of public finances over the last two decades, a considerable
increase in public investment might even not be politically feasible. Moreover, since the signing of the Stability and Growth Pact - requiring EMU Member States’ budgetary positions close to balance or in surplus and a government debt ratio declining steadily to about 60% of GDP - the room for new policy initiatives has been seriously reduced.

In order not to jeopardise the reduction of government debt, higher public investments should therefore preferably be financed from the budgetary margin created by increases in the primary surplus and declining debt interest payments. Even then, the long-run benefits of public investments should still be balanced against the benefits – in terms of lower interest payments for instance – of faster debt reduction. Moreover, the government should be aware that the current budgetary room is partly created by a strong upswing in economic activity and might disappear quickly as the economy slips into a new recession.

**Structural labour market reform.** Our findings in chapter 4 that high and persistent unemployment in Europe is not due to strong hysteresis effects but more likely reflects adjustment to an increased natural rate, suggests that the battle against unemployment can only be won if governments focus on structural labour market reform aimed at bringing down the natural rate. Often cited reform measures are (i) a more flexible employment protection legislation, (ii) reduction of generous unemployment benefits, (iii) reduction of minimum wages, (iv) decentralisation of wage bargaining, (v) training of structurally unemployed workers, etc. (Calmfors, 1998; Wyplosz, 2000; Elmeskov et al., 1999). Based on the results outlined in section 4, infrastructure investments appear to fail as a candidate in bringing down the natural rate of unemployment. As noted before, however, these estimates are based on a broad public capital stock concept. Carefully selected investment projects (supra) might be beneficial in terms of employment.

Without the ambition to provide a thorough investigation of the route to go for labour market reform to be successful – which is beyond the scope of this dissertation – public investment can also play an important role as attendant measure. The main reason is that structural reforms are unlikely to pay off in a negative macroeconomic environment. During a recession for instance, the willingness of employees and unions to accept reform measures will be very low since – at least in the short-run - the potential benefits do not outweigh the risks of a more flexible labour market (see e.g. Allsopp and Vines, 2000; Wyplosz, 2000). In terms of the analysis in chapter 4, structural reform brings down the natural rate of unemployment. A fast
reduction in the actual rate of unemployment might require positive shocks to economic activity as this speeds up convergence to the lower natural rate. In this respect, the growth stimulating effect of public investment can be used to raise the short-run benefits of structural reform. Increases in public investment are preferred over increases in current expenditures as the former measure is expected to bring about – in addition to its direct aggregate demand effect - indirect demand effects resulting from the increased productivity of private sector factors of production.

6. DIRECTIONS FOR FUTURE RESEARCH

Finally, let us outline some possible directions for future research. First, concerning the analysis in chapter 1, the calculation of multifactor productivity relies on the assumption that technological progress is not biased towards certain factors of production, i.e. technological progress is assumed to be Hicks-neutral. If this assumption fails to hold in reality – technical progress is for instance labour augmenting – the calculation is obviously flawed. Note that with Cobb-Douglas production, labour-augmenting, capital-augmenting and Hicks-neutral technical progress are essentially the same. Allowing for factor-biased technical progress in the calculation of multifactor productivity therefore requires specifying a more flexible production function. Haskel and Slaughter (1998) for instance measure skill-biased technical progress as the change in skilled labour’s cost share – derived from a translog cost function – that is not explained by changes in factor prices.

Second, the ability of the analysis in chapter 2 to identify the proposed steady-state relationship as a cointegrating vector relies on the observation that the impact on equilibrium output of the upward shift in the private investment rate occurring in the 1980s has been cancelled out by the negative shock to the public investment rate over more or less the same period. In future research, it might be interesting to model these shocks endogenously by including the driving factors behind private and public investment.

Third, as noted above the very small impact of output on labour market performance is most probably due to an overestimation in the dynamic cost function of the speed of adjustment of factor shares toward their long-run equilibrium. Therefore, modelling the short-run dynamics of the cost function using alternative specifications is worth considering in future research. An alternative dynamic specification is for instance proposed by Hall and Nixon (1999). They
believe that firms face costs from adjusting the actual level of factor input rather than from the adjustment of factor shares. Therefore, they create a dynamic model from costly adjustment of factor volumes in stead of factor shares. A second available modelling strategy is the restricted cost function approach of Allen (1994). In this approach, costs are minimised conditional on a given capital stock. From these conditional estimates, the long-run optimum - allowing for adjustment of all the factors of production - can be derived using the Le Chatelier-Samuelson principle.

Fourth, chapter 4 suggests that the strong persistence in European unemployment rates is due to persistence of infrequent large shocks rather than to full hysteresis. These infrequent large shocks are interpreted as extreme adverse shocks to the unemployment rate, implying jumps in the direction of an exogenous natural rate. An alternative interpretation would be that a strong rise in actual unemployment – whatever its cause – may provoke a permanent rise in the long-run equilibrium rate because it triggers fiscal and social policy responses that negatively affect the structural characteristics of the labour market (see also Everaert and Heylen, 1999). Fiscal policy responses may follow from the negative effects of unemployment on the government’s financial balance, e.g. social security receipts and expenditures. To the extent that these negative effects force the government to raise taxes, an increase in the equilibrium rate of unemployment might arise. Further, observing the strong rise in unemployment, the government may for social reasons enhance the generosity of the unemployment benefit system or the early retirement system (OECD, 1994). Alternatively, the government may create more public sector jobs. In the short run these policy actions probably exert favourable effects. In the long run, however, the effects may be negative. Once the more generous system has been introduced, it may for political reasons be very hard to turn it back. Gradually, changes are then to be expected in the behaviour of private sector wage bargainers (higher wage claims) and unemployed workers (reduced search intensity or willingness to accept job offers) with potentially permanent adverse effects on employment. These fiscal and social policy responses provide a new perspective on the observed high unemployment persistence in Europe, a proposition worth further investigation in future research.
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Public capital and productivity growth
- Evidence for Belgium, 1953-1996 -

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ABSTRACT: This paper analyses the impact of public capital on multifactor productivity in Belgium making use of single-equation cointegration analysis on annual data for the period 1953-1996. Instead of fitting a deterministic trend to capture the underlying technological progress, patent statistics are used as a proxy. From the estimated long-run equilibrium between public capital and productivity, we estimate an error-correction model to check for the direction of causality. The results support a strong positive relationship with causality running from public capital to productivity.

KEYWORDS: Public capital, multifactor productivity, technological progress, patents.

1. INTRODUCTION

Since Aschauer (1989a), the potential negative impact of reduced public investment on productivity growth has been a hot topic in the economic literature. Observing that the US productivity slowdown “is matched, or slightly preceded, by a precipitous decline in additions to the net stock of public non-military structures and equipment”, Aschauer expands the conventional aggregate Cobb-Douglas production function to include the public capital stock. He provides empirical evidence that a 1% decrease in the ratio of public to private capital decreases multifactor productivity in the US by 0.39%. Taking a panel of seven countries, Aschauer (1989b) shows that this strong, positive correlation continues to hold in a broader sample of countries.

Inspired by Aschauer, a large body of empirical research investigating the link between public capital and private sector productivity has arisen\(^1\). Initially, Aschauer’s findings were confirmed, see e.g. Munnell (1990a, 1990b). Later, however, they have come under severe

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** Forthcoming in Economic Modelling (2001).

\(^1\) See Gramlich (1994) and Sturm et al. (1997) for excellent surveys of the literature.
attack for the estimated output elasticity of public capital implied an implausibly high return to public capital (see e.g. Aaron, 1990). The rate of return to core infrastructure implied by Aschauer’s (1989a) results equals almost 150%, while Munnell’s (1990a) results point to a rate of return to all public capital of about 60% (Hurst, 1994).

Alarmed by these high figures, a lot of defects in Aschauer’s methodology have been identified. A first important problem emerges from the observation that all variables included in the production function show stochastic non-stationary behaviour. The finding of a unit root makes Aschauer’s results, derived using level data, suspicious due to possible spurious correlation (see e.g. Tatom, 1991). Second, it is often argued that the established positive correlation could be evidence that multifactor productivity has a significant positive impact on public capital rather than the other way around, i.e. reverse causation (Eisner, 1991). Third, Hulten and Schwab (1993) argue that a production function is likely to be part of a system in which both input and output variables are endogenously determined. This makes the results from estimating a single-equation production function potentially liable to simultaneous equation bias.

Despite the fact that many have identified these important problems, they have not yet been properly taken care of. Using annual data for Belgium over the period 1953-1996, the goal of this paper is to reconsider these problems in a production function approach similar to the one initially applied by Aschauer. However, instead of following the traditional approach of including a linear time trend to capture the underlying technological progress, we use patent statistics as a proxy. To make sure that our results indicate a long-run equilibrium relationship rather than follow from spurious regression, we run cointegration tests drawing on Engle and Granger (1987). Since in a cointegrating framework, standard Granger causality tests are no longer valid, we proceed to analyse the direction of causality by estimating an error-correction model (ECM).

In order to check the robustness of our results, we have changed the set-up of the regression analysis in two different ways. First, to correct for possible simultaneous equation bias, we implement the fully modified least squares estimation procedure suggested by Phillips and Hansen (1990). Second, we allow for changes in the construction of public capital stock data.
The remainder of this paper is organised as follows. The next section briefly outlines the production function approach. Section three discusses the empirical methodology of the paper. The results are presented in the fourth section. The final section summarises.

2. **SPECIFICATION OF THE MODEL**

The analysis of the productivity impact of public infrastructures is usually done by adding public capital \((G)\) to private sector factors of production, capital \((K)\) and labour \((L)\), in an aggregate production function,

\[
Y_t = A_t f \left( K_t, L_t, G_t \right)
\]

(2.1)

where \(Y\) equals total private sector output and \(A\) captures technological progress. Assuming the production function to be adequately described by a Cobb-Douglas specification,

\[
Y_t = A_t K_t^{e_k} L_t^{e_l} G_t^{e_g}
\]

(2.2)

and rewriting in logarithms yields\(^2\):

\[
y_t = a_t + e_k k_t + e_l l_t + e_g g_t
\]

(2.3)

By definition, multifactor productivity can be calculated as:

\[
\text{mfp}_t = y_t - s_k k_t - s_l l_t
\]

(2.4)

with \(s_k\) and \(s_l\) denoting the shares of the private sector inputs, capital and labour respectively, in private sector output. Under constant returns to private inputs and perfect competition, profit maximisation implies that private factors of production earn their marginal products: \(e_l\) and \(e_k\) will be equal to \(s_l\) and \(s_k\) respectively. Constant returns over private sector inputs, and therefore increasing returns over all inputs, are motivated from the possibility of considerable economies of scale resting behind the provision of public capital (Aschauer, 1989a). The expression for multifactor productivity can then be rewritten as:

\[
\text{mfp}_t = y_t - e_k k_t - e_l l_t = a_t + e_g g_t
\]

(2.5)

\(^2\) Lower-case variables denote logarithms.
However, if congestion effects are severe enough, they may lead to constant returns over all inputs (Aschauer, 1989a). Since public capital is an unpaid factor of production, the assumption of constant returns to all inputs implies that paying private factors of production their marginal product would not exhaust total output. One possible way out of this problem is to assume that all benefits from the contribution of public capital to production are distributed among private factors of production proportionally to their output elasticities, i.e. factor shares are proportionally related to output elasticities (Aschauer, 1989a):

\[ s_l = \lambda \, e_l \quad \text{and} \quad s_k = \lambda \, e_k \quad \text{with} \quad \lambda > 1 \quad (2.6) \]

This assumption enables us to find a simple expression for multifactor productivity:

\[ mfp_t = a_t + e_g \left( g_t - s_k \, k_t - s_l \, l_t \right) \quad (2.7) \]

Note that the two alternative specifications are nested in the following ‘general’ specification which is a weighted average of (2.5) and (2.7):

\[ mfp_t = \gamma \left( a_t + e_g \, g_t \right) + (1 - \gamma) \left[ a + e_g \left( g_t - s_k \, k_t - s_l \, l_t \right) \right] \\
= a_t + e_g \, g_t - (1 - \gamma) e_g \left( s_k \, k_t + s_l \, l_t \right) \quad (2.8) \]

If \( \gamma = 0 \), the production function exhibits constant returns over all inputs, while \( \gamma = 1 \) points to constant returns to private sector factors of production only.

3. DATA SELECTION AND EMPIRICAL SPECIFICATION

3.1. Data selection

Data concerning gross capital stocks over the period 1953-1996 were kindly provided by the Belgian Federal Planning Bureau. The public capital stock includes roads, buildings, educational facilities, etc. owned by federal, regional and local authorities as well as by the semi-governmental social security institutions. It does not include public enterprises (e.g. railways, harbours, ...). These are included in the private capital stock. Residential buildings, owned by the households, are excluded from the private sector capital stock. Data on private sector output and factor shares are drawn from the OECD’s ‘Economic Outlook’. Private sector employment is taken from the OECD’s ‘Business Sector Data Base’. Since data taken from
OECD sources are only available from 1960 onward, we have reconstructed these series for 1953-59 based on national accounts data provided by the Belgian Federal Planning Bureau.

### 3.2. Empirical specification

Before choosing the appropriate econometric methodology, it is very important to analyse the non-stationary behaviour of the variables of interest. Besides the traditional Dickey-Fuller tests, the order of integration is tested for applying an alternative testing procedure proposed by Kwiatkowski *et al.* (1992).

The results from running these tests (see appendix A) clearly show that all variables exhibit a unit root, making Aschauer’s results - derived using level data - suspicious due to possible spurious correlation. A popular way to handle this kind of non-stationarity is to estimate the model in *first-differences*. However, this approach only leads to very ambiguous results. Estimation in first-differences removes all trend components, putting heavy weight on high-frequency disturbances. Essentially, one analyses the effect of public capital growth in one year on productivity growth during the same year (Munnell, 1992). By contrast, economic theory suggests the relationship between multifactor productivity and public capital to be at much lower frequencies. First-differencing would remove this low-frequency component.

Much more relevant than simply taking first-differences is the concept of *cointegration*, analysing the existence of a long-run equilibrium relationship between stochastic non-stationary variables. However, only some authors have actually elaborated on this view. On the one hand, Tatom (1991) finds no evidence of cointegration in the US. Sturm and de Haan (1995) come to the same conclusion for both the US and the Netherlands. On the other hand, Bajo-Rubio and Sosvilla-Rivero (1993) find a clear cointegrating relationship in Spain. Furthermore, there are some problems related to these studies. First, Tatom includes energy prices in his specification of the production function. Since prices belong more to a cost than to a production function, the interpretation of his results is not straightforward. Second, sticking to Aschauer’s model, Sturm and de Haan include a linear time trend to capture

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4. Another clear advantage of cointegration over first-differencing is that the estimated coefficients converge much faster to their true population values (the OLS estimator is said to be super consistent). This greater power may be critical given the relatively small sample size.
technological progress. They also include the capacity utilisation rate to capture short-term disturbances. Besides the fact that within the context of cointegration, short-term disturbances are adequately captured by an error-correction model, both variables simply cannot cointegrate with multifactor productivity for they are not integrated processes (see Stock and Watson, 1988). Moreover, including time as one of the regressors implies looking for cointegration in linearly detrended data. This is clearly in contradiction with the results from the unit root tests and with the notion of long-run equilibrium. Third, besides including capacity utilisation Bajo-Rubio and Sosvilla-Rivero also completely ignore the underlying technological progress.

The studies mentioned above are all based on single-equation cointegration techniques. Alternatively, some authors employ Johansen’s maximum likelihood procedure (Johansen, 1988), estimating cointegrating relationships in vector autoregressive error-correction models. From a theoretical point of view, this system based approach is more satisfactory for besides treating all variables endogenously it also allows for multiple cointegrating relationships. In light of the unit root tests reported in appendix A, it suffers from a serious drawback though. The results point to the presence of I(2) components in some of the variables. Although we make a strong argument that these variables are in fact I(1), this does not take away the problem that they might behave as I(2) processes in the small sample under investigation. Ho and SØrensen (1994) show that Johansen’s estimation procedure performs badly in the presence of near-I(2) variables. The Engle-Granger methodology in contrast does not suffer from this kind of anomaly.

Given the problems identified above, we revise the production function approach for Belgium and check for cointegration using the single-equation Engle-Granger cointegration methodology. A serious problem concerning the estimation of (2.8) is that technology (ₐₜ) is not directly measurable. For reasons outlined above, we cannot proxy this process by a linear trend, as was originally proposed by Aschauer. Instead, we use patent statistics as an approximation. Since patent data are readily available, related to inventive activity and based on objective standards, they have been used extensively to get some intuition on the underlying rate of technological change (see e.g. Griliches, 1990; or Verspagen, 1996).

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5 Nelson and Kang (1981) show that a ‘linearly detrended random walk is likely to exhibit spurious periodicity’.
Following a large body of research we opt for patents granted by the US Patent and Trademark Office. The large outlet and objective standards used by this office are generally thought to enhance the economic relevance of the data. Instead of using data on patenting to Belgian firms only, we withhold patents granted to both domestic and foreign firms. Underlying this choice are the ideas that (i) total patenting better reflects the world’s total knowledge stock (Griliches, 1990) and (ii) that all economies, especially those very open to international trade, have to a large degree access to this world knowledge pool (see e.g. Mankiw, 1995; Coe and Helpman, 1995).

Data on the total number of granted patents are accumulated into stock data by applying a perpetual inventory method with a quasi-logistic mortality function, assuming a fixed average lifetime. Based on the statutory period for which a patent is granted, our first guess is to fix the average patent lifetime at 18 years, $p(18)$. Since a lot of patents may be scrapped before they reach this age, we have constructed two alternative patent stock series. The first, $p(18s)$, continues to use a patent lifetime of 18 years but fits a right-skewed mortality function. This imposes the restriction that a larger fraction of the patents are discarded before the expiring of the patent grant. The second, $p(10)$, reduces the average lifetime to 10 years.

Figure 1 provides some intuition about the possible relationship between multifactor productivity and the three alternative patent stock measures. Especially $p(18)$ seems to be a relevant proxy for the rate of technological progress. A less strong relationship emerges for the other two proxies.

It must be acknowledged though that the use of patent statistics as a proxy for the aggregate stock of knowledge is not beyond dispute. In a recent survey article Keely and Quah (1998) are highly critical. First, a significant fraction of research, e.g. research by academic and government scientists, is not driven by the incentive to patent innovations for appropriating rent. Second, patents are not the only form of appropriation (and thus not always necessary). Alternative forms are e.g. secrecy, lead time and movement down the learning curve. Third, there is no clear relation between the number of patents and their relevance for aggregate production. Many patents are essentially worthless, they are never cited. Also, countries may lack the ability to use the technology that has been developed internationally.
Alternative proxies for knowledge in the production function are (foreign and domestic) R&D capital stocks. For example, Coe and Helpman (1995) and Coe, Helpman and Hoffmaister (1997) use these variables in empirical analyses of the evolution of multifactor productivity in OECD and developing countries respectively. R&D capital stocks may correct for the second shortcoming of patent statistics and if public sector R&D is included also for the first\(^\text{6}\). Further, domestic R&D may be a good indicator for a country’s ability to adapt (world-wide) technology.

Our motivation to use patents, granted to both Belgian and foreign firms, rather than R&D capital stocks is triple. First, R&D data availability is limited. To the best of our knowledge no data are available for the 1950s and early 1960s. For many countries, e.g. Belgium, data are available only since the 1970s (see also Coe and Helpman, 1995). Using R&D capital stocks would thus seriously shorten our sample period. Second, although R&D data might be preferable from a theoretical point of view, an empirical test discussed in appendix B suggests that the patent stock \(p(18)\) is in the long-run related to the ‘world’ R&D capital stock. Third, since Belgium is a very open economy with a highly educated workforce, it is undoubtedly allowed to assume that the country - as such - is able to adapt world-wide technology. There

\(^{6}\) Note though that the data used in many studies, e.g. Coe et al. (1995, 1997) only concern business sector R&D.
is as a consequence no need to include the domestic R&D capital stock as an explanatory variable.

Assuming that patent statistics are proportional to the true technological process, we obtain the following empirical specification:

\[ mfp_t = \beta_0 + \beta_p p_t + e_g g_t - e_g (1 - \gamma) (s_k k_t + s_l l_t) + \mu_t \]  \hspace{1cm} (3.1)

with \( p \) denoting the natural logarithm of the stock of patents.

Note that equation (3.1) does not allow the coefficients on \( k_t \) and \( l_t \) to be estimated freely. Rather, both variables have been combined into one private sector input variable. This approach, also applied by Ford and Poret (1991a), has been criticised for imposing theoretical restrictions that are not tested for (see e.g. Sturm et al., 1997). However, the imposed restriction is motivated from technical rather than economic considerations. An important shortcoming of the Engle-Granger methodology is that it can only handle a single cointegrating vector. However, in considering a five-dimensional system, which would be the case if we included private sector inputs separately, the number of cointegrating relationships may be up to four. The Engle-Granger methodology is then no longer valid\(^7\). Therefore, we have tried to restrict the dimension of the system as much as possible\(^8\).

### 4. Empirical results

#### 4.1. Engle-Granger cointegration

Table 1 reports the results from using the Engle-Granger cointegration methodology. In order to check the relevance of patent statistics as a proxy for technological progress, we first estimate our model under four alternative specifications of the Cobb-Douglas production

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\(^7\) An alternative approach that deals with this problem is the Johansen cointegration technique (Johansen, 1988), which allows for multiple cointegrating vectors. However, if multiple cointegrating vectors are found, one isolated equilibrium relationship does not necessarily have a direct economic interpretation for it is no longer uniquely identified. In this case, economic theory has to be imposed in order to identify the cointegrating space. Since these exactly identifying restrictions cannot be tested for, this approach also implies imposing economic theory that cannot be tested for.

\(^8\) Tentative estimations based on the unrestricted model indeed yielded estimates that did not allow a straightforward economic interpretation, raising our suspicion about the ability of the Engle-Granger methodology to identify a production function in the larger system.
function (regressions (1)-(4)). The estimation method is non-linear least squares (NLLS). Simply regressing productivity on the bundle of private sector inputs and public capital (regression (1)) yields a clear example of the spurious regression problem. Although we obtain highly significant estimates, the residuals from the cointegrating regression are clearly non-stationary, invalidating the NLLS results.

Table 1  The productivity effect of public capital in Belgium (1953-96)\textsuperscript{a}

<table>
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<th>Dependent var.</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tr>
<td>( mfp_t )</td>
<td>1.44</td>
<td>1.57</td>
<td>1.85</td>
<td>2.41</td>
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<td>1.80</td>
</tr>
<tr>
<td>( p )</td>
<td>0.52</td>
<td>0.45</td>
<td>0.62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e_g )</td>
<td>0.54</td>
<td>0.31</td>
<td>0.44</td>
<td>0.56</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>1.29</td>
<td>0.47</td>
<td>0.62</td>
<td>0.46</td>
<td>1.00\textsuperscript{f}</td>
<td>0.00\textsuperscript{f}</td>
</tr>
<tr>
<td>( \text{Adj. } R^2 )</td>
<td>0.988</td>
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<td>0.993</td>
<td>0.992</td>
<td>0.993</td>
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</tr>
<tr>
<td>( DW \textsuperscript{b} )</td>
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<td>0.81</td>
<td>0.80</td>
<td>0.70</td>
<td>0.73</td>
<td>0.85</td>
</tr>
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</table>

Cointegration tests

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<tbody>
<tr>
<td>( c )</td>
<td>-2.18</td>
<td>-4.05</td>
<td>-3.31</td>
<td>-2.87</td>
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<td>-4.34</td>
</tr>
<tr>
<td>5% critical values</td>
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<td>-4.36</td>
<td>-4.36</td>
<td>-4.36</td>
<td>-3.94</td>
<td>-3.94</td>
</tr>
<tr>
<td>10% critical values</td>
<td>-3.60</td>
<td>-4.05</td>
<td>-4.05</td>
<td>-4.05</td>
<td>-3.60</td>
<td>-3.60</td>
</tr>
</tbody>
</table>

Testing constant returns to scale in regression (2)

\( \gamma=0 \)  \( \gamma=1 \)

<table>
<thead>
<tr>
<th>( t )-statistic</th>
<th>1.26</th>
<th>-1.41</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bootstrapped critical values</td>
<td>5%</td>
<td>2.07</td>
</tr>
<tr>
<td>10%</td>
<td>1.64</td>
<td>-1.03</td>
</tr>
<tr>
<td>Bootstrapped value of ( \gamma )</td>
<td>0.40</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Notes: \textsuperscript{a} \( t \)-statistics in parentheses.
\textsuperscript{b} \( DW \) denotes the Durbin-Watson \( d \)-statistic.
\textsuperscript{c} Critical values are taken from MacKinnon (1991).
\textsuperscript{f} Restricted coefficients.
The results improve significantly when the patent stock, based on a lifetime of 18 years, \( p(18) \), is added as an additional explanatory variable (regression (2)). The residuals are found to be stationary at the 10% level of significance, giving weak evidence in favour of a cointegrating relationship. Both the patent stock constructed by fitting a right-skewed mortality function (regression (3)) and the one based on a lifetime of 10 years (regression (4)) yield spurious results. We therefore opt to work with \( p(18) \).

As a further step in analysing the impact of public capital on private sector performance, we test whether constant returns to scale prevail over all inputs (\( \gamma=0 \)) or over private sector inputs (\( \gamma=1 \)) only. Unfortunately, the point estimate of 0.47 for \( \gamma \) in regression (2) does not allow us to discriminate between the two alternative specifications as 0.47 is not significantly different from zero (1.26) nor from one (-1.41).

Note that the standard critical values from the \( t \)-distribution are derived from asymptotic theory, which might be invalidated in the small sample currently at hand. In order to investigate the small sample properties of the \( t \)-statistic, we bootstrapped cointegrated regression (2) following a procedure proposed by Maddala and Kim (1998). This procedure amounts to generating 10 000 \( mfp \) series - using an error resampling method - under the hypothesis that \( \gamma=0 \). In each replication, we estimate regression (3.1) based on the generated series and calculate the \( t \)-statistic for \( \gamma=0 \). Ordering the obtained \( t \)-statistics results in the small sample distribution of \( \gamma=0 \) relevant to the data set used. The same procedure is repeated for the hypothesis \( \gamma=1 \). The critical values for a one-sided test for \( \gamma=0 \) against \( \gamma>0 \) and \( \gamma=1 \) against \( \gamma<1 \) obtained from this bootstrapping procedure are reported at the bottom of table 1. The hypothesis of constant returns to scale to all inputs (\( \gamma=0 \)) can clearly not be rejected while constant returns to private sector inputs (\( \gamma=1 \)) can be rejected at about the 6% level of significance. The reason why the asymptotic distribution does not allow for a straightforward discrimination between the two alternative hypotheses can be found in the small sample bias of the estimate for \( \gamma \). Even with \( \gamma=0 \) imposed in generating \( mfp \) series, the median value of \( \gamma \) from the bootstrapped sample equals 0.40 (see table 1). Note that this value is remarkably close to our estimate of \( \gamma \) in regression (2). A similar - although smaller - upward bias is found under the hypothesis that \( \gamma=1 \).
As a final check, the model is estimated under both alternative restrictions. Imposing constant returns to private inputs (regression (5)) yields spurious results, i.e. the NLLS results are invalidated since the null hypothesis of no cointegration cannot be rejected. Imposing constant returns to all inputs in contrast (regression (6)) gives strong evidence in favour of a positive cointegrating relationship between multifactor productivity and public capital. A unit root in the residuals can now be rejected well below the 5% level of significance. The estimated output elasticity of public capital equals 0.29 and is highly significant.\(^9\) The average rate of return\(^10\) from public capital investment implied by regression (6) lies around 27%. Although this estimate is still higher than the values usually obtained for the return to private sector investments, it is clearly more plausible than the estimates obtained by Aschauer (1989a) and Munnell (1990a).

### 4.2. Robustness of the results

Underlying regressions (1)-(2) in table 2 is the Phillips-Hansen (1990) fully-modified least squares estimation procedure, which is explicitly designed to correct for the joint dependence between aggregate time series. As argued by Hulten and Schwab (1993), the fact that a production function is likely to be part of a larger system in which both input and output variables are endogenously determined makes the results from single-equation regressions potentially liable to simultaneous equation bias. The Phillips-Hansen estimation procedure deals with this kind of bias through semi-parametric corrections for serial correlation and endogeneity, yielding asymptotically median-unbiased estimators (Phillips and Hansen, 1990). As shown in table 2, the results from implementing this procedure are highly similar to the results from the Engle-Granger methodology, suggesting that there is no significant simultaneous equation bias.

Regressions (3)-(6) in table 2 follow from using revised estimates of the public capital stock. These estimates implement some small changes to the assumptions underlying the perpetual inventory method used by the Belgian Federal Planning Bureau. In most cases, the outcome of

\(^9\) The results from bootstrapping regression (6) to obtain the small sample distribution of \(e_r=0\) are not reported for (i) no small sample bias was detected and (ii) the small sample critical values were found to be somewhat lower compared to the asymptotic ones, leaving the conclusions from table 1 unaffected.

\(^10\) If the rate of return to public capital is set equal to its marginal product, it can be calculated as \(\partial Y/\partial G = e_r Y/G\).
### Table 2  Productivity effect of public capital in Belgium, robustness tests (1953-96)\(^a\)

<table>
<thead>
<tr>
<th>Dependent var.</th>
<th>Phillips-Hansen procedure(^b)</th>
<th>Longer lifetime of public assets</th>
<th>Shorter lifetime of public assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>constant</td>
<td>mfp(_t)</td>
<td>mfp(_t)</td>
<td>mfp(_t)</td>
</tr>
<tr>
<td></td>
<td>1.71 (6.18)</td>
<td>1.88 (7.04)</td>
<td>1.58 (4.83)</td>
</tr>
<tr>
<td>(\beta_{p18})</td>
<td>0.65 (7.25)</td>
<td>0.61 (10.38)</td>
<td>0.57 (5.82)</td>
</tr>
<tr>
<td>(e_g)</td>
<td>0.27 (5.48)</td>
<td>0.30 (5.19)</td>
<td>0.30 (5.20)</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>-0.02 (-0.05)</td>
<td>0.00 (0.81)</td>
<td>0.33 (0.81)</td>
</tr>
<tr>
<td>Adj. (R^2)</td>
<td>0.994</td>
<td>0.994</td>
<td>0.993</td>
</tr>
<tr>
<td>DW(^c)</td>
<td>0.99</td>
<td>1.00</td>
<td>0.79</td>
</tr>
<tr>
<td>Cointegration tests</td>
<td>DF</td>
<td>DF</td>
<td>DF</td>
</tr>
<tr>
<td>(t)-statistic(^d)</td>
<td>-3.72</td>
<td>-3.97</td>
<td>-4.01</td>
</tr>
<tr>
<td>5% critical values</td>
<td>-4.36</td>
<td>-3.94</td>
<td>-4.36</td>
</tr>
<tr>
<td>10% critical values</td>
<td>-4.05</td>
<td>-3.60</td>
<td>-4.05</td>
</tr>
</tbody>
</table>

Notes:  
\(^a\) \(t\)-statistics in parentheses.  
\(^b\) the truncated lag is set equal to 3. The results are insensitive to alternative lag values.  
\(^c\) DW denotes the Durbin-Watson \(d\)-statistic.  
\(^d\) critical values are taken from MacKinnon (1991).  
\(^r\) restricted coefficients.

such an inventory method is very sensitive to the choice of the period during which assets are kept in the capital stock. This choice concerns two assumptions: one about the average service life of assets and one about the distribution of liquidations around this average. Usually, the latter problem is adequately tackled by simulating retirement patterns based on some kind of mortality function. In Belgium, retirement distributions are compiled using a quasi-logistic bell-shaped mortality function. Fitting alternative, equally plausible mortality functions\(^{11}\) leads to highly similar results (not reported). In contrast, the size of the public capital stock is found to be very sensitive to alternative specifications of average service lives. Unfortunately, reliable information on this matter is hardly available (OECD, 1993). A simple comparison of

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\(^{11}\) Plausible alternatives are delayed-linear retirement, simultaneous exit and a wide range of bell-shaped mortality functions. Linear retirement and declining-balance functions suffer some important drawbacks and
service lives used in a number of OECD countries (see e.g. OECD, 1993) reveals a considerable variability, raising doubts about the reliability of the assumed lifetimes. Therefore, we construct two new public capital stock series. Regressions (3)-(4) are derived from assuming longer service lives, while regressions (5)-(6) are based on a shorter average lifetime. The results are found to be fairly insensitive to these alternative specifications. Only in the case of shorter asset lifetimes, the output elasticity of public capital is somewhat higher.

Note that, again using bootstrapped small sample distributions\textsuperscript{12}, the estimates of $\gamma$ still point to constant returns to all inputs in regression (3) ($t_{\gamma=0} = 0.81; t_{\gamma=1} = -1.64$) but to constant returns to private sector inputs in regression (5) ($t_{\gamma=0} = 2.61; t_{\gamma=1} = -0.50$). Regressions (4) and (6) are therefore estimated under the restrictions $\gamma=0$ and $\gamma=1$ respectively.

4.3. Causality tests from error-correction models

The most recurrent criticism states that the results from estimating a single-equation production function in no way guarantee causality to run from public investment to private sector productivity. In fact, the direction of causality might very well run the other way around, i.e. reverse causation (Eisner, 1991; Tatom, 1991). Public capital is argued to be a normal good, giving rise to the expectation that the public capital stock increases with society growing wealthier, i.e. with increasing multifactor productivity (Hurst, 1994). Alternatively, the levelling off in productivity growth implies a reduction in GDP growth, which in its turn negatively affects the budgetary position. To the extent that fiscal contraction is called for, public investment might by the first victim, reflecting “\textit{the political reality that it is easier to cut back or postpone investment spending than it is to cut current expenditures.”} (Oxley and Martin, 1991).

Performing a standard Granger causality test, Tatom (1993) indeed finds evidence that multifactor productivity growth uni-directionally causes public capital formation in the US. Granger causality tests try to detect the direction of causality between variables based on the analysis of lead-lag relationships. This boils down to estimating a \textit{vector autoregression}

\textsuperscript{12}The critical values from bootstrapping regressions (3) and (5) in table 2 are highly similar to the ones reported in table 1 and are therefore not repeated here.
(VAR) and testing whether lags of one variable enter significantly into the equation for the current value of the other.

The issue is somewhat more complicated in a cointegrating framework, though. Although there exists a long-run equilibrium, the system does not need to be in equilibrium at any instant of time, i.e. in the short-run there may be disequilibrium as measured by the error term from equation (3.1). The Granger representation theorem (Engle and Granger, 1987) states that the fact that a set of variables is cointegrated implies that there must exist a valid error-correction mechanism that drives the variables back to equilibrium. A simple version of such an error-correction model (ECM) is given by:

\[
\begin{align*}
\Delta mfp_t &= \alpha_{10} + \sum_{l} \alpha_{12}(l)\Delta mfp_{t-l} + \sum_{l} \alpha_{13}(l)\Delta g_{t-l} + \sum_{l} \alpha_{14}(l)\Delta \mu_{t-l} + \sum_{l} \alpha_{15}(l)\sum_{i} (s_k l_{t-i} + s_i l_{t-i}) + \xi_{1t}, \\
\Delta g_t &= \alpha_{20} + \sum_{l} \alpha_{22}(l)\Delta mfp_{t-l} + \sum_{l} \alpha_{23}(l)\Delta \mu_{t-l} + \sum_{l} \alpha_{24}(l)\Delta \mu_{t-l} + \sum_{l} \alpha_{25}(l)\sum_{i} (s_k l_{t-i} + s_i l_{t-i}) + \xi_{2t},
\end{align*}
\]

Besides pure innovations and autoregressive components, the short-run dynamics of at least one of the variables is influenced by deviations from long-run equilibrium ($\mu_{t-1}$). In this context, $\alpha_{11}$ and $\alpha_{21}$ can be interpreted as speed of adjustment parameters. Large values of these adjustment parameters imply high responsiveness to equilibrium errors.

Since at least one of the adjustment parameters must significantly differ from zero when variables are cointegrated, the standard Granger causality test is inappropriate to conduct causality tests for it omits an important part of the true data generating process. Even more important to note is that the error-correction representation allows for causality between public capital and multifactor productivity resulting from their long-run equilibrium relationship. Causality will run from public capital to multifactor productivity if $\alpha_{11}$ is significantly different from zero. In this case, part of the change in multifactor productivity is generated by the need to move into alignment with the trend in public capital, as implied by the cointegrating relationship. Clearly, such a causality relationship cannot be detected by the standard Granger test procedure.
Table 3 reports the results of causality tests based on estimates of an error-correction model. A lag structure of one was found to be optimal in all specifications. Regressions (1)-(2) take the residuals from regression (6) in table 1 as equilibrium errors. Regressions (3)-(4) are based on regression (2) from table 2, which implements the Phillips-Hansen estimation procedure. Regressions (5)-(6) finally, are based on regression (6) from table 2, which was derived using a shorter average lifetime of public assets.

<table>
<thead>
<tr>
<th>Dep. var.</th>
<th>From Engle-Granger procedure</th>
<th>From Phillips-Hansen procedure</th>
<th>Assuming a shorter public asset lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.02 (2.43)</td>
<td>-0.00 (-0.85)</td>
<td>-0.02 (-2.54)</td>
</tr>
<tr>
<td>$\hat{\mu}_{t-1}$</td>
<td>-0.67 (-3.90)</td>
<td>-0.08 (-1.62)</td>
<td>-0.66 (-3.97)</td>
</tr>
<tr>
<td>$\Delta mfp_{t-1}$</td>
<td>-0.10 (-0.62)</td>
<td>-0.03 (-0.76)</td>
<td>-0.11 (-0.67)</td>
</tr>
<tr>
<td>$\Delta g_{t-1}$</td>
<td>1.06 (4.09)</td>
<td>1.01 (14.53)</td>
<td>1.00 (3.90)</td>
</tr>
<tr>
<td>$\Delta p(18)_{t-1}$</td>
<td>-0.06 (-0.29)</td>
<td>0.04 (0.75)</td>
<td>-0.02 (-0.11)</td>
</tr>
<tr>
<td>$\Delta (s_k + s_l)_{t-1}$</td>
<td>0.79 (2.29)</td>
<td>0.08 (0.84)</td>
<td>0.75 (2.22)</td>
</tr>
</tbody>
</table>

| Adj. R² | 0.42 | 0.91 | 0.42 | 0.91 | 0.46 | 0.91 |
| DW      | 2.10 | 1.85 | 2.11 | 1.92 | 2.09 | 1.88 |

Notes:  

1. $t$-statistics in parentheses.  
2. DW denotes the Durbin-Watson $d$-statistic.

The results clearly show that the critique of reverse causation does not hold within the cointegrating framework. Causality from public capital to multifactor productivity derives from both the adjustment towards long-run equilibrium and short-run dynamics in all specifications. Multifactor productivity is found not to cause public capital.

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13 The lag structure was determined starting with a maximum lag length of 4. Lags were then eliminated using F-tests and system specification tests.
5. CONCLUSION

This paper analyses the impact of public capital on private sector productivity in Belgium over the period 1953-1996. Three important problems have been addressed. First, given the stochastic non-stationary behaviour of the variables traditionally included in the analysis, the estimates are based on cointegration techniques. Second, technology is not simply taken to follow some deterministic trend. Instead, an alternative proxy is constructed by applying a perpetual inventory method to the total number of patents granted by the US Patent and Trademark Office. Third, the direction of causality is tested for by using an error-correction model. In contrast to standard Granger causality tests, this approach allows for causality resulting from the estimated long-run equilibrium.

The estimates reveal a significant positive relationship between public capital and private sector productivity. The estimated output elasticity of capital lies around 0.29. In our ‘best’ specification, the results point to a cointegrating relationship. The results are shown to be fairly insensitive to corrections for a possible endogeneity bias and to small changes in the assumptions underlying the construction of public capital stock data. Moreover, the results from estimating an error-correction model show that causality runs from public capital to multifactor productivity.
APPENDIX A: UNIT ROOT TESTS

We test for the order of integration of the variables by means of two alternative testing procedures, i.e. augmented Dickey-Fuller (ADF) tests and Kwiatkowski et al (KPPS) tests. The first test takes a unit root as the null hypothesis, the second has (trend-) stationarity as the relevant null hypothesis.

Table 4 Augmented Dickey-Fuller unit root test (1953-96)

<table>
<thead>
<tr>
<th>Series</th>
<th>$\tau^b_t$</th>
<th>$\tau^c_t$</th>
<th>Series</th>
<th>$\tau^b_t$</th>
<th>$\tau^c_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$mfp$</td>
<td>$p=0$</td>
<td>-0.16</td>
<td>$\Delta mfp$</td>
<td>$p=0$</td>
<td>-6.83*</td>
</tr>
<tr>
<td>$g$</td>
<td>$p=1$</td>
<td>-1.18</td>
<td>$\Delta g$</td>
<td>$p=0$</td>
<td>-1.63</td>
</tr>
<tr>
<td>$(s_kl^t+s_l^t)$</td>
<td>$p=1$</td>
<td>-2.28</td>
<td>$\Delta (s_kl^t+s_l^t)$</td>
<td>$p=0$</td>
<td>-3.71*</td>
</tr>
<tr>
<td>$p_{18}$</td>
<td>$p=2$</td>
<td>-2.70</td>
<td>$\Delta p_{18}$</td>
<td>$p=0$</td>
<td>-2.64</td>
</tr>
<tr>
<td>$p_{18s}$</td>
<td>$p=2$</td>
<td>-2.64</td>
<td>$\Delta p_{18s}$</td>
<td>$p=0$</td>
<td>-2.52</td>
</tr>
<tr>
<td>$p_{10}$</td>
<td>$p=1$</td>
<td>-2.82</td>
<td>$\Delta p_{10}$</td>
<td>$p=0$</td>
<td>-2.25</td>
</tr>
<tr>
<td>$gi$</td>
<td>$p=0$</td>
<td>-1.00</td>
<td>$\Delta gi$</td>
<td>$p=0$</td>
<td>-5.76*</td>
</tr>
<tr>
<td>$pi$</td>
<td>$p=0$</td>
<td>-3.06</td>
<td>$\Delta pi$</td>
<td>$p=0$</td>
<td>-8.87*</td>
</tr>
</tbody>
</table>

Notes: 

- The lag length is denoted by $p$.
- Based on regression $\Delta y_t = \alpha + \theta x_{t-1} + \sum_1^p \beta \Delta y_{t-1} + \gamma x_t + \epsilon_t$. The MacKinnon critical values for the rejection of a unit root equal -3.52 and -3.19 at the 5% and 10% levels of significance respectively.
- Based on regression $\Delta y_t = \alpha + \theta x_{t-1} + \sum_1^p \beta \Delta y_{t-1} + \epsilon_t$. The MacKinnon critical values equal -2.93 and -2.60 at the 5% and 10% levels respectively.
- Significant at the 5% level.

The results from running ADF tests clearly point to the presence of a unit root in the levels of all variables (see the upper part of table 4). First-differencing the series removes the non-stationary components in the case of multifactor productivity and the bundle of private sector factors of production. However, the public capital stock and patent stock measures appear to remain non-stationary after taking first-differences.

14 In order to pin down the appropriate number of lagged differences, we started with a relatively long lag length and then used traditional t-tests and/or F-tests to assess whether lags could be omitted. After having selected a tentative specification, additional tests to check for autocorrelation and heteroskedasticity were conducted.

15 Since standard ADF unit root tests are based on the assumption of at most a single unit root, one should in fact start with testing the null hypothesis of a unit root in the first-differences (I(2)-ness in levels), conditional on the fact that the series is at least I(1) in levels. If the null hypothesis is rejected the next step should be to test for I(1)-ness with I(0) as the alternative (Dickey and Pantula, 1987). This means that in order to be theoretically correct, table 4 should be read from the right to the left.
The finding of different orders of integration, i.e. I(1) and I(2), seems to rule out the possibility of cointegration between some of the variables. However, the results from the ADF unit root tests should not necessarily be taken for granted. Capital stock data are constructed based on a perpetual inventory method, implying:

\[ x_t = \sum_{i=0}^{m} s_i I_{t-i} \]  \hspace{1cm} (A.1)

with \( x_t \) denoting a capital stock measure, \( s_i \) the share of assets installed in period \( t-i \) that are still productive at time \( t \), \( I_{t-i} \) the investments in period \( t-i \) and \( m \) the maximum lifetime of the asset. Backward substitution yields:

\[ x_t = I_t + s_1 x_{t-1} + \sum_{i=2}^{\infty} \Phi_i x_{t-i} \]  \hspace{1cm} (A.2)

with \( \Phi_i \) being some complex function of \( s \), constituting the parameters of a stable autoregressive process. Equation (A.2) shows that the process generating capital stock series equals the sum of a stationary auto-regressive (AR) process and the process generating investments. The bottom part of table 4 provides clear evidence that public sector investment \((g_i)\) and the number of granted patents \((p_i)\) are both I(1), i.e. first-differences are stationary. This ‘proofs’ that the processes generating public capital and patent stock measures should be I(1) as well.\(^{16}\)

A possible reason why non-stationarity of the first-differences of some of the variables cannot be rejected can be seen from rewriting equation (A.1). Assuming that \( I_t \) is generated by a unit root process:

\[ I_t = I_{t-1} + e_t, \]  \hspace{1cm} (A.3)

it follows from (A.1) that:

\[ x_t = x_{t-1} + \sum_{i=0}^{m} s_i e_{t-i}. \]  \hspace{1cm} (A.4)

Equation (A.4) shows that the data-generating process of capital stock data contains important moving average (MA) components, potentially causing a bias in the results reported above. Since any MA model with an invertible polynomial can be represented by an infinite AR

\[^{16}\text{For a similar argument, considering the process generating the change in the private sector capital stock, see Ford and Poret (1991b).}\]
model, the ADF test can easily be extended to allow for such processes\textsuperscript{17}. However, including a long lag length may seriously reduce the power of the test. A useful alternative in this case is the Phillips-Perron test. Rather than including lagged differences, Phillips and Perron (1988) allow serially correlated and heteroscedastic errors but adjust Dickey-Fuller $t$-statistics based on heteroscedasticity and autocorrelation consistent (Newey-West) standard errors. The main advantage of this procedure is that it does not require the estimation of additional autoregressive parameters, which consumes a larger number of degrees of freedom. The results (not reported) are however not fundamentally different from the ADF tests.

An important shortcoming of ADF and Phillips-Perron tests is that a unit root is taken to be the null hypothesis. Since both tests generally have low power against possible alternatives, a unit root is only rejected if there is considerable evidence against it. Kwiatkowski \textit{et al.} (1992) have developed an alternative test which takes (trend-) stationarity as the null hypothesis. The idea is to express a series as the sum of a deterministic trend, a random walk and a stationary error term. The null hypothesis can be rejected if the variance of the random walk component is significantly different from zero. In order to correct for serial correlation in the error terms, Kwiatkowski \textit{et al.} (1992) proceed along the lines suggested by Phillips and Perron (1990).

Table 5 reports the results of the KPSS test under two alternative null hypotheses, i.e. stationarity around a linear trend and level-stationarity. Following Kwiatkowski \textit{et al.} (1992) we consider values of $q$ (needed in the calculation of Newey-West standard errors) up to eight, which is a compromise between size and power of the test\textsuperscript{18}. The upper part of table 5 clearly shows that none of the variables is stationary in levels. With a lag truncation parameter set equal to four, we can also reject the null hypothesis of trend-stationarity at the 5% level of significance for all variables except the bundle of private sector factors of production for which trend-stationarity can only be rejected at the 10% level. For higher values of $q$, the evidence in favour of a unit root is weaker. Given the low power of the test for high values of $q$, it seems not unreasonable to use a significance level higher than the usual 5%, though. The lower part of table 5 provides evidence that the first-differences of all variables are trend or

\textsuperscript{17} In a finite sample, an AR($\infty$) can be approximated by an AR($n$) model with $n$ appropriately large.

\textsuperscript{18} In a sample of about 50 observations, the test has reasonable power for $q=4$, while in order for the test to have more or less correct size, $q$ should be raised to about 12.
level stationary. Only for the differences of the public capital stock, this evidence is weak, i.e. level-stationarity is only rejected in specifications with \( q \) larger than 4.

**Table 5** Kwiatkowski et al. unit root test (1953-96)

<table>
<thead>
<tr>
<th></th>
<th>Lag Truncation parameter</th>
<th></th>
<th></th>
<th>Lag Truncation parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 2 4 6 8</td>
<td></td>
<td>0 2 4 6 8</td>
<td></td>
</tr>
<tr>
<td>( mfp )</td>
<td>0.95* 0.35* 0.23* 0.18* 0.15*</td>
<td>( mfp )</td>
<td>4.29* 1.51* 0.95* 0.71* 0.58*</td>
<td></td>
</tr>
<tr>
<td>( g )</td>
<td>0.89* 0.32* 0.21* 0.17* 0.14</td>
<td>( g )</td>
<td>4.40* 1.54* 0.97* 0.72* 0.59*</td>
<td></td>
</tr>
<tr>
<td>( (s_{ik+sl_i}) )</td>
<td>0.47* 0.17* 0.12 0.10 0.09</td>
<td>( (s_{ik+sl_i}) )</td>
<td>4.27* 1.52* 0.97* 0.73* 0.61*</td>
<td></td>
</tr>
<tr>
<td>( p^{18} )</td>
<td>0.89* 0.31* 0.20* 0.15* 0.13</td>
<td>( p^{18} )</td>
<td>4.28* 1.50* 0.94* 0.71* 0.58*</td>
<td></td>
</tr>
<tr>
<td>( p^{18s} )</td>
<td>0.96* 0.33* 0.21* 0.16* 0.14</td>
<td>( p^{18s} )</td>
<td>4.11* 1.46* 0.92* 0.70* 0.58*</td>
<td></td>
</tr>
<tr>
<td>( p^{10} )</td>
<td>0.77* 0.27* 0.18* 0.14 0.13</td>
<td>( p^{10} )</td>
<td>3.81* 1.38* 0.90* 0.69* 0.58*</td>
<td></td>
</tr>
<tr>
<td>( \Delta mfp )</td>
<td>0.12 0.12 0.11 0.11 0.11</td>
<td>( \Delta mfp )</td>
<td>0.64* 0.53* 0.43 0.38 0.34</td>
<td></td>
</tr>
<tr>
<td>( \Delta g )</td>
<td>0.93* 0.34* 0.22* 0.18* 0.15*</td>
<td>( \Delta g )</td>
<td>2.18* 0.78* 0.50* 0.38 0.32</td>
<td></td>
</tr>
<tr>
<td>( \Delta (s_{ik+sl_i}) )</td>
<td>0.15* 0.09 0.08 0.07 0.08</td>
<td>( \Delta (s_{ik+sl_i}) )</td>
<td>0.15 0.09 0.08 0.07 0.08</td>
<td></td>
</tr>
<tr>
<td>( \Delta p^{18} )</td>
<td>0.48* 0.20* 0.14 0.12 0.11</td>
<td>( \Delta p^{18} )</td>
<td>0.52* 0.22 0.15 0.12 0.10</td>
<td></td>
</tr>
<tr>
<td>( \Delta p^{18s} )</td>
<td>0.40* 0.17* 0.12 0.10 0.10</td>
<td>( \Delta p^{18s} )</td>
<td>0.51* 0.21 0.15 0.13 0.12</td>
<td></td>
</tr>
<tr>
<td>( \Delta p^{10} )</td>
<td>0.42* 0.18* 0.13 0.11 0.11</td>
<td>( \Delta p^{10} )</td>
<td>0.59* 0.25 0.18 0.15 0.14</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

- Asymptotic critical values at the 10%, 5% and 1% level of significance are respectively equal to 0.119, 0.146 and 0.176 (see Kwiatkowski et al., 1992).
- Asymptotic critical values at the 10%, 5% and 1% level of significance are respectively equal to 0.347, 0.463 and 0.574 (see Kwiatkowski et al., 1992).
APPENDIX B: RELATION BETWEEN PATENT AND R&D CAPITAL STOCKS

Without the ambition to provide a thorough analysis of the relationship between R&D and patents, we show that our patent stock variable $p(18)$ is strongly correlated with the ‘world’ R&D capital stock. The latter variable, $rd(w)$, is calculated as the natural logarithm of the sum of business sector and government R&D capital stocks of the US, Germany, France, the UK and Japan. R&D capital stocks are calculated from R&D expenditure data (Source: OECD’s Basic Science and Technology Statistics) following the methodology of Coe and Helpman (1995). Data on R&D expenditures are available for all five countries from 1970 onward. The relationship between the two measures of technological progress is estimated using cointegration analysis. Instead of the commonly used two-step (Engle-Granger, 1987) procedure, which relies on low power DF unit root tests, we use the single-step procedure that checks for cointegration based on the $t$-ratio of the error-correction term in a conditional error-correction model. This single-step procedure has been shown to be much more efficient (see e.g. Kremers et al., 1992). Especially in the small sample under investigation (1970-1995), this might be an important advantage over the two-step procedure.

Estimating an ECM(1) yields the following results (with $t$-values in parentheses),

$$
\Delta p(18)_t = 0.92 - 0.29(p(18)_{t-1} - 0.44rd(w)_{t-1}) + 0.08\Delta p(18)_{t-1} - 1.14\Delta rd(w)_{t-1} \quad (B.1)
$$

(4.66) (-4.85) (25.34) (0.47) (-3.75)

$R^2=0.87 \quad DW=2.23$

The $t$-value (-4.85) of the error-correction term is highly significant, pointing to a cointegrating relationship between our patent stock variable and the ‘world’ R&D capital stock.
REFERENCES


Balanced growth and public capital
- An empirical analysis with I(2)-trends in capital stock data -

GERDIE EVERAERT*

August 2000

ABSTRACT: In the large literature that tries to estimate the contribution of public capital formation to economic growth, two potentially serious problems have remained largely unaccounted for. First, technological progress, which is one of the most important determinants of long-run growth, is mostly ignored or improperly modelled. Second, due to slow capital accumulation, the capital stock series usually included in the empirical analysis often exhibit I(2)-components, implying the conditions underlying the standard cointegrated VAR methodology to be unsatisfied. This paper uses the long-run growth properties of the neoclassical model to identify the rate of technological growth as a common stochastic trend in the data. The I(2)-trends in capital stocks are being used to model the medium-term adjustment path towards the long-run steady-state of output which is determined by private and public investment behaviour. Using Belgian data for the period 1953-96, the analysis supports Aschauer’s hypothesis that the decline in public capital investment has lowered the balanced growth path of real output. In contrast to Aschauer’s results, the output elasticity of public capital is found to be only a fraction 0.4 of the output elasticity of private capital.

KEYWORDS: Public capital, economic growth, cointegration, I(2).

1. INTRODUCTION

In a series of influential papers Aschauer (1989a, 1989b, 1989c) shows that the decline in public capital observed in a major part of the OECD countries since the early 1970s may be, to a large extent, responsible for the slowdown of productivity growth, which set in at about the same time. Expanding the conventional aggregate Cobb-Douglas production function with the public capital stock, Aschauer provides empirical evidence that a 1% decrease in the ratio of public to private capital decreases US multifactor productivity by 0.39%.

In the large literature that sprung from Aschauer’s work, a lot of possible defects in the initial methodology have been identified. One major problem emerges from the fact that all variables included in the production function show stochastic non-stationary behaviour. The finding of a unit root makes Aschauer’s results, derived using level data, suspicious due to possible spurious correlation (see e.g. Tatom, 1991).

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In trying to deal with this kind of non-stationarity, some authors have proceeded to check for cointegration in the production function estimates using the residual-based ADF method in the sense of Engle and Granger (1987) or the maximum likelihood estimation procedure developed by Johansen (1988). Unfortunately, the results point in opposite directions. On the one hand, Tatom (1991) finds no evidence of cointegration in the US. Sturm and de Haan (1995) come to the same conclusion for both the US and the Netherlands. On the other hand, Bajo-Rubio and Sosvilla-Rivero (1993) find a clear cointegrating relationship in Spain. Applying the Johansen technique on data for the US, the UK, France and Germany, Clarida (1993) also finds a strong impact of public capital on multifactor productivity.

One problem that has been unremittingly ignored in all these studies is the treatment of the underlying rate of technological progress, which is an important determinant of economic growth. In fact, from the neoclassical growth model we learn that the per capita growth rates of output and capital are exclusively determined by the rate of technological progress once the economy has reached its balanced growth path. Therefore, direct estimation in a cointegration framework of a production function omitting technology yields coefficients that cannot be interpreted as output elasticities, i.e. the coefficients on capital stocks will be biased upwards for in the long-run capital and output grow at more or less the same rate.

Note that this remark is closely related to the critique that the strong result reported by Aschauer might be caused by reverse causation. This argument relies on the assumption that public investment is an endogenous variable, i.e. the public capital stock grows with increasing output. To the extent that technological shocks are first reflected in higher economic growth, which in its turn affects the accumulation of factors of production, omitting technological growth may indeed induce an important reverse causation problem, reflected in an upward bias on the coefficient of public capital.

The reason why technological progress is easily omitted is that it is not a directly measurable variable. Some authors, e.g. Sturm and de Haan (1995), try to capture technological progress by including a linear trend in the production function. However, this implies that technological progress is modelled as a purely deterministic process, which is clearly in contradiction with the results from unit root tests, indicating a stochastic rather than a linear long-run trend in output. In an attempt to deal with this problem, Everaert and Heylen (2000) use patent statistics – cumulated into a stock variable - as an approximation. Including this
measure, they find a cointegrating relationship between output, private factors of production and public capital with an output elasticity of public capital around 0.29 and causality running from public capital to output. However, an important defect of this approach is the sensitivity of the results to the choice - to some extent arbitrary - of patent lifetime that one has to make to accumulate patent stock data.

Crowder and Himarios (1997) have proposed an alternative approach that relies on the neoclassical ideas that (i) technological progress determines the long-run growth of output and capital stocks and (ii) the production function is a ‘period-by-period’ constraint that describes the short- to medium-term behaviour of the variables. Empirically, the approach boils down to analysing whether output and capital stocks cointegrate subject to the balanced growth restrictions from the neoclassical model, i.e. capital/output ratios are stationary stochastic processes. If the restrictions are valid, the common stochastic trend in the data can be identified as technological progress. After filtering out this common trend from output and capital stock series, the production function can be estimated from the stationary data as a short-run constraint. Applying this methodology to US data, the authors find a strong confirmation of Aschauer’s result.

Although the feature that technological progress is determined endogenously from the long-run behaviour of data is very appealing, the approach has a number of drawbacks. A first potential problem is the implicit assumption that the economy is on - or moves to - a fixed balanced growth path. Shifts in investment behaviour imply shifts in the steady-state of the economy, though. If a structural reduction in public investment has indeed occurred, the public capital/output ratio will fail to be a stationary stochastic process. A second problem – also present in other studies using cointegrating analysis - is caused by the time series properties of capital stock data. The smooth adjustment of the capital stock to its steady-state level implies that the growth rate of the capital stock exhibits non-stationary behaviour in the relatively small samples usually available. Although the perpetual inventory method used in their construction implies that capital stock series should be I(1) in theory, it might therefore be statistically more appropriate to treat them as I(2).

The purpose of this paper is to analyse whether the drastic reduction of public investment in Belgium during the fiscal consolidation episodes of the 1980s and 1990s has reduced the long-run output capacity of the Belgian economy. We use data for the period 1953-96. The
analysis is inspired by the methodology proposed by Crowder and Himarios in that the long-run growth properties of the neoclassical model are being used to identify the rate of technological growth as a common stochastic trend in the data. The empirical development of this idea differs in three crucial respects, though. First, instead of trying to test/impose the restriction that capital/output ratios are stationary stochastic processes, we test a more general balanced growth relation between output, investments and capital stock growth rates implied by the neoclassical growth model. Second, the output elasticities do not need to be estimated from a short-run production function in a second step but can be calculated from the long-run equilibrium relation. Third, the analysis takes into account the smooth adjustment of capital stocks series by modelling them as I(2) variables.

The remainder of the paper is organised as follows. The next section discusses the implications of a simple neoclassical growth model with stochastic technological progress in terms of cointegrating relations and common trends driving the variables and briefly confronts these implications with the data. Section 3 outlines the cointegrated VAR methodology both under the assumption of I(1) and I(2) processes being present in the data vector. Section 4 investigates the integration and cointegration properties of a VAR including output and capital stock series. Section 5 transforms the system into an I(1) equivalent including output, investments and capital stock growth rates and discusses its properties. The final section summarises and outlines some directions for future research.

2. AN ECONOMETRIC REPRESENTATION OF THE NEOCLASSICAL GROWTH MODEL

One of the nice features of neoclassical growth models is that they have strong implications concerning the long-run behaviour of the economy. It is a well-known result that once the economy has converged to its steady-state growth path, the growth rates of per capita output, investment, consumption and capital stocks should all be equal to the exogenous rate of technological progress. This common deterministic trend makes the ‘great ratios’ of consumption, investment and capital to output constant along the balanced growth path. King et al. (1991) point out that when uncertainty is added to the long-run behaviour of the economy - i.e. technological progress has a stochastic rather than a deterministic data generating process - output, consumption, investment and capital stocks exhibit common stochastic trends, implying the ‘great ratios’ to become stationary stochastic processes.
Figure 1  Data in levels and first-differences (Belgium, 1953-96)

Sources: OECD statistical compendium 1998/1 and Belgian Federal Planning Bureau.

Figure 1 plots the logarithms of real output ($Y$), gross real private capital ($K$), gross real public capital ($G$), real private investment ($Ik$) and real public investment ($Ig$). At first sight output
and capital stocks share a broadly similar upward trend. Investments are clearly more volatile. The left hand part of figure 2 plots the private capital/output ratio (ln(K)-ln(Y)) and the public capital/output ratio (ln(G)-ln(Y)). Neither ratio shows strong signs of mean reverting behaviour. Since the neoclassical growth model only predicts stationarity when the economy is on its steady-state growth path, this finding should not necessarily be interpreted as a falsification of the model, though. One possible explanation for the apparent non-stationarity of capital/output ratios is that the economy has been hit by shocks causing permanent shifts in the steady-state.

In order to fix ideas about which variables may cause the steady-state to move, this section first outlines a simple neoclassical growth model with stochastic technological growth. The implications of this model are then interpreted in terms of cointegrating relations and common trends driving the variables.

Consider the following Cobb-Douglas production function, characterised by constant returns to scale over all inputs and diminishing returns to private and public capital:

\[ Y_t = K_t^\alpha G_t^\beta (AL_t)^{(1-\alpha-\beta)}, \quad 0 < \alpha + \beta < 1, \alpha > 0, \beta \geq 0, \quad (2.1) \]

with \( Y, K \) and \( G \) as defined above and \( L \) and \( A \) denoting labour input and labour-augmenting technological progress, respectively. Uncertainty is introduced by assuming that effective labour \((AL_t)\) is generated by the following logarithmic random walk with drift:

\[ \ln(\text{AL}_t) = g + \ln(\text{AL}_{t-1}) + \varepsilon_{\lambda t}. \quad (2.2) \]

The drift term \( g \) determines the average rate of growth in \( AL_t \). Temporary deviations from this average are captured by the error term \( \varepsilon_{\lambda t} \). Given that \( K \) and \( G \) are generated as:

\[ \Delta K_t = Ik_t - \delta_k K_t \quad \text{and} \quad \Delta G_t = Ig_t - \delta_g G_t, \quad (2.3) \]

with \( \delta_k \) and \( \delta_g \) denoting depreciation rates, it is straightforward to show that for given levels of investment, the steady-state capital stocks – for which is required that \( \Delta\ln(K/AL) = \Delta\ln(G/AL) = \Delta\ln(Y/AL) = 0 \) - should satisfy\(^2\):

\[ \ln(K_t^*) = \ln(Ik_t) - \ln(g + \delta_k) \quad \text{and} \quad \ln(G_t^*) = \ln(Ig_t) - \ln(g + \delta_g) \quad (2.4) \]

\(^1\) For simplicity, the rate of growth in effective labour will be called technological progress in the remainder of the paper.

\(^2\) Steady-state values of variables are indicated with a *. 
Private and public investment are assumed to be generated as:

\[
\ln(I_k) = \ln(S_k) + \ln(Y)
\]

and

\[
\ln(I_g) = \ln(S_g) + \ln(Y)
\]

(2.5)

where \(S_k\) and \(S_g\) denote private and public investment rates, respectively, which are given by the stochastic processes:

\[
\ln(S_k) = \ln(S_{k-1}) + \epsilon_{sk,t}
\]

and

\[
\ln(S_g) = \ln(S_{g-1}) + \epsilon_{sg,t}
\]

(2.6)

Inserting (2.5) in (2.4) shows that in the steady-state the economy should satisfy:

\[
\ln(K^*) - \ln(Y^*) = \ln(S_k) - \ln(g + \delta_k)\quad \text{and} \quad \ln(G^*) - \ln(Y^*) = \ln(S_g) - \ln(g + \delta_g)
\]

(2.7)

Equation (2.7) shows that shocks to technology (\(\epsilon_{At}\)) have a ‘balanced’ impact on output and capital stocks, leaving the equilibrium ratios of capital to output unaffected. Possible sources of a permanent upward (downward) shift in the steady-state capital to output ratio are (i) a positive (negative) shock to the investment rates (\(\epsilon_{sk,t}\) and \(\epsilon_{sg,t}\)) and (ii) a lower (higher) average growth rate of technology (\(g\)). The positive effect of positive shocks to investment rates on capital/output ratios derives from the model’s assumption of diminishing marginal products to private and public capital. In this case, higher investment raises output less than it raises the capital stock. A lower average growth rate of effective labour increases the equilibrium capital stocks for given level of investments (see equation (2.4)), implying higher capital/output ratios.

**Figure 2** Private and public capital/output ratios and investment rates (Belgium, 1953-96)

![Graphs showing capital/output ratios and investment rates](image)

**Sources:** See figure 1.
Equation (2.7) now allows us to interpret the apparent non-stationary behaviour of capital/output ratios. During the 1960s, weak upward trends in investment rates (see figure 2, right hand side) counteract an increase in the average rate of productivity growth, leaving capital/output ratios more or less constant. In the mid 1970s a significant shift in the steady-state occurs. Combined with more or less stable investment rates, the productivity slowdown has induced a strong increase in both the private and the public capital/output ratio. The further increase in \( \ln(K) - \ln(Y) \) results from a very strong increase in the private sector investment rate over the period 1984-90. The strong decline in \( \ln(G) - \ln(Y) \) in contrast is caused by a significant reduction in public investment from the early 1980s onward, caused mainly by the drastic fiscal consolidation programs of the 1980s and the 1990s.

Due to the shifts in the steady-state of the economy, capital/output ratios clearly fail to be of any guidance as equilibrium relations in the subsequent cointegrating analysis. Alternatively, the steady-state level of output \( \ln(Y^*) \) can be expressed as a function of steady-state capital stocks by substituting out effective labour from the production function (2.1) using the standard neoclassical growth model’s result that once the economy has converged to its steady-state growth path, output grows at a rate equal to the rate of growth in effective labour, i.e. \( \ln(Y^*) - \ln(AL) \) is constant \( \omega^* \) along the balanced growth path:

\[
\ln(Y^*) = -\frac{1-\alpha-\beta}{\alpha+\beta}\omega^* + \frac{\alpha}{\alpha+\beta} \ln(K^*) + \frac{\beta}{\alpha+\beta} \ln(G^*)
\]  

(2.8)

Equivalently, steady-state output can be expressed as a function of investments by substituting out \( \ln(K^*) \) and \( \ln(G^*) \) from (2.4):

\[
\ln(Y^*) = \pi^* + \frac{\alpha}{\alpha+\beta} \ln(I_k) + \frac{\beta}{\alpha+\beta} \ln(I_g)
\]

(2.9)

with \( \pi^* = -\left( \frac{\alpha}{\alpha+\beta} \ln(g+\delta_k) + \frac{\beta}{\alpha+\beta} \ln(g+\delta_g) + \frac{1-\alpha-\beta}{\alpha+\beta} \omega^* \right) \)

Equations (2.8) and (2.9) assume that capital stocks are at their steady-state level. Since capital takes time to accumulate, these equilibrium relations will only hold in the (very) long run. A medium term equilibrium relation can be derived (see appendix A) as:

\[
\ln(Y_t) = \ln(Y^*) - \frac{\alpha}{g+\delta_k} [\Delta \ln(K_t) - g] - \frac{\beta}{g+\delta_g} [\Delta \ln(G_t) - g]
\]

(2.10)
Equation (2.10) states that the medium term production capacity equals the long-run production capacity if the economy is on its balanced growth path, i.e. \( \Delta \ln(K_t) = g \) and \( \Delta \ln(G_t) = g \). If the economy is outside its steady state, the medium-term production capacity will converge towards the equilibrium with each period’s equilibrium error being proportional to \( (\Delta \ln(K) - g) \) and \( (\Delta \ln(G) - g) \). If the public capital stock for instance is below the steady-state level implied by the government’s investment behaviour, the growth rate of the public capital stock will exceed its steady-state growth rate, \( \Delta \ln(G_t) > g \), implying \( \ln(Y_t) < \ln(Y^*_t) \).

Notice that – like the capital/output ratios - the long-run and medium-term equilibrium conditions in (2.9) and (2.10), respectively, fail to be stable cointegrating relations in the presence of shocks shifting the steady-state of the economy. Both shocks to investment rates, i.e. \( \varepsilon_{Sk,t} \) and \( \varepsilon_{Sg,t} \), and shifts in the average rate of productivity growth imply shifts in the constant term \( \pi^* \). Anticipating the empirical analysis in section 5, the impact on equilibrium output of the upward shift in the private investment rate occurring in the 1980s appears to have been cancelled out by the negative shock to the public investment rate over more or less the same period, leaving \( \pi^* \) unaffected. In fact this implies that \( \varepsilon_{Sg,t} \approx - \alpha/\beta \varepsilon_{Sk,t} \). Shifts in the average rate of productivity growth will be modelled by allowing for a broken linear trend.

Inspired by the ‘scenario analysis’ introduced in Juselius (1999), the remainder of this section interprets the implications of the model in terms of cointegrating relations and common trends driving the variables. Two distinct cases, corresponding to the long-run equilibrium under (2.8) and the medium-term equilibrium under (2.10) respectively, are considered.

**Case 1: Capital stock series are I(1).** In the (very) long run, capital stocks converge towards their steady-state level, implying \( \Delta \ln(K_t) \) and \( \Delta \ln(G_t) \) to be stationary processes around a constant level \( g \). Equation (2.8) shows that in this case, conditional on the fact that \( \omega^* \) is constant, there exists a stable long-run relation between output and capital stocks, i.e. \( (\ln(Y_t) - b_1 \ln(K_t) - b_2 \ln(G_t)) \sim I(0) \). With three variables and one cointegrating relation, two common trends are driving the system (see e.g. King et al., 1991). This data generating process can be represented as:

\[
\begin{bmatrix}
\ln(Y_t) \\
\ln(K_t) \\
\ln(G_t)
\end{bmatrix} = 
\begin{bmatrix}
d_{11} & d_{12} \\
d_{21} & d_{22} \\
d_{31} & d_{32}
\end{bmatrix}
\begin{bmatrix}
\sum u_{1t} \\
\sum u_{2t}
\end{bmatrix} + 
\begin{bmatrix}
g_1 \\
g_2 \\
g_3
\end{bmatrix} t + X_t.
\]  

(2.11)
The linear growth in the data is accounted for by the trend component $t$ while $X_0$ captures the constant terms and interventions in the system.

The most obvious interpretation of (2.11) is that the two common trends represent a trend in technology and a trend in investment. In this case, we expect to find $d_{11}=d_{21}=d_{31}=1$ and $d_{12}=0$. The latter condition states that output does not load on the trend in investments, i.e. the impact on output of shocks to private and public investment rates have cancelled out to leave output unaffected. This interpretation of the common trends specification implies that private and public capital are complements, i.e. a reduction in public investment for instance has no impact on output as the private sector reacts by increasing its investments. An alternative possibility is that $\Sigma \mu_{j1}$ and $\Sigma \mu_{j2}$ represent independent I(1) trends in private and public investments, respectively. Equation (2.5) shows that in this case $\Sigma \mu_{j1}$ and $\Sigma \mu_{j2}$ are combinations of technological shocks ($\xi_{A,t,i}$) and shocks to the private ($\xi_{sk,t,i}$) and the public ($\xi_{sg,t,i}$) investment rate respectively. In this interpretation, shocks to technology must affect both $\Sigma \mu_{j1}$ and $\Sigma \mu_{j2}$ in the same way. The unit long-run impact of technological shocks on output than implies that $d_{11}+d_{12}=1$. As output must be unaffected by opposite shocks to private and public investment rates for (2.8) to be a cointegrating relationship, $d_{11}\xi_{sk,t}$ should be equal to $-d_{12}\xi_{sg,t}$. This interpretation therefore implies $d_{11}=\alpha/(\alpha+\beta)$, $d_{12}=\beta/(\alpha+\beta)$, $d_{21}=d_{22}=1$ and $d_{22}=d_{31}=0$.

**Case 2: Capital stock series are I(2).** The plots of $\Delta \ln(K_t)$ and $\Delta \ln(G_t)$ in figure 1 show that capital stock growth rates are not characterised by strong mean reverting behaviour. This slow adjustment of capital stocks towards their steady-state levels makes it statistically more appropriate to treat $\ln(K_t)$ and $\ln(G_t)$ as I(2)-processes in the relatively short sample under investigation. In this case, cointegrating analysis is unable to pick up the long-run equilibrium under (2.8) or (2.9) as a cointegrating relationship. The data generating process can now be represented as:

\[
\begin{bmatrix}
\ln(Y_t) \\
\ln(I_{K,t}) \\
\ln(I_{G,t}) \\
\ln(K_t) \\
\ln(G_t) \\
\Delta \ln(K_t) \\
\Delta \ln(G_t)
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
c_{41} & 0 \\
c_{52} & \sum \sum u_{3i} \\
\sum \sum u_{4i} \\
\sum \sum u_{5i}
\end{bmatrix} + 
\begin{bmatrix}
d_{11} & d_{12} & d_{13} & d_{14} \\
d_{21} & d_{22} & d_{23} & d_{24} \\
d_{31} & d_{32} & d_{33} & d_{34} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & c_{41} & 0 \\
0 & 0 & 0 & c_{52}
\end{bmatrix} + 
\begin{bmatrix}
g_1 \\
g_2 \\
g_3 \\
g_4 \\
g_5
\end{bmatrix} + \xi + X_0 \quad (2.12)
\]
The slow adjustment of capital stocks introduces two I(2) components in the system, i.e. \(\Sigma \mu_{3i}\) and \(\Sigma \mu_{4i}\) being the I(2) trends in private and public capital stocks respectively. In this case \((\ln(Y_t) - b_1\ln(K_t) - b_2\ln(G_t)) \sim I(2)\) and \((\ln(Y_t) - b_1\ln(I_k_t) - b_2\ln(I_g_t)) \sim I(1)\). As the two I(2) components are essentially independent, there is no linear combination of the variables that is able to ‘kill’ the I(2)-trends. Therefore, capital stocks have to enter the model in first differences. Substituting \(\ln(Y^*)\) in equation (2.10) from the long-run equilibrium condition in (2.9) yields a stable (medium-term) cointegrating relationship:

\[
\ln(Y_t) = \pi^* + \frac{\alpha}{\alpha + \beta} \ln(I_k_t) + \frac{\beta}{\alpha + \beta} \ln(I_g_t) - \frac{\alpha}{g + \delta_k} [\Delta \ln(K_t) - g] - \frac{\beta}{g + \delta_g} [\Delta \ln(G_t) - g]
\]

(2.13)

Important to note here is that investments and capital stocks cannot both enter the model as endogenous variables for the capital stock can be calculated from its past realisation using identity (2.3) and the current level of investments. Therefore \(\Delta \ln(K)\) and \(\Delta \ln(G)\) enter the model as exogenous variables.\(^3\) Implicitly, this implies that \(\Delta \ln(K)\) and \(\Delta \ln(G)\) are assumed to be two driving trends in the system from the outset.

From equation (2.12), it is clear that the I(1)-trends in \(\Delta \ln(K)\) and \(\Delta \ln(G)\), i.e. \(\Sigma \mu_{3i}\) and \(\Sigma \mu_{4i}\) respectively, capture shocks to investment rates and technology that, due to slow capital accumulation, are not yet transmitted into output, i.e. we expect to find \(d_{13}=d_{14}=0\). Therefore, \(\Sigma \mu_{1i}\) and \(\Sigma \mu_{2i}\) must now be interpreted as the accumulation of shocks that have already been transmitted in output. As the contemporaneous effect of investments on the capital stock is very small, shifts in investment rates enter the system mainly through shocks to \(\Delta \ln(K)\) and \(\Delta \ln(G)\). As capital accumulates, \(\Delta \ln(K)\) and \(\Delta \ln(G)\) slowly return to the average rate of productivity growth \(g\), ‘forcing’ \(\Sigma \mu_{1i}\) and \(\Sigma \mu_{2i}\) to accumulate to the long-run I(1)-trends in investments and output once \(\Delta \ln(K) = \Delta \ln(G) = g\). Shocks to technology have both a direct and an indirect effect on output. The direct effect stems from the term \((1-\alpha-\beta)\ln(AL_t)\) in the production function while the indirect effect again results from the slow adjustment of capital stocks to the new long-run trend in investments. Essentially, the system in (2.12) equals the long-run structure described in (2.11), augmented with a medium-term adjustment path which has been modelled by introducing \(\Delta \ln(K)\) and \(\Delta \ln(G)\) as additional stochastic trends.

\(^3\) Equivalently, one can choose to model \(\Delta \ln(K)\) and \(\Delta \ln(G)\) endogenously, treating \(\ln(I_k)\) and \(\ln(I_g)\) as exogenous variables.
3. COINTEGRATED VAR METHODOLOGY

Our baseline empirical model is a \( p \) dimensional VAR of order \( q \):

\[
X_t = \mu_0 + \sum_{i=1}^{q} \Pi_{iti} + \Phi D_t + \varepsilon_t, \quad t = 1, \ldots, T,
\]

where \( X_t \) is a \( p \times 1 \) data vector, \( D_t \) contains intervention dummies and \( \mu_0 \) is a constant. A particularly useful methodology for analysing long-run relationships in a system like (3.1) has been suggested by Johansen (1988) and extended in Johansen and Juselius (1990). They propose a reparameterization of the VAR under (3.1) in a vector error-correction model (VECM):

\[
\Delta X_t = \mu_{j0} + \sum_{j=1}^{q} \Pi_{itj} \Delta X_{t-j} - \Pi X_{t-1} + \Phi D_t + \varepsilon_t, \quad t = 1, \ldots, T,
\]

where \( \Pi_{itj} = - \sum_{i=j+1}^{q} \Pi_j \) and \( \Pi = I - \sum_{i=1}^{q} \Pi_i \).

Since \( X_{t-1} \) is the only level term in equation (3.2), \( \Pi \) is the only matrix that contains information about the long-run relationships among the variables. Three cases can be distinguished. First, if \( \Pi \) has full rank all variables are stationary. Second, if \( \Pi = 0 \), all variables are integrated of order one but they are not cointegrated. Third, if \( \Pi \) has reduced rank \( r \) (\( 0 < r < p \)), there exist \( r \) independent linear combinations that transform the data from I(1) to I(0), i.e. there are \( r \) cointegrating relationships. In this case, \( \Pi \) can be written as the product of a \( p \times r \) matrix \( \alpha \) and a \( r \times p \) matrix \( \beta' \):

\[
\Pi = \alpha \beta'
\]

with \( \beta' X_{t-1} \) representing the \( r \) cointegrating relationships and \( \alpha \) measuring the speed of adjustment towards the long-run equilibrium. In a \( p \)-dimensional system with \( r \) cointegrating relations, \( (p-r) \) common stochastic trends determine the long-run behaviour of the variables.

Since \( X_t \) is very likely to contain I(2) variables, a more convenient reparametrization of (3.1) is (see also Johansen, 1992):

\[
\Delta^2 X_t = \mu_0 + \sum_{k=1}^{q-2} \Psi_{ik} \Delta^2 X_{t-k} - \Gamma \Delta X_{t-1} - \Pi X_{t-1} + \Phi D_t + \varepsilon_t
\]
where $\Psi_k = -\sum_{j=k+1}^{a-1} \Gamma_j$ and $\Gamma = I - \sum_{j=1}^{a-1} \Gamma_j$. The hypothesis that $X_t$ is I(2) consists of two reduced rank conditions (Johansen, 1992):

$$\Pi = \alpha \beta'$$

$$\alpha_\perp \Gamma_\perp = \xi \eta'$$

(3.5)

where $\eta$ and $\xi$ are $(p-r)\times s_1$ matrices and $(p-r) > s_1$. Johansen (1995) shows that as in the I(1) model, the reduced rank of $\Pi$ implies that there exist $r$ independent linear combinations that are stationary while the remaining $(p-r)$ relations can only become stationary by differencing. The crucial difference is that the $r$ stationary combinations can now be decomposed in $r_0$ direct cointegrating vectors and $r_1$ polynomial cointegrating vectors. Direct cointegrating vectors are linear combinations of the levels of the data that reduce the order of integration from two all the way down to zero while polynomial cointegrating vectors involve combinations of the levels of the data that only transform the process from I(2) to I(1) but cointegrate with linear combinations of the differenced process $\Delta X_t$ to achieve an I(0) process. Direct cointegrating relations can be interpreted as static long-run equilibria while polynomial cointegrating vectors correspond to dynamic long-run steady-state relations (Juselius, 1999). The $(p-r)$ non-cointegrating relations can be decomposed in $s_1$ first order and $s_2$ second order non-stationary relations. If $s_1=(p-r)$, $\alpha_\perp \Gamma_\perp$ is of full rank so that there are no I(2) trends in the data.

4. THE EMPIRICAL MODEL

The heuristic analysis in section 2 suggested that slow capital accumulation might cause the cointegration analysis to fail picking up equation (2.8) as a stable long-run equilibrium relationship. In this section this hypothesis is tested more formally by analysing the integration and cointegration properties of the data vector:

$$X_t = [\ln(Y), \ln(K), \ln(G)], \ t = 1953-1996,$$

using the cointegrated VAR methodology outlined in section 3. The VAR model is estimated with an unrestricted constant to allow for linear growth in the data and two unrestricted step dummies to account for possible breaks in this linear growth rate. Given a mild increase in the average rate of productivity growth ($g$) in the beginning of the 1960s, the first step dummy ($D60$) is specified to be 1 in 1960-1973, 0 otherwise. In order to capture the strong slowdown
of productivity growth in the mid 1970s, the second dummy \((D74)\) is 1 in 1974-1996, being 0 otherwise.

**Specification tests.** Prior to the cointegrating analysis, the appropriate order \((q)\) of the VAR needs to be determined. To do so, the unrestricted VAR under (3.1) was estimated starting with a relatively long lag-length \((q=5)\) and then applying system specification tests (not reported) to assess whether lags can be eliminated. The results of these tests point out that a reduction of the system to \(q=2\) cannot be rejected. To check the statistical adequacy of the maintained specification, table 1 reports some multivariate and univariate misspecification tests. The results show that the normality condition of the residuals is not violated with no clear signs of autocorrelation nor ARCH effects.

**Table 1** Specification tests and rank determination

<table>
<thead>
<tr>
<th><strong>Multivariate diagnostic tests</strong></th>
<th>(\text{Ln}(Y))</th>
<th>(\text{Ln}(K))</th>
<th>(\text{Ln}(G))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual autocorrelation (LM_1) (\chi^2(9) = 8.17[0.52]) &amp; 1.06[0.59] &amp; 0.75[0.69]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normality (LM_4) (\chi^2(9) = 4.22[0.90]) &amp; 3.55[0.17] &amp; 2.46[0.30]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normality (Jarq. Bera) (LM) (\chi^2(6) = 3.66[0.72]) &amp; -0.390 &amp; -0.180 &amp; 0.050</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.390 &amp; -0.180 &amp; 0.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>-0.511 &amp; -1.095 &amp; 0.459</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.017 &amp; 0.003 &amp; 0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARCH (\chi^2(2) = 0.05[0.98]) &amp; 1.06[0.59] &amp; 0.75[0.69]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normality (Jarq. Bera) (\chi^2(2) = 1.97[0.37]) &amp; 3.55[0.17] &amp; 2.46[0.30]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Univariate diagnostic tests and descriptive statistics**

<table>
<thead>
<tr>
<th>(\text{Ln}(Y))</th>
<th>(\text{Ln}(K))</th>
<th>(\text{Ln}(G))</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH (\chi^2(2) = 0.05[0.98]) &amp; 1.06[0.59] &amp; 0.75[0.69]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normality (Jarq. Bera) (\chi^2(2) = 1.97[0.37]) &amp; 3.55[0.17] &amp; 2.46[0.30]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.390 &amp; -0.180 &amp; 0.050</td>
<td></td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>-0.511 &amp; -1.095 &amp; 0.459</td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.017 &amp; 0.003 &amp; 0.003</td>
<td></td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.457</td>
<td>0.870</td>
</tr>
</tbody>
</table>

**Cointegrating rank tests**

<table>
<thead>
<tr>
<th>Eigenvalues of the (\Pi)-matrix</th>
<th>0.614</th>
<th>0.122</th>
<th>0.084</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trace test</td>
<td>49.14</td>
<td>9.19</td>
<td>3.70</td>
</tr>
<tr>
<td>Roots of the companion matrix (R) unrestricted (r = 2)</td>
<td>(r = 1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\rho_1)</td>
<td>0.991</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>(\rho_2)</td>
<td>0.991</td>
<td>0.894</td>
<td>1.000</td>
</tr>
<tr>
<td>(\rho_3)</td>
<td>0.833</td>
<td>0.894</td>
<td>0.855</td>
</tr>
<tr>
<td>(\rho_4)</td>
<td>0.833</td>
<td>0.791</td>
<td>0.787</td>
</tr>
<tr>
<td>(\rho_5)</td>
<td>0.011</td>
<td>0.049</td>
<td>0.031</td>
</tr>
<tr>
<td>(\rho_6)</td>
<td>0.011</td>
<td>0.049</td>
<td>0.031</td>
</tr>
</tbody>
</table>
Cointegrating and integrating properties. The next step is to determine the number of cointegrating vectors. If capital stock series have a near I(2) component, we expect to find \((r=0, s_1=1, s_2=2)\). The cointegrating rank \((\hat{r})\) equals the number of characteristic roots or eigenvalues of the \(\Pi\)-matrix that differ from zero. While the number of unit roots in the \(\Pi\) matrix equals \(p-r=s_1+s_2\), each I(2) trend produces an additional root in the full model, i.e. the number of unit roots in the characteristic polynomial of the VAR equals \(s_1+2s_2\). Therefore, a comparison of the number of roots of the characteristic polynomial with the number of roots in the \(\Pi\) matrix gives a first indication about whether I(2) components are present in the data (Juselius, 1997).

Table 1 reports the estimated eigenvalues of the \(\Pi\) matrix together with the characteristic roots of the process for the unrestricted VAR and the VAR under the restrictions \(r = 2\) and \(1\). Whatever the value chosen for \(r\), at least one and possibly two large roots remain in the system. This evidence supports the suggestion in section 2 that the slow adjustment of capital stocks introduces two I(2) components in the model.

<table>
<thead>
<tr>
<th></th>
<th>(p-r)</th>
<th>(r)</th>
<th>(Q(s_1,r))</th>
<th>(Q(r))</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
<td>99.88</td>
<td><strong>44.21</strong></td>
<td>37.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70.87</td>
<td>51.35</td>
<td>38.82</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>22.90</td>
<td>12.68</td>
<td>11.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>36.12</td>
<td>22.60</td>
<td>15.41</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>11.03</td>
<td>12.93</td>
<td>2.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12.93</td>
<td>3.76</td>
<td></td>
</tr>
</tbody>
</table>

Table 2  Testing the joint hypothesis \(H(s_1,r)\)

<table>
<thead>
<tr>
<th></th>
<th>(s_2)</th>
<th>(3)</th>
<th>(2)</th>
<th>(1)</th>
<th>(0)</th>
</tr>
</thead>
</table>

Note: critical values are printed in italics.

With possible I(2) trends in the data, a formal test procedure for the joint determination of the number of cointegrating vectors and the number of I(2) trends among the \(p-r\) common trends can be found in Paruolo (1996). Since in the I(2) specification an unrestricted constant cumulates twice to obtain quadratic trends in the levels of the series, the constant enters the empirical model with a zero restriction on the quadratic component. Likewise, an unrestricted step dummy cumulates twice to obtain a broken quadratic trend in \(X_t\). In order to obtain the desired broken linear trend, \(D60\) and \(D74\) enter in first differences. The test statistics are
Balanced growth and public capital

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reported in table 2. The 95% quantiles, taken from Paruolo (1996), are given in italics. Note that these critical values can only serve as a rough approximation for they were generated for a model without dummies. Therefore, they are very unlikely to be exact in our case. The standard test procedure starts reading from the upper left-hand corner of table 2 to the right, continuing on the subsequent rows until one finds the first clear acceptance. Consistent with our a priori hypothesis, \((r=0, s_1=1, s_2=2)\) is the first acceptable case, i.e. the near \(I(2)\) component in capital stock series prevents the cointegration analysis to pick up the long-run equilibrium condition \((2.8)\) as a stable cointegrating vector.

5. Transformation to an \(I(1)\) model

Given the \(I(2)\) component in capital stock series, the model is transformed along the lines suggested in section 2. The data vector now becomes:

\[
X_t = \left[ \ln(Y), \ln(Ik), \ln(Ig), \Delta \ln(K), \Delta \ln(G) \right], \ t = 1953-1996.
\]

Since investments and capital stocks cannot both enter the model endogenously, \(\Delta \ln(K)\) and \(\Delta \ln(G)\) enter the model as exogenous variables. This exogeneity restriction is imposed by letting \(X_1\) be partitioned as \(X_1 = (X_{1,t}, X_{2,t})\), with \(X_{1,t}\) containing the 3 endogenous variables and \(X_{2,t}\) the 2 exogenous driving trends, i.e.

\[
X_{1,t} = \left[ \ln(Y), \ln(Ik), \ln(Ig) \right],
\]

\[
X_{2,t} = \left[ \Delta \ln(K), \Delta \ln(G) \right],
\]

and reformulating the system under \(3.2\) in a conditional error-correction model:

\[
\Delta X_{1,t} = \mu_{t,0} + \sum_{j=1}^{q-1} \Gamma_{1,j} \Delta X_{t-j} - \alpha_1 \beta' X_{t-1} + \Omega \Delta X_{2,t} + \Phi_1 D_t + \epsilon_{1,t} \tag{5.1}
\]

where \(\alpha_1\) is a \((3 \times r)\) matrix of error-correction coefficients and \(\Gamma_{1,j}\) and \(\Omega\) are parameter matrices of order \((3 \times 5)\) and \((3 \times 2)\) respectively. The VAR model is again estimated with an unrestricted constant and two unrestricted step dummies \((D60, D74)\).

**Specification tests.** The results of the system specification tests (not reported) to determine the appropriate lag length of the VAR point to \(q=2\). The univariate misspecification tests reported in table 3 show that this specification is statistically acceptable. Moreover, the considerably higher \(R^2\)-value of the output equation - compared to the original model in
section 4 - shows that the transformed model is now much more able to explain the variation in output.

Table 3  Specification tests and rank determination

<table>
<thead>
<tr>
<th>Multivariate diagnostic tests</th>
<th>Ln(Y)</th>
<th>Ln(lk)</th>
<th>Ln(lg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual autocorrelation LM&lt;sub&gt;1&lt;/sub&gt;</td>
<td>$\chi^2(9) = 6.97[0.64]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM&lt;sub&gt;4&lt;/sub&gt;</td>
<td>$\chi^2(9) = 2.98[0.96]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normality LM</td>
<td>$\chi^2(6) = 7.22[0.30]$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Univariate diagnostic tests and descriptive statistics</th>
<th>Ln(Y)</th>
<th>Ln(lk)</th>
<th>Ln(lg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH</td>
<td>$\chi^2(2) = 1.37[0.50]$</td>
<td>0.40[0.82]</td>
<td>2.37[0.31]</td>
</tr>
<tr>
<td>Normality (Jarq. Bera)</td>
<td>$\chi^2(2) = 1.65[0.44]$</td>
<td>3.00[0.22]</td>
<td>4.45[0.11]</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.374</td>
<td>-0.572</td>
<td>-0.420</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>0.201</td>
<td>-0.118</td>
<td>0.937</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.012</td>
<td>0.005</td>
<td>0.029</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.756</td>
<td>0.993</td>
<td>0.941</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cointegrating rank tests</th>
<th>R unrestricted</th>
<th>$r = 2$</th>
<th>$r = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalues of the $\Pi$-matrix</td>
<td>0.610</td>
<td>0.385</td>
<td>0.114</td>
</tr>
<tr>
<td>Trace test</td>
<td>63.44</td>
<td>24.89</td>
<td>4.96</td>
</tr>
<tr>
<td>Roots of the companion matrix $\rho$</td>
<td>$R$ unrestricted</td>
<td>$r = 2$</td>
<td>$r = 1$</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>1.008</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>0.887</td>
<td>0.969</td>
<td>1.000</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>0.119</td>
<td>0.136</td>
<td>0.356</td>
</tr>
<tr>
<td>$\rho_4$</td>
<td>0.119</td>
<td>0.136</td>
<td>0.113</td>
</tr>
<tr>
<td>$\rho_5$</td>
<td>-0.009</td>
<td>0.056</td>
<td>0.113</td>
</tr>
<tr>
<td>$\rho_6$</td>
<td>-0.386</td>
<td>-0.330</td>
<td>-0.112</td>
</tr>
</tbody>
</table>

Cointegrating properties. The estimated eigenvalues of the $\Pi$ matrix together with the characteristic roots of the process for the unrestricted VAR and the VAR under the restrictions $r = 2$ and 1 are reported in the bottom of Table 3. The critical values of the trace-test are not reported as they are not correct in the case of breaks in the deterministic component of the model. From the analysis in section 2, we expect to find one cointegrating relationship. The two large roots in the characteristic polynomial on the one hand support this choice of $r=1$. The relatively large value of the trace test on the second eigenvalue on the other hand suggests that there might be two cointegrating relations ($r = 2$). Under the choice $r = 2$ a large root remains present in the system, though. Restricting $r = 1$ removes all unit roots.
Consistent with the scenario analysis in section 2, these findings support one cointegrating vector.

As noted in section 2, the medium-term equilibrium condition in (2.13) fails to be a stable cointegrating relation in the presence of shocks shifting the steady-state of the economy. While shifts in the average rate of productivity growth have been modelled by including two step dummies, the finding of a stable cointegrating relation suggests that the impact on equilibrium output of the upward shift in the private investment rate - occurring in the 1980s - has been cancelled out by the negative shock to the public investment rate over more or less the same period. Modelling intervention dummies at the timing of the shifts in investment rates did only yield minor improvements and are therefore left out.

**Time series properties of the data.** Table 4 includes tests that serve as an investigation of the time series properties of the data and as an exploratory inspection of the role of the variables in the system under the maintained hypothesis that \( r = 1 \). The test for stationarity checks whether variables are I(0) by testing whether the corresponding unit vector is included in the cointegrating space. The test is \( \chi^2 \) -distributed with \( (p-r) \) degrees of freedom. The results indicate that none of the variables is stationary by itself.

<table>
<thead>
<tr>
<th>Test for stationarity</th>
<th>5% cv.</th>
<th>( \ln(Y) )</th>
<th>( \ln(Ik) )</th>
<th>( \ln(Ig) )</th>
<th>( \Delta \ln(K) )</th>
<th>( \Delta \ln(G) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi^2(4) )</td>
<td>9.49</td>
<td>33.63</td>
<td>34.32</td>
<td>35.51</td>
<td>25.53</td>
<td>32.91</td>
</tr>
<tr>
<td>Test for long-run exclusion</td>
<td>( \chi^2(1) )</td>
<td>3.84</td>
<td>18.34</td>
<td>17.81</td>
<td>14.74</td>
<td>12.33</td>
</tr>
<tr>
<td>Test for weak exogeneity</td>
<td>( \chi^2(1) )</td>
<td>3.84</td>
<td>7.25</td>
<td>3.91</td>
<td>5.01</td>
<td>-</td>
</tr>
</tbody>
</table>

The null hypothesis in the test for long-run exclusion is formulated as a zero row in \( \beta \), implying the absence of a long-run relationship between the variable under consideration and the remaining variables in the system. The test is \( \chi^2 \)-distributed with \( r \) degrees of freedom. The test statistics in table 4 show that the null-hypothesis can be rejected for all variables, meaning that none of the variables can be excluded from the cointegrating space without loss of information. The significance of \( \Delta \ln(K) \) and \( \Delta \ln(G) \) again shows that the slow adjustment of capital stocks blurs the long-run equilibrium relation between output and investment, making the cointegrating analysis unable to detect it in the small sample under investigation.
Finally, the test for long-run weak exogeneity investigates whether a variable does not adjust to equilibrium errors. The null-hypothesis is formulated as a zero row restriction on $\alpha$ (Johansen and Juselius, 1990). The test is $\chi^2$-distributed with $r$ degrees of freedom. Using the 5% critical value, weak exogeneity must be rejected for each of the three variables that are allowed to be endogenously determined. Note that the test statistic for private investment is close to its critical value, though, suggesting that $\ln(I_k)$ might be an exogenous trend.

**Cointegrating properties.** With $r = 1$, there is no need to impose identifying restrictions on the cointegrating space. The unrestricted cointegrating vector and the adjustment coefficients are reported in table 5. Equation (2.13) shows that the coefficients in the cointegrating vector cannot straightforwardly be interpreted in terms of output elasticities of private and public capital as they only provide us with estimates of the relative size of output elasticities. The results suggest that the output elasticity of public capital is about a fraction 0.4 of the output elasticity of private capital. Assuming that the output elasticity of private capital is about 0.33, these estimates imply an output elasticity of public capital of about 0.14. Given the relative value of $\alpha$ and $\beta$, the coefficients on $\Delta \ln(K)$ and $\Delta \ln(G)$ indicate that $\delta_k \approx g + 2\delta_g$ which is broadly consistent with the values observed in reality, i.e. $\delta_k \approx 3\%$ and $\delta_g \approx 0.5\%$. Notice that although $0.642 + 0.270 \approx 1$, the restriction implied by equilibrium condition (2.13):

$$H_1: \beta' = [1, a, -(1 + a), \ast, \ast, \ast],$$

is rejected by the data $\chi^2(1) = 9.19 [0.00])$, indicating that on average investments have grown somewhat faster than output.

**Table 5**  The normalised cointegrating vector and adjustment parameters

<table>
<thead>
<tr>
<th>Equation</th>
<th>Variables</th>
<th>Weights $\alpha$</th>
<th>Eigenvectors $\beta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \ln(Y)$</td>
<td>$-0.588$</td>
<td>(0.179)</td>
<td>$1.000$</td>
</tr>
<tr>
<td>$\Delta \ln(I_k)$</td>
<td>$0.164$</td>
<td>(0.076)</td>
<td>$-0.047$</td>
</tr>
<tr>
<td>$\Delta \ln(I_g)$</td>
<td>$0.943$</td>
<td>(0.418)</td>
<td>$-0.047$</td>
</tr>
</tbody>
</table>

*Note: standard errors are printed in parentheses.*

The estimated cointegrating relationship is graphed in figure 3. Taking into account the shifts in the constant of the cointegrating relationship (the dashed lines in the graph) due to shifts in the average rate of productivity growth, this vector appears to be very stationary.
Moving average representation. In a system with three endogenous and two exogenous variables, the choice of $r = 1$ implies that four common trends are determining the medium-term behaviour of the variables. These common trends and the variable’s loadings to these trends can be extracted from the VECM by inverting (5.1) to obtain the moving average representation:

$$
\Delta X_{1,t} = C(L)\left(\epsilon_{1,t} + \mu_{1,0} + \Phi_1D_t - \alpha_1\beta_2X_{2,t-1}\right) + B(L)\Delta X_{2,t}
$$

(5.2)

where $C(L)$ and $B(L)$ are infinite lag polynomials and $\beta_2$ a $(2 \times 1)$ vector including the coefficients in the cointegration vector $\beta$ related to the exogenous variables. Provided $\alpha_1 \perp \Gamma_{11} \beta_1 \perp$ has full rank $(p-r)$, the process $X_{1,t}$ has the representation:

$$
X_{1,t} = X_{1,0} + \sum_{i=1}^{r} (\epsilon_{1,t} + \mu_{1,0} + \Phi_1D_t) + C_1(L)(\epsilon_{1,t} + \mu_{1,0} + \Phi_1D_t - \alpha_1\beta_2X_{2,t-1}) + B(L)X_{2,t}
$$

(5.3)

with $X_{1,0}$ capturing the initial conditions, $C = \beta_1 \left(\alpha_{11}^\top \Gamma_{11} \beta_1\right)^{-1} \alpha_{11}^\top$ being the matrix of long-run multipliers and $C_1(L) = (C(L) - C(1))/(1 - L)$ (see Johansen, 1991). Since $\alpha_{11} \perp \beta_{11}$ are the $p \times (p-r)$ orthogonal complements to $\alpha$ and $\beta$, the long-run impact matrix $C$ has reduced rank $(p-r)$ and can be rewritten as the product of two $p \times (p-r)$ matrices, $C = \beta_{11} \alpha_{11}^\top$, with

$$
\beta_{11} = \beta_1 \left(\alpha_{11}^\top \Gamma_{11} \beta_1\right)^{-1}. \alpha_1^\top \sum_{i=1}^{r} \epsilon_i
$$
can now be interpreted as the $(p-r) = 2$ common stochastic trends in the system, with $\beta_{11}$ being the weights to these common trends.
The moving average representation of the process in (5.3) shows that besides the two endogenously determined common trends, the medium-run stochastic behaviour of the endogenous variables is also influenced by the exogenous trends in $X_{2,t}$, representing the slow adjustment of capital stocks, with weighting matrix equal to the matrix of long-run multipliers of the lag polynomial $\left( B(L) - C_1(L) \alpha \beta_2 L \right)$.

### Table 6  Common trends, trend-loadings and the long-run impact matrix

<table>
<thead>
<tr>
<th>Common Trends</th>
<th>Loadings</th>
<th>Coefficients</th>
<th>Long-run impact matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Ln}(Y)$</td>
<td>0.19</td>
<td>0.36</td>
<td>$\Sigma \varepsilon_y$</td>
</tr>
<tr>
<td>$\text{Ln}(I_k)$</td>
<td>-0.12</td>
<td>0.63</td>
<td>$\Sigma \varepsilon_{ik}$</td>
</tr>
<tr>
<td>$\text{Ln}(I_g)$</td>
<td>1.00</td>
<td>-</td>
<td>$\Sigma \varepsilon_{ig}$</td>
</tr>
</tbody>
</table>

| $\text{Ln}(Y)$ | 0.480 (0.166) | 1.014 (0.390) | 0.122 (0.081) | $\Sigma \varepsilon_y$ |
| $\text{Ln}(I_k)$ | 0.246 (0.126) | 1.302 (0.296) | -0.073 (0.061) | $\Sigma \varepsilon_{ik}$ |
| $\text{Ln}(I_g)$ | 1.192 (0.413) | 0.662 (0.972) | 0.628 (0.203) | $\Sigma \varepsilon_{ig}$ |

Note: standard errors are printed in parentheses.

The coefficients of the common trends and the loadings to these trends by the three endogenous variables are reported in the first part of table 6. Before turning to the interpretation of these trends, it should be noted that due to the inclusion of $\Delta \ln(K)$ and $\Delta \ln(G)$ as exogenous driving trends, a straightforward interpretation of the two endogenous common trends might be very arduous. As noted in section 2, shifts in investment rates and shocks to technological growth enter the system mainly and/or partly through shocks to $\Delta \ln(K)$ and $\Delta \ln(G)$. As capital accumulates, $\Delta \ln(K)$ and $\Delta \ln(G)$ slowly return to the average rate of productivity growth $g$, ‘forcing’ $\Sigma \mu_{ik}$ and $\Sigma \mu_{ig}$ to accumulate to the long-run I(1)-trends in investments and output once $\Delta \ln(K) = \Delta \ln(G) = g$. In fact this specification implies that shocks to investments have been exogenously introduced in the system. Subsequently, the long-run trends are allowed to build up endogenously. Clearly, it can be discussed whether shocks to for instance the public investment rate that were introduced in the system through exogenous.
shocks to $\Delta \ln(G)$ will also give rise to subsequent shocks to public investments to cumulate into the long-run trend $I(1)$ trend.

Insofar as a sensible interpretation is possible, the two endogenously determined common trends seem to represent the long-run trends in public and private investment respectively. The driving trend in public investments, the first common trend, is essentially a combination of shocks to output and public sector investments while the driving trend in private investment, the second common trend, is dominated by shocks to private investments. Notice that public investments also have a high loading on the trend in private investment. The long-run impact matrix $C$ reported in the lower part of table 6 indicates that the long-run impact of shocks to private investments on public investments is not statistically significant, though.

Consistent with the exogeneity tests in table 4, the $C$-matrix shows that private investments are mainly exogenously driven with only a minor impact of shocks to output. Public investments are both ‘endogenously’ determined by output movements and ‘exogenously’ by shifts in the public investment rate. The loadings of output to the trend in private and public capital equal 0.84 and 0.19 respectively. The long-run impact of shocks to public investment on output appears to be only borderline significant, though.

The third and the fourth common trend reported in the upper part of table 6 are the medium-term exogenous trends in $\Delta \ln(K)$ and $\Delta \ln(G)$ respectively. Recall that in $\Delta \ln(K)$ and $\Delta \ln(G)$ capture the proportion of shocks to investments that, due to slow capital accumulation, are not yet transmitted into output. Consistent with this interpretation, the trend loadings in table 6 show that $\Delta \ln(K)$ and $\Delta \ln(G)$ are the medium-term driving trend in $\ln(I_k)$ and $\ln(I_g)$ respectively. The small loading of output on these medium-term trends is probably due to business cycle fluctuations. The higher loading of $\ln(I_g)$ on its driving trend $\Delta \ln(G)$ is due to the lower depreciation rate of public capital, implying public capital accumulation to take more time, i.e. a larger proportion of shocks to public investments is not yet transmitted into output. Notice also the negative loading of $\ln(I_g)$ on $\Delta \ln(K)$, which might suggest that there is some medium-term complementary between public and private investments. Nevertheless, the long-run impact matrix $C$ reported in the lower part of table 6 suggests that the long-run trends in private and public investment are largely independent.

---

4 Notice that instead of normalising on the vectors in $\alpha_{\perp}$, we normalised on the loading matrix $\beta_{\perp}$ to obtain a unit loading of private and public investments on their driving trend, i.e. the second and the first common
Parameter constancy. As a final specification check, figure 4 reports recursively calculated test statistics for the stability of the $\beta$ vector. The first model is estimated using the first 24 years (1953-1976) of the sample. The model is then repeatedly re-estimated adding one extra observation at a time. Beta_Z is constructed using recursive estimates of both short-run and long-run parameters. Beta_R only uses recursive estimates of the long-run parameters. Both test statistics were scaled such that 1 corresponds to the 5% critical value. The results show that, holding short-run parameters fixed at their sample values, the coefficients in the cointegrating vector can be considered stable over the sample period.

**Figure 4** Recursive test statistics for the test of a constant cointegration space

![Graph showing recursive test statistics](image)

*Note:* 1 is the 5% critical value.
6. Conclusion

This paper asks the question of whether the drastic reduction of public investment in Belgium during the fiscal consolidation episodes of the 1980s and 1990s has reduced the long-run output capacity of the Belgian economy. Two potentially important problems in the literature that tries to estimate the contribution of public capital to long-run economic growth have been addressed. First, most studies estimating production functions including public capital as one of the explanatory variables ignore or misspecify the underlying rate of technological progress. Standard neoclassical growth models show that the long-run growth in most macroeconomic series is primarily determined by technological progress, though. As technology is not a directly observable variable, I follow the underlying idea in the methodology proposed by Crowder and Himarios (1997) to let the rate of technological growth be endogenously determined as a common stochastic trend in the data using the long-run growth properties of the neoclassical model.

Second, slow capital accumulation implies that capital stock series include a near I(2)-component in the small samples usually available to the researcher. The standard I(1) cointegrated VAR methodology is no longer valid in this case. Moreover, as the I(2)-trends in private and public capital are independent, no linear combination of the variables is able to kill these I(2) components in an I(2) cointegrated model. Therefore, capital stock series enter the analysis in first differences. Inspired by the neoclassical growth model, the balanced growth path of output is now being determined by private and public investments with private and public capital stock growth rates capturing the medium-term slow adjustment towards this steady-state.

Using Belgian data for the period 1953-96, the analysis supports Aschauer’s hypothesis that the decline in public capital investment has lowered the balanced growth path of real output. In contrast to Aschauer’s results, the output elasticity of public capital is found to be only a fraction 0.4 of the output elasticity of private capital. Assuming that the output elasticity of private capital is about 0.33, these estimates imply an output elasticity of public capital of about 0.14.
Equation (2.1) and its steady-state counterpart imply:

\[ \ln (Y_t) - \ln (Y_t^*) = \alpha \ln (K_t - \ln (K_t^*)) + \beta \ln (G_t - \ln (G_t^*)) \]  
\[ (A.1) \]

Substituting out \( \ln(K_t^*) \) and \( \ln(G_t^*) \) from (2.4) yields:

\[ \ln (Y_t) - \ln (Y_t^*) = -\alpha \ln \left[ \frac{I_k}{K_t} \frac{1}{g + \delta_k} \right] - \beta \ln \left[ \frac{I_g}{G_t} \frac{1}{g + \delta_g} \right] \]  
\[ (A.2) \]

Noting that:

\[ \frac{I_k}{K_t} \frac{1}{g + \delta_k} = 1 \quad \text{and} \quad \frac{I_g}{G_t} \frac{1}{g + \delta_g} = 1, \]  
\[ (A.3) \]

equation (A.2) can be rewritten as:

\[ \ln (Y_t) - \ln (Y_t^*) = -\alpha \left[ \frac{I_k}{K_t} \frac{1}{g + \delta_k} - 1 \right] - \beta \left[ \frac{I_g}{G_t} \frac{1}{g + \delta_g} - 1 \right] \]  
\[ (A.4) \]

Rearranging terms:

\[ \ln (Y_t) - \ln (Y_t^*) = -\frac{\alpha}{g + \delta_k} \left[ I_k - (g + \delta_k) \right] - \frac{\beta}{g + \delta_g} \left[ I_g - (g + \delta_g) \right], \]  
\[ (A.5) \]

and substituting for \( \Delta \ln(K_t) \) and \( \Delta \ln(G_t) \) from equation (2.3) yields:

\[ \ln (Y_t) = \ln (Y_t^*) - \frac{\alpha}{g + \delta_k} \left[ \Delta \ln (K_t) - g \right] - \frac{\beta}{g + \delta_g} \left[ \Delta \ln (G_t) - g \right]. \]  
\[ (A.6) \]
REFERENCES


Public capital and labour market performance in Belgium

GERDIE EVERAERT and FREDDY HEYLEN*

September 2000

ABSTRACT: This paper investigates the output and labour market effects of public capital formation in Belgium. In earlier work Everaert and Heylen (2000) have found significant, positive effects of public capital on multifactor productivity within a single equation production function framework. In this paper we revisit and extend this work within a broader model. This model explains - among other variables - private output, private employment and unemployment, private capital formation, wage bargaining and price setting. For each of these endogenous variables we specify error correction processes, to be estimated simultaneously. Model simulations show (strongly) positive effects of public investment on private sector output and capital formation. Public infrastructures and private employment are found to be substitutes. The simulations also show that the initial strong negative impact of higher public investment spending on the government budget is largely undone as the increase in the public capital stock starts feeding through into higher economic growth.

KEYWORDS: Public capital, structural model, employment, output growth, government budget.

1. INTRODUCTION

Following Aschauer (1989a) a huge literature has studied the macroeconomic effects of the decline in public investment that has taken place in most OECD countries since the early 1970s. Most research has concentrated on output and productivity effects. As is well known by now, the results of this literature are quite controversial. Estimating an aggregate production function, Aschauer (1989a, 1989b) found highly positive and significant effects of infrastructure on productivity growth in the US and the G7. In more recent work, however, many authors have criticised these results both on theoretical and methodological grounds. Major issues that have been raised concern the non-stationarity of the data, the direction of causality, possible misspecification of the production function, the endogeneity of other factors of production (i.e. private labour and capital) and the correct empirical proxy for technical progress. Dealing with (some of) these issues some authors confirm Aschauer’s ‘public capital hypothesis’, others reject it (see e.g. Sturm et al., 1997, for a broad survey).1

* The authors are grateful to the participants at the conference on “Macroeconomic Transmission Mechanisms: Empirical Applications and Econometric Methods” (18-20 May, 2000, Copenhagen).

1 Another wave of criticism on Aschauer’s seminal work concerns the correct way to model the relationship between public capital and output and productivity. Instead of adding public capital as an additional input variable in a production function, some have included public capital in cost or profit functions. Others have estimated VAR’s or included government investment spending in cross-section growth regressions. Still others have estimated multi-equation structural econometric models.
Also taking a production function approach, Everaert and Heylen (2000) confirm the public capital hypothesis for Belgium. They find a significant output elasticity of public capital equal to 0.29.

The focus of this paper is on the relationship between public capital formation and the labour market, in particular employment, in Belgium. Figure 1 shows a remarkable correlation. The figure depicts the evolutions of the trend yearly growth rate of the real public capital stock and of the private sector employment rate, i.e. the percentage of people at working age with a private sector job. In line with observations for many other countries, one can see that the former peaked around 1970, after which a substantial reduction set in. Quite remarkably, with a lag of only a few years the private employment rate made almost the same dive. It fell from about 41% in 1973-75 to 33% ten years later. Since 1987 the employment rate has shown a weak upward trend, disturbing the correlation with the growth rate of public capital. To a very important extent, however, this upward trend is due to a growing share of part time employment. Redefined in full-time equivalents, the employment rate was not higher in 1996 than it was in 1986 (Nationale Bank van België, 1999).

**Figure 1** Public capital and employment in Belgium, 1961-1996

*Sources:* OECD statistical compendium 1999/1 and Belgian Federal Planning Bureau.

Whether the correlation shown in figure 1 is - at least partially - due to a causal relation running from public capital to employment or whether it is a mere coincidence, driven by the evolution of other variables, is the main research question in this paper. In contrast to its
economic growth and productivity effects, the influence of public capital on employment has received much less attention in the literature. If there were relevant findings, these generally resulted from the estimation of private sector cost functions and/or factor share or input demand equations from which can be derived whether labour is a substitute or a complement to public capital. Most studies find that public capital reduces private sector costs. Many - but certainly not all - studies find that public capital acts as a substitute for intermediate inputs and often also labour. As for the relation between public and private capital most studies conclude that it is complementary (see again Sturm et al., 1997, for a survey).

The perspective of this paper is broader. We specify a simple structural model for the Belgian economy explaining - among other variables - private sector output and costs, private employment and unemployment, private capital formation, wage bargaining, price setting and aggregate demand. The model allows for three channels of influence of public capital on private employment: (i) direct complementary or substitution effects for given output, (ii) indirect effects on real wages, due to changes in labour productivity and/or the unemployment rate, (iii) indirect effects caused by changes in aggregate demand and the output level. Once estimated, the model is used to simulate private sector performance under an alternative public investment policy. More specifically, we analyse how the economy would have evolved if the strong decline in public investment during the period 1982-89 had not occurred.

Two other motivations underlie this paper. First, in recent years a growing literature has tried to explain the success or failure of fiscal consolidation programmes. Most often, this literature has taken a cross-country perspective (see e.g. Alesina and Perotti, 1995; McDermott and Wescott, 1996; Heylen and Everaert, 2000). This paper allows a time series test for Belgium of the well-known hypothesis that to be successful, fiscal consolidation should not rely on cutting public investment. Second, the estimated model provides a robustness cheque on our findings for the output elasticity of public capital in Everaert and Heylen (2000).

Some, but not much, similar work has been done before. Westerhout and van Sinderen (1994) estimate a model for - among other variables - output growth, employment growth and the private investment ratio in the Netherlands in 1958-89. Their results show that the decrease of government investment spending since the 1970s has undermined each of these three dependent variables.
The remainder of the paper is as follows. Section 2 presents our long-run structural model. Section 3 introduces dynamics through error-correction specifications. Section 4 presents the estimates of the dynamic model for the Belgian economy using annual data for 1965-96. In order to detect the effects of public capital formation on economic growth and the labour market, section 5 reports the results of a simulation experiment. Section 6 concludes and outlines directions for future research.

2. THE MODEL

Our model describes a representative, imperfectly competitive firm, operating in a small open economy. This firm decides on its required input volumes (labour, capital and materials) to produce a certain level of output, taking factor prices, the current state of technology and the public capital stock as given. The output level is determined by expected demand, which is itself a function of the prices set by the firm on the domestic market and abroad. To maximise its profits, the firm determines these prices as mark-ups on marginal cost. As far as costs are concerned, the cost of capital and the price of materials are exogenous. Wages, however, are endogenous. They are bargained between unions and employers. Finally, depending on prices, actual demand will be determined through the demand side of the economy. The following subsections describe production, input prices (mainly wage formation), output price setting and the aggregate economy’s supply and demand side.

2.1. Production technology and factor demand

Imagine a small open economy consisting of \( N \) identical private sector firms. We consider one of these firms. Its real gross output \( (X) \) is produced by services from private capital \( (K) \), employed labour \( (E) \), imported intermediate inputs (materials, \( M \)) and public capital \( (KG) \). The production function is of the form

\[
X = AF\left( E, K, KG, M \right)
\]

with \( A \) representing the stance of technology. The firm minimises total cost

\[
C = P_K E + P_K K + P_M M
\]

subject to the production function under (1). This results in a cost function of the type
with $P_i$ the vector of variable factor prices. Public capital and the stock of technology are treated as unpaid fixed inputs\(^2\).

Consider the transpose flexible functional form as an adequate approximation of the cost function under (2).

\[
\ln(C) = \alpha_0 + \sum_i \alpha_i \ln(P_i) + \sum_i \sum_j \alpha_{ij} \ln(I_i) + \frac{1}{2} \sum_i \sum_j \sum_k \alpha_{ijk} \ln(I_i) \ln(P_j) \\
+ \frac{1}{2} \sum_j \sum_i \alpha_{ij} \ln(I_i) \ln(J_j) + \frac{1}{2} \sum_i \sum_j \sum_k \alpha_{ijk} \ln(I_i) \ln(P_j) \\
+ \frac{1}{2} \sum_i \sum_j \alpha_{ij} \ln(I_i) \ln(P_j)
\]

with $i$ and $j$ ranging over the domain of variable inputs ($E, K, M$) and $I$ and $J$ ranging over the domain of gross output ($X$) and fixed inputs ($KG, A$). The translog cost function captures all relevant characteristics of the underlying production function (i.e. duality) only if a number of conditions are satisfied:

(i) Symmetry

\[
\alpha_{ij} = \alpha_{ji}, \quad \alpha_{ii} = \alpha_{II}, \quad \alpha_{IJ} = \alpha_{JI}
\]  

(ii) Linear homogeneity in factor prices

Factor price homogeneity ensures that - holding fixed inputs and output constant - a proportional increase in factor prices does not affect factor shares, i.e. only relative prices matter in the optimisation process.

\[
\sum_i \alpha_i = 1, \quad \sum_j \alpha_{ij} = 0, \quad \sum_i \alpha_{il} = 0
\]

Symmetry and linear homogeneity in factor prices ensure that the sum of factor shares in total cost adds up to unity ($\alpha_E + \alpha_K + \alpha_M = 1$).

(iii) Positivity

The cost function must be positive for positive input prices and a positive level of output.
(iv) **Monotonicity**

The cost function must be increasing in input prices and in the level of output.

(v) **Concavity in Factor Prices**

Concavity in factor prices requires negative semi-definiteness of the matrix of second order partial derivatives of the cost function with respect to prices, which ensures that we are minimising – instead of maximising - cost.

Since globally imposed curvatory conditions imply strong restrictions on the elasticities of substitution (Diewert and Wales, 1987), only restrictions (i) and (ii) are imposed in the estimation of the translog function. The results are checked for conformity with restrictions (iii)-(v) after estimation.

Assumptions concerning returns to scale in the underlying production function imply a further set of restrictions on the cost function. As shown by Conrad and Unger (1987), if the production function under (1) is homogeneous of degree $\lambda$ in all tangible factors - variable $(E,K,M)$ and fixed $(KG)$ - the cost function is almost homogeneous of degree $1/\lambda$ in $X$ and $KG$. In case of homogeneity of the production function, this statement implies (see Conrad and Unger, 1987, for proof):

$$\frac{\partial \ln(C)}{\partial \ln(X)} + \frac{1}{\lambda} \frac{\partial \ln(C)}{\partial \ln(KG)} = \frac{1}{\lambda}$$

(vi) **Homogeneity of the underlying production function and constant returns to scale**

From equation (6), it can be derived that homogeneity of the underlying production function is ensured by the following set of restrictions

$$\alpha_X + \frac{1}{\lambda} \alpha_{KG} = \frac{1}{\lambda}$$

$$\alpha_h, X + \frac{1}{\lambda} \alpha_{h,KG} = 0 \quad h = E, K, M, X, KG, A$$

**Constant returns to scale** over all inputs require the further restriction that $\lambda$=1. Note that the homogeneity restrictions under (7) imply $\lambda$ to be constant over the sample period. If these restrictions are not satisfied, $\lambda$ can still be calculated but varies with prices and output. Homogeneity of the production function and constant returns to scale are not imposed but tested for after estimation.
Using Shephard’s lemma, *factor demand equations* for variable inputs can straightforwardly be derived by taking partial derivatives of the cost function with respect to factor prices.

\[
E = \frac{\partial \ln(C)}{\partial P_E} = S_E = \frac{E.P_E}{C} = \frac{\partial \ln(C)}{\partial \ln(P_E)} = \left[ \alpha_E + \sum_i \alpha_{E,i} \ln(P_i) + \sum_i \alpha_{E,i} \ln(I) \right]
\] (8a)

\[
K = \frac{\partial \ln(C)}{\partial P_K} = S_K = \frac{K.P_K}{C} = \frac{\partial \ln(C)}{\partial \ln(P_K)} = \left[ \alpha_K + \sum_i \alpha_{K,i} \ln(P_i) + \sum_i \alpha_{K,i} \ln(I) \right]
\] (8b)

\[
M = \frac{\partial \ln(C)}{\partial P_M} = S_M = \frac{M.P_M}{C} = \frac{\partial \ln(C)}{\partial \ln(P_M)} = \left[ \alpha_M + \sum_i \alpha_{M,i} \ln(P_i) + \sum_i \alpha_{M,i} \ln(I) \right]
\] (8c)

with \(S_i\) (\(i=E, K, M\)) denoting the cost shares of variable input factors. The adding-up property of factor shares (restrictions (i) and (ii)) implies that one of the factor demand equations is redundant and must be deleted from the system. The parameters of the deleted equation can be recovered after estimation from the restrictions placed on the system.

Under conditions (i)-(v), the cost function is dual to the production function and all relevant characteristics of the latter can be derived from (3). A first important aspect is the elasticity of substitution between input factors. A commonly used concept, measuring the elasticity of substitution between inputs \(i\) and \(j\), is the Allen-Uzawa elasticity of substitution, \(\sigma_{ij}\). This elasticity turns out to be the ratio of the cross-price elasticity, \(\varepsilon_{ij}\), and the factor share, \(S_j\):

\[
\sigma_{ij}^A = \frac{CC_{ij}}{C_i C_j} = \frac{\varepsilon_{ij}}{S_j} = \frac{\alpha_{ij} - S_i (1 - S_j)}{S_i^2}, \quad \sigma_{ij} = \frac{CC_{ij}}{C_i C_j} = \frac{\varepsilon_{ij}}{S_j} = \frac{\alpha_{ij} + S_i S_j}{S_i S_j},
\] (9)

with \(C_i\) being the derivative of the cost function with respect to the input price \(P_i\) and \(C_{ij}\) the derivative of \(C_i\) with respect to the input price \(P_j\). Blackorby and Russell (1989) argue that (9) is not a measure of the ease of substitution in the case of more than two inputs. They propose to use the Morishima elasticity of substitution instead, which is defined as the derivative of the optimal quantity ratio \(i/j\) with respect to the appropriate price ratio \(P_i/P_j\) under the requirement that only the \(i\)th price in the ratio \(P_i/P_j\) varies:

\[
\sigma_{ij}^M = \frac{PC_{ij}}{C_j} - \frac{PC_{ij}}{C_i} = \varepsilon_{ij} - \varepsilon_{ij}, \quad (9')
\]

In contrast to the Allen-Uzawa elasticities, the Morishima elasticity is asymmetric. This asymmetry results from the fact that the log derivative of the ratio of inputs \(j/i\) with respect to \(P_i/P_j\) depends on whether \(\Delta(P_i/P_j)\) is due to \(\Delta P_i\) or \(\Delta P_j\). The Allen-Uzawa concept does not take this feature into account.
Next, we can determine the willingness-to-pay for public capital by calculating its shadow price as:

\[
P_{KG}^s = -\frac{\partial \ C}{\partial \ KG} = -\frac{C}{KG} \left( \frac{\partial \ln(C)}{\partial \ KG} \right) = -\frac{C}{KG} \varepsilon_{C,KG}
\]  

(10)

The shadow price \( P_{KG}^s \) measures the reduction in the representative firm’s total cost due to one additional unit of public capital. Further, the elasticity of variable input factors with respect to public capital equals

\[
\varepsilon_{i,KG} = \frac{\partial \ ln(\alpha_{i,KG})}{\partial \ ln(KG)} = \frac{\alpha_{i,KG}}{S_i} \frac{P_{KG}^s \cdot KG}{C} = \frac{\alpha_{i,KG}}{S_i} - S_{KG}
\]

(11)

The first term on the right-hand side measures the so-called bias of public infrastructure on factor cost shares. If \( \alpha_{i,KG} > 0 \), the cost share of variable input \( i \) is increasing with the provision of public capital. If \( \alpha_{i,KG} < 0 \), public capital is factor \( i \) saving. The second term of equation (11) can be labelled the shadow cost share of public infrastructure \( S_{KG} \). For a positive shadow price, it measures the decline in all private input quantities (and thus the firm’s costs) to produce a given output \( X \) when public infrastructure increases.

The output elasticity of public capital, \( \varepsilon_{X,KG} \), can be calculated as

\[
\varepsilon_{X,KG} = \frac{\partial \ln(X)}{\partial \ln(KG)} = -\frac{\varepsilon_{C,KG}}{\varepsilon_{C,X}}
\]

(12)

where \( \varepsilon_{C,X} \) measures the cost flexibility of output: \( \varepsilon_{C,X} = \frac{\partial \ln(C)}{\partial \ ln(X)} \).

In a similar way, the output elasticity of a variable input factor \( i \) can be calculated by dividing its cost share \( (S_i) \) through by \( \varepsilon_{C,X} \).

2.2. Input price formation

When the firm decides on the volume of inputs to be used in production, input prices \( P_M, P_K \) and \( P_E \) are given. \( P_M \) concerns the price level of imported intermediate inputs, expressed in domestic currency. It is assumed exogenous. The nominal user cost of capital is a function of the real long-term interest rate \( (R) \), the depreciation rate for private capital \( (\delta_k) \) and the price of capital goods \( (P_{inv}) \). Each of these three determinants are assumed to be exogenous.

\[
P_K = (R + \delta_k) P_{inv}
\]

(13)
The nominal user cost of labour is a function of the nominal gross wage ($W$), which has been bargained ex-ante with a union, and taxes on labour to be paid by the firm/employer ($t_{bs}$).

$$P_E = (1 + t_{bs})W$$

(14)

The nominal gross wage is assumed to result from the maximisation of the following Nash-bargaining maximand:

$$\Omega = U^z \Pi$$

(15)

with: $U$ the union’s utility function, $\Pi$ the firm’s real profit and $z$ the relative bargaining power of the union. The union’s utility typically depends on employment and the real consumption wage to be earned in the firm (in excess of its outside counterpart). Taking this into account, equation (15) can be rewritten as

$$\Omega = E^\rho z \left( \frac{W(1-t_w)}{P_c} - \frac{H(1-t_w)}{P_c} \right)^z \Pi$$

(15’)

with $E$ employment, $\rho$ the union’s relative preference for employment (versus real income), $t_w$ the tax rate on the gross wage to be paid by workers and $P_c$ the aggregate consumer price level. The variable $H$ is the nominal gross value of the expected outside income of a worker who loses his job in the firm$^3$. It is defined as

$$H = \left[ 1 - f(u) \right] \bar{W} + f(u)B$$

(16)

with: $\bar{W}$ the aggregate nominal gross (private sector) wage, $B$ the nominal unemployment benefit and $u$ the aggregate unemployment rate. $f(u)$ indicates the probability that a worker who loses his job in the firm remains unemployed within the relevant period, $1-f(u)$ indicates the probability that he finds another job. The probability to remain unemployed is a positive function of the aggregate unemployment rate (see e.g. Layard et al., 1991, and Nixon and Urga, 1999, for highly similar specifications).

As we show in appendix B, maximising $\Omega$ with respect to the wage $W$ (both in logs) and assuming that in long-run equilibrium $W = \bar{W}$, allows to derive the following equilibrium wage equation$^4$:

---

$^3$ It is assumed that there is no hiring by the public sector.

$^4$ For other assumptions underlying this equation, see appendix B.
\[
\ln W = \beta_0 + \ln\left(\frac{P_X}{P_C}\right) + \beta_p \ln(\frac{P_f}{P_X}) + \beta_q \ln(Q) - \beta_u \ln(u) + \beta_n \ln(\frac{B}{W}) \\
- \beta_{tbs} \ln(1 + t_u) + \beta_n \ln(\tau) + \beta_z(z) \tag{17}
\]

where \( Q \) is the productivity of labour, \( \pi \) is the profit rate, \( P_X \) the firm’s output price and \( B/W \) the unemployment benefit replacement rate. The productivity of labour is measured as real value added (\( Y \)) per worker, the profit rate as real profit per unit of real value added. Algebraically

\[
Q = \frac{Y}{E} \quad \text{with} \quad Y = X - M \tag{17'}
\]

\[
\pi = \frac{\Pi}{Y} \tag{17''}
\]

Finally, \( z \) reflects the union’s bargaining strength. For the long-run, it is theoretically expected that \( \beta_q = \beta_{tbs} = 1 \). Further, given that in Belgium wages are automatically indexed to consumer prices, we also expect \( \beta_p = 1 \). All other parameters are expected to be positive.

2.3. Price setting

The representative firm sells its real output (\( X \)) on both the domestic market (\( X_d \)) and abroad (\( X_f \)) at prices \( P_d \) and \( P_f \) respectively. Prices are set to maximise the (nominal) profit function:

\[
\Pi_a = P_d X_d + P_f X_f - C(X, P, KG, A) \tag{18}
\]

subject to:

\[
X = X_d + X_f
\]

\[
X_d = X_d \left( \frac{P_d}{P}, EXP_d \right) \tag{19}
\]

\[
X_f = X_f \left( \frac{P_f}{P}, EXP_w \right) \tag{20}
\]

with: \( P = P(P_d, P_{im}) \)

\[
\frac{\partial X_d}{\partial \left( \frac{P_d}{P} \right)}, \frac{\partial X_d}{\partial \left( \frac{P_f}{P} \right)} < 0, \quad \frac{\partial X_f}{\partial EXP_d}, \frac{\partial X_f}{\partial EXP_w} > 0
\]

Equations (19) and (20) indicate the demand functions for output. The output levels \( X_d \) and \( X_f \) that the firm can (and will) produce are determined by expected demand and relative prices.
EXP_d^e is expected real aggregate domestic demand, EXP_w^e stands for expected real world imports. Further, P is the aggregate price of goods sold on the domestic market. It is a weighted average of the aggregate domestic price level of domestic producers (P_d) and the price of imported final products (P_{im}). Finally, P^* is the price of foreign final products on the world market. Both P_{im} and P^* are expressed in domestic currency.

The crucial assumption that we make is that the representative firm has market power. Profit maximising price setting on the domestic and world markets leads to the well-known mark-up conditions, where \( e_d \) and \( e_f \) denote the absolute value of the price elasticity of demand for output at home and on the world market. Algebraically,

\[
\begin{align*}
P_d &= \frac{\varepsilon_d}{(\varepsilon_d - 1)} \frac{\partial C}{\partial X} = \mu_d \frac{\partial C}{\partial X} \quad \text{with } \varepsilon_d > 1 \\
P_f &= \frac{\varepsilon_f}{(\varepsilon_f - 1)} \frac{\partial C}{\partial X} = \mu_f \frac{\partial C}{\partial X} \quad \text{with } \varepsilon_f > 1
\end{align*}
\]

Theoretically, there are good reasons for the price elasticity of demand - and thus the mark-up on marginal cost - to depend on the business cycle. The sign of this relation is unclear, however. Further, there may be a positive effect from competitiveness (relative prices) on the mark-up (Layard et al., 1991). We shall come back to this in the empirical section.

Once \( P_d \) and \( P_f \) are known, the firm’s (weighted average) output price \( P_X \) can be defined as:

\[
P_X = \psi P_d + (1-\psi)P_f \quad \text{with } \psi = \frac{X_d}{X} = \frac{X_d}{X_d + X_f}
\]

2.4. Aggregate supply and demand

The assumption of identical firms makes it relatively easy to move to the aggregate economy. For many variables the ‘aggregate’ level is equal to what has been determined at the level of the representative firm. This is the case for wages \( (W) \) and prices \( (P_d, P_f, P_X) \). Other variables like private sector output, value added, employment and input volumes are at the aggregate level equal to \( N \) times their size decided by the individual firm. For these variables, where ambiguity could arise, we shall indicate the aggregate variable with a bar. So,
\[
\tilde{X}_d = NX_d, \quad \tilde{X}_f = NX_f, \quad \tilde{X} = \tilde{X}_d + \tilde{X}_f = NX, \quad \tilde{Y} = NY
\] (24)

\[
\tilde{E} = NE, \quad \tilde{K} = NK, \quad \tilde{M} = NM
\] (25)

For other variables, which typically belong to the aggregate level, like real aggregate domestic demand, real private consumption, real government consumption, real imports of goods and services, the unemployment rate, aggregate labour supply, etc. confusion seems quite unlikely. To keep notation simple, we will not add bars for these variables.

Equation (26) indicates the equilibrium of supply and demand for aggregate private output.

\[
\tilde{X} = EXP_d + (\tilde{X}_f - IM)
\] (26)

where \(IM\) stands for the imported aggregate volume of final goods and services.

Supply has been identified earlier in equations (1), (19)-(22) and (24). Firms produce output in response to expected demand, given relative prices. When planned actual demand deviates from the level expected by the firm (and thus from produced output), stocks will change, forcing ex-post actual demand to adjust.

Equation (27) identifies (ex-post) real aggregate domestic demand (\(EXP_d\)) as the sum of real private consumption expenditures (\(C_p\)), real non-wage government consumption (\(C_g\)) and real aggregate gross investment (\(I\)).

\[
EXP_d = C_p + C_g + I
\] (27)

Among the demand variables, non-wage government consumption will be considered exogenous. The other variables are endogenous. Real private consumption (\(C_p\)) is assumed mainly (i.e. in the long-run) to be a function of real household disposable income (\(Y_{dis}/P_c\)).

\[
C_p = C_p \left( \frac{Y_{dis}}{P_c} \right)
\] (28)

In the short-run changes in consumer confidence, which we consider to be determined by the evolution of the (lagged) aggregate unemployment rate, may also matter.
Assuming that wages and unemployment benefits face the same tax rate, \( Y_{dis} \) can be specified in greater detail as

\[
Y_{dis} = Y_{d0} + (1 - t_w) \left[ (\bar{E}W + \bar{E}_gW_g) + (L_s - \bar{E} - \bar{E}_g)B \right] + (1 - t_c)Y_c
\]

with:
- \( L_s \) aggregate labour supply
- \( \bar{E}_g \) government employment
- \( W_g \) nominal gross government wage
- \( Y_c \) capital income received by households
- \( t_c \) tax rate on capital income

In this equation \( Y_{d0} \) is a function of – among others – transfers other than unemployment benefits and self-employment income. Assuming for simplicity that (i) households use their savings to invest in shares and in long-term bonds and (ii) households receive a fixed fraction \((G_1)\) of after-tax business sector profits as dividends, capital income received by households can be calculated as:

\[
Y_c = G_1(1 - t_{bd})\bar{\Pi}_n + R_n(B_H)_{-1}
\]

with \( t_{bd} \) direct taxes on business sector profits, \( R_n \) the nominal long-term interest rate and \( B_H \) the stock of bonds held by households. \( B_H \) is calculated as \( B_H = (B_H)_{-1} + \Gamma_2(Y_{dis} - C_pP_c) \), with \( \Gamma_2 \) capturing the share of households’ savings invested in bonds.

The consumer price level is a weighted average of the domestic output price level and the price of imported goods and services.

\[
P_c = \left(1 + t_{ind}\right) \left[ \frac{X_d}{X_d + IM} P_d + \frac{IM}{X_d + IM} P_{im} \right]
\]

where \( t_{ind} \) indicates the rate of indirect taxes. Real aggregate gross investment includes real fixed capital formation by the private business sector \( (I_k) \), the government \( (I_g) \) and households \( (I_h) \), as well as changes in stocks \( (I_s) \).

\[
I = I_k + I_g + I_h + I_s
\]
\[ \ddot{K} = (1 - \delta_k) \dot{K} + I_k \]  

Equation (33)

Equation (34) explains real aggregate imports of final goods and services \( (IM) \) as a function of real domestic demand \( \exp_d \) and the relative price \( \frac{P_d}{P_{im}} \). Imports are expected to rise in both arguments.

\[ IM = IM(\exp_d, \frac{P_d}{P_{im}}) \]  

Equation (34)

Equation (35) describes total real imports.

\[ Mt = IM + M \]  

Equation (35)

The model for the demand side is closed by assuming that real exports are determined by real world imports \( (\exp_w) \) and by the price of exports \( P_f \) relative to the price of foreign competitors on the world market \( P^* \).

\[ \ddot{X}_f = X(\exp_w, \frac{P_f}{P^*}) \]  

Equation (36)

2.5. Government budget balance

The three equations in (37) describe the nominal government budget balance as the difference between nominal revenues \( (T) \) and nominal expenditures \( (G) \). The former are mainly determined by the sum of taxes on gross wages in the business sector, taxes on the unemployed, indirect taxes on nominal domestic demand, direct taxes on business sector profits and taxes on households’ capital income. The latter mainly reflect government wages, nominal public non-wage consumption, nominal public investment, unemployment benefits and interest payments on the outstanding debt. \( T_0 \) captures other revenue categories, e.g. taxes on other capital income, taxes on wages earned in the government sector. \( G_0 \) captures other spending categories, e.g. transfers other than unemployment benefits.

\[ \text{Budget} = T - G \]

\[ T = T_0 + (t_{bs} + t_w)W \dot{E} + t_{en}B(Ls - \dot{E} - \dot{E}_g) + \frac{t_{end}}{1 + t_{end}} \left( P_i \left( C_p + C_g \right) + P_{im}I_g \right) + t_{cap} \Pi_n + t_s Y_r \]  

Equation (37)

\[ G = G_0 + W_g \dot{E}_g + P_i C_g + P_{im} I_g + B(Ls - \dot{E} - \dot{E}_g) + RGD \]  

with \( GD (=GD.i-Budget) \) denoting nominal gross government debt.
3. **Dynamic Specification of the System**

The long-run static model described in section 2 relates $n = 25$ endogenous variables to $k = 31$ conditioning variables through $n_1 = 9$ long-run equilibrium relations and $n_2 = n-n_1 = 16$ identities. The parameters of the identities are known and do not need estimation. The empirical implementation of the stochastic long-run equilibrium relations requires a model of disequilibrium adjustment. Ideally one would like to estimate a vector error-correction model (VECM) - treating all variables endogenously - and then test whether exogeneity restrictions and restrictions on the long-run cointegrating vectors implied by the $n_1$ structural equations are valid in practice. Given the large dimension of the system ($n_1+k = 40$) relative to the small dimension of the sample (32 yearly observations), estimation of the full VECM or even a conditional ECM – i.e. conditional on the $k$ exogenous variables - is infeasible though. Moreover, the conditions for integrability of the factor demand system impose a rather specific structure on the dynamic cost function and dynamic factor demand equations.

Greenslade et al. (1999) provide Monte Carlo evidence that in a situation where a fairly rich model needs to be estimated with a limited data set, imposing exogeneity restrictions at the earliest possible stage through the use of economic theory – rather than treating the model as a pure statistical artefact – can yield enormous benefits. Therefore, we specify separate error-correction models for each of the four sectors of the economy – i.e. factor demand and price setting by firms, wage bargaining, total domestic demand and the external sector – conditioning on the $k$ exogenous variables and the endogenous variables determined in the other sectors of the economy. By doing so, we implicitly impose a recursive structure on a more general conditional VECM for the $n_1$ endogenous variables.

In subsection 3.1, we introduce a dynamic cost function that allows for a consistent derivation of a set of inter-related dynamic factor demand equations in general error-correction form. Subsection 3.2 presents single equation ECMs for the remaining endogenous variables ($P_d$, $P_f$, $W$, $C_p$, $X_f$, $IM$).

---

5 From now on, all variables are aggregate measures. For notational purposes, the ‘bars’ that we have introduced in equations (24)-(25) are dropped.

6 Alternatively, the short-run cost function can be specified conditional on a given capital stock, i.e. the restricted cost function approach of Allen (1994). From these conditional estimates, the long-run optimum – allowing for full adjustment of all the factors of production - can be derived. In this approach, dynamics are not explicitly modelled, though.
3.1 Dynamic Cost Function

Assuming that $S_t$ and $S^*_t$ are three-dimensional vectors containing effective and equilibrium cost shares respectively, a generalized error-correction mechanism for the short-run actual cost shares $S_t$ can be expressed as:

$$\Delta S_t = A \Delta S_t^* + (A + B)(S^*_t - S_{t-1}),$$

where $A$ and $B$ are three-dimensional square matrices containing adjustment parameters.

The singularity of the system of factor demand equations requires a number of restrictions on the adjustment parameters. Sufficient conditions under which the adding-up property of factor demands is satisfied are (i) $t' A = m t'$ and (ii) $t' B = b t'$, where $t$ is a unit vector and $m$ and $b$ are scalars. Notice that if one wants to estimate the dynamic system under (38), the adding-up property of factor shares again implies that one of the share equations is redundant and should be deleted from the system. As a result, the short-run adjustment parameters in $B$ are not individually identified, i.e. as $b$ is not known, only the ratios of $b_{11}/b_{21}$ for example are identified. In order to solve this problem, Urga (1996) and Allen and Urga (1999) suggest to estimate a dynamic cost function - which satisfies the necessary conditions for integrability of the factor demand system - of the type:

$$\Delta \ln (C)_i = m \Delta \ln (C^*_t)_i + (1 - m) \sum_{j} n S_{t,j-1} \Delta \ln (P_{1,t}) + b \left( \ln (C^*_t)_{i-1} - \ln (C)_{i-1} \right) + \sum_{j} b_{ij} \left( S^*_{t,j-1} - S_{t,j-1} \right) \Delta \ln (P_{it}) + \text{constant}_i,$$

jointly with the system of factor demand equations. Notice that cost functions may vary in the specification of the time-varying constant term. Applying Shephard’s lemma once more, the corresponding dynamic factor demand equations can straightforwardly be derived from (39) as:

$$\Delta S_{i,t} = m \Delta S^*_{i,t} + \sum_{j} n k_{ij} \left( S^*_{j,t-1} - S_{j,t-1} \right), \quad i = E,M,K$$

where $k_{ij} = m + b - \sum_{j} b_{ij}$ and $k_{ij} = b_{ij}$. The adding-up property of factor shares again implies that one of the share equations is redundant and should be deleted from the system. Identification of the 9 parameters in the $B$-matrix requires 2 additional restrictions. Without loss of generality, we shall impose $Bt = bt$. 
Differentiating the short-run factor demand equations with respect to input prices yields the short-run (Allen-Uzawa) substitution elasticities:

\[
\sigma_{ii}^{sr} = \frac{m \alpha_{ii} - S_i (1 - S_i)}{S_i^2}, \quad \sigma_{ij}^{sr} = \frac{m \alpha_{ij} + S_i S_j}{S_i S_j}
\]  

(41)

The Le Chatelier principle states that, in absolute values, the short-run own price elasticities must be smaller than their long-run counterparts. Using (9) and (41), this condition can be reformulated as \(\alpha_i (m-1) > 0\) (Allen and Urga, 1999). For long-run factor demands being sufficiently elastic (i.e. \(|\sigma_{ii}| > (1-S_i)|\)), we have that \(\alpha_i < 0\) which requires \(m < 1\). Otherwise, if \(|\sigma_{ii}| < (1-S_i)|\), \(\alpha_i > 0\) implies \(m > 1\) for the Le Chatelier principle to hold.

By differentiating the dynamic cost function with respect to output, one can also derive short-run marginal cost, which could be assumed to form the basis of the firms’ short-run domestic pricing behaviour. Following Allen (1997) we did not put any restrictions, derived from the dynamic specification of the cost function, on the dynamics of the price setting equation for besides marginal cost short-run prices are very likely to be affected by other elements like for instance price adjustment costs. The condition that prices are set as a mark-up on marginal cost has been imposed on the long-run structure though. The specification of the ECM for \(P_d\) and \(P_f\) can be found in section 3.2.

### 3.2. Single equation ECMs

The single equation conditional error-correction model that we estimate for domestic prices, wages, private consumption, imports and exports is of the type:

\[
\delta_1(L) \Delta y_{i,t} = \mu_i + \delta_2(L) \Delta x_{i,t} + \alpha_i \left[ y_{i,t-1} - \beta' x_{i,t-1} \right] + \epsilon_{i,t}, \quad i = (P_d, P_f, \ln W, \ln C, \ln X_f - \ln IM)(42)
\]

where \(y_{i,t}\) is the endogenous variable under consideration, \(x_{i,t}\) is a vector of explanatory variables assumed to be relevant for explaining \(y_{i,t}\), \(\epsilon_{i,t}\) is a stationary error term and \(\delta_1(L)\) and \(\delta_2(L)\) are lag polynomials of order \(p\) and \(q\) respectively. Note that to eliminate the impact of the steady increase in openness of the Belgian economy, we estimate the ratio of exports over imports of final goods and services (i.e. \(X_f/IM\)) instead of equations (34) and (36) separately. The implicit assumption underlying this choice is that the increasing openness of the Belgian economy affects imports and exports with the same magnitude.
Following Kremers et al. (1992), we test the null-hypothesis of no cointegration in the long-run equations by testing whether $\alpha_i = 0$ in (42). Under the null-hypothesis of no cointegration the $t$-test for $\alpha_i = 0$ has a non-normal distribution. Kremers et al. (1992) suggest using the MacKinnon critical values associated with the comparable Dickey-Fuller test (DF) in the Engle-Granger two-step procedure as a first approximation.

One obvious possible severe drawback of the single-equation ECM-estimator is that it is necessary to assume that $x_t$ is weakly exogenous. In the current setting, this boils down to imposing from the outset (i) weak exogeneity of the $k$ conditioning variables and (ii) recursiveness of the conditional ECM for the endogenous variables. Although this assumption results in empirical tractability of the model outlined in section 2, it should be stressed that inference from the conditional ECM may be invalid if this assumption were violated in practise. A Monte-Carlo study by Inder (1993) suggests however that even in the presence of endogenous explanatory variables, the ECM-estimator might give good estimates and valid $t$-statistics.

4. **EMPIRICAL ANALYSIS**

The dynamic model outlined in section 3 has been estimated for the Belgian economy using three-stage least squares (3SLS). The data are annual from 1965 to 1996. Data sources are described in appendix A. Note that as a measure for the stock of knowledge ($A$) we use cumulated data for patents granted by the US Patent and Trademark Office (for details and justification, see Everaert and Heylen, 2000). The maximum lag length of the error-correction models was fixed at two.

4.1. **Dynamic cost function and dynamic factor demands**

Parameter estimates of the dynamic cost function and related dynamic factor share equations are presented in table 1. The results show that the cost function is theoretically well-behaved, i.e. the conditions that it should be positive for positive input prices and positive output, monotonically non-decreasing in input prices and output and concave in input prices are satisfied. Note that symmetry and linear homogeneity in factor prices were imposed from the outset. The restriction of homogeneity of the production function stated in equation (7) was rejected by the data and therefore not imposed. Further, the residual diagnostics reported in
the right-hand side of table 1 show that the cost function is also empirically well-behaved, i.e. the residuals from the cost function and the factor share equations do not show clear signs of non-normality or autocorrelation. The high $R^2$-values show that the model is able to explain most of the variance in the data.

**Table 1** Parameter estimates (3SLS) and residual diagnostics of the dynamic cost function and factor share equations (1965-96) $^{a,b}$

<table>
<thead>
<tr>
<th>Long-run parameters</th>
<th>Coef</th>
<th>Variable</th>
<th>Value</th>
<th>stdv</th>
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</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>14.890</td>
<td>Cst</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_E$</td>
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<td>$\ln(P_M/P_K)$</td>
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<td>$\ln(P_M/P_K)$</td>
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<td>$\ln(KG)$</td>
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</tr>
<tr>
<td>$\alpha_A$</td>
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<td>$\ln(A)$</td>
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<td></td>
</tr>
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<td>$\ln^2(X)$</td>
<td>(0.31)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{KG,KG}$</td>
<td>-1.41</td>
<td>$\ln^2(KG)$</td>
<td>(0.67)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Short-run adjustment</th>
<th>Coef</th>
<th>Variable</th>
<th>Value</th>
<th>stdv</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>1.02</td>
<td>$\Delta \ln \left( C^*_{t} \right)$</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>1.24</td>
<td>$\left( \ln \left( C^*<em>{t} \right) - \ln \left( C</em>{t-1} \right) \right)$</td>
<td>(0.19)</td>
<td></td>
</tr>
<tr>
<td>$B_{EM}$</td>
<td>0.55</td>
<td>$(S_{M,t-1}^* - S_{M,t-1}) \ln(P_{E,t})$</td>
<td>(0.11)</td>
<td></td>
</tr>
<tr>
<td>$b_{E,K}$</td>
<td>0.93</td>
<td>$(S_{K,t-1}^* - S_{K,t-1}) \ln(P_{E,t})$</td>
<td>(0.12)</td>
<td></td>
</tr>
<tr>
<td>$b_{M,E}$</td>
<td>0.74</td>
<td>$(S_{E,t-1}^* - S_{E,t-1}) \ln(P_{M,t})$</td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>$b_{M,K}$</td>
<td>0.60</td>
<td>$(S_{K,t-1}^* - S_{K,t-1}) \ln(P_{M,t})$</td>
<td>(0.12)</td>
<td></td>
</tr>
<tr>
<td>$b_{K,E}$</td>
<td>0.74</td>
<td>$(S_{E,t-1}^* - S_{E,t-1}) \ln(P_{K,t})$</td>
<td>(0.07)</td>
<td></td>
</tr>
<tr>
<td>$b_{K,M}$</td>
<td>0.78</td>
<td>$(S_{M,t-1}^* - S_{M,t-1}) \ln(P_{K,t})$</td>
<td>(0.07)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Residual diagnostics $^{c}$</th>
<th>Coef</th>
<th>Variable</th>
<th>Value</th>
<th>stdv</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost function</strong></td>
<td></td>
<td>Normality</td>
<td>$\chi^2(2) = 1.86[0.39]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Serial correlation</td>
<td>$\chi^2(4) = 3.36[0.50]$</td>
<td></td>
</tr>
<tr>
<td><strong>Employment cost share</strong></td>
<td></td>
<td>Normality</td>
<td>$\chi^2(2) = 0.69[0.71]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Serial correlation</td>
<td>$\chi^2(4) = 5.60[0.23]$</td>
<td></td>
</tr>
<tr>
<td><strong>Material cost share</strong></td>
<td></td>
<td>Normality</td>
<td>$\chi^2(2) = 0.68[0.71]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Serial correlation</td>
<td>$\chi^2(4) = 5.19[0.27]$</td>
<td></td>
</tr>
</tbody>
</table>

- Standard errors in parentheses, $p$-values between square brackets.
- All variables are normalised by subtracting their sample mean.
- Jarque-Bera test for residual normality and Box-Pierce test for fourth-order serial correlation.

The adjustment parameters are reported in the right-hand side of table 1. The main coefficient of adjustment ($m$) is estimated to be 1.02. Given the positive sign of the $\alpha_i$’s, $m$ being numerically larger than 1 ensures that the Le Chatelier principle is satisfied. Note that $m$ is statistically not greater than one, though, implying that there is only a small difference between short-run and long-run elasticities. Basically, this means that there is not much
rigidity in factor shares, implying short-run costs to be only a small fraction above optimal long-run costs in the aftermath of shocks shifting the factor shares. A similar result was obtained by Nixon and Urga (1999). They conclude that despite the fact that the dynamic specification allows for different speeds of adjustment in individual factor shares, the common immediate adjustment coefficient $m$ imposes a significant element of common response to shocks. Although the long-run cost function was found to be both theoretically and empirically well-behaved, $m$ not being significantly larger than one suggests that there might be some misspecification in the short-run structure of the system.

Table 2  Price elasticities, Allen-Uzawa and Morishima elasticities of substitution

<table>
<thead>
<tr>
<th>Price elasticities ($\varepsilon_{ij}$)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Row($i$) wrt col($j$)</td>
<td>$E$</td>
<td>$M$</td>
<td>$K$</td>
</tr>
<tr>
<td>$E$</td>
<td>-0.23 (0.09)</td>
<td>0.26 (0.09)</td>
<td>-0.02 (0.08)</td>
</tr>
<tr>
<td>$M$</td>
<td>0.24 (0.09)</td>
<td>-0.36 (0.11)</td>
<td>0.29 (0.11)</td>
</tr>
<tr>
<td>$K$</td>
<td>-0.01 (0.03)</td>
<td>0.10 (0.04)</td>
<td>-0.27 (0.08)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Allen elasticities of substitution ($\sigma_{ij}^A$)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Row($i$) wrt col($j$)</td>
<td>$E$</td>
<td>$M$</td>
<td>$K$</td>
</tr>
<tr>
<td>$E$</td>
<td>-0.53 (0.20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>0.58 (0.21)</td>
<td>-0.87 (0.27)</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>-0.05 (0.18)</td>
<td>0.71 (0.27)</td>
<td>-1.90 (0.53)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Morishima elasticities of substitution ($\sigma_{ij}^M$)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Row($i$) wrt col($j$)</td>
<td>$E$</td>
<td>$M$</td>
<td>$K$</td>
</tr>
<tr>
<td>$E$</td>
<td>-</td>
<td>0.48 (0.17)</td>
<td>0.23 (0.10)</td>
</tr>
<tr>
<td>$M$</td>
<td>0.62 (0.20)</td>
<td>-</td>
<td>0.46 (0.14)</td>
</tr>
<tr>
<td>$K$</td>
<td>0.25 (0.10)</td>
<td>0.57 (0.18)</td>
<td>-</td>
</tr>
</tbody>
</table>

* Evaluated at the sample mean, standard deviations in parentheses.

Given that the duality conditions hold, the parameter estimates in table 1 can be utilised to uncover the characteristics of the underlying production function. Table 2 presents own- and cross-price elasticities, Allen-Uzawa elasticities of substitution and Morishima elasticities of substitution, all evaluated at the sample mean. As the differences between the short- and long-run elasticities are very small, only the long-run values are reported. The own-price elasticities are all negative, being about -0.25 for labour and capital and -0.36 for imported materials. The elasticities of substitution show that all variable input factors are Allen substitutes except for capital and labour which appear to be independent. Table 2 also includes estimates of the Morishima elasticities, for it was argued that they are a more
accurate measure of substitutability. These elasticities are reported such that each row $i$ shows how the ratio of inputs ($j/i$) responds to a change in the price $P_i$. All variable input factors are now shown to be substitutes.

Table 3 reports output elasticities, a measure of returns to scale, the shadow price of public capital and the elasticities of variable inputs with respect to public capital, all evaluated at the sample mean. Let us start with the impact of public capital on private sector production cost and factor demand. The statistically significant positive shadow price of public capital shows that public infrastructure is cost saving. The estimates imply that an increase in the public capital stock by 1 EURO reduces long-run private sector cost by 0.24 EURO. The negative sign of $\alpha_{E,KG}$ in the labour demand equation suggests a negative bias of public capital on the labour cost share. The positive shadow price of public capital enlarges its negative impact on private employment. As a result, an increase in the public capital stock by 1%, for a given output, reduces private sector employment by about 0.32%, indicating a substitutive relationship. At first sight, this substitution effect may appear large. Note, however, that in 1996 a 70% increase in public investment was required to raise public capital with 1%. The private capital cost share on the other hand is clearly increasing in the public capital stock ($\alpha_{K,KG} = -\alpha_{E,KG} - \alpha_{M,KG} = 0.09 > 0$). The net effect of public capital on private capital remains positive, i.e. an 1% increase in the public capital stock raises private sector capital by 0.33%. Note that the large standard error suggests that this effect is not very accurately measured, though.

Noting that the shadow cost share of public capital ($S_{KG}$) equals the elasticity of private cost with respect to public capital, a measure of returns to scale ($\lambda$) can be calculated by inserting the cost elasticities $\varepsilon_{C,X}$ and $\varepsilon_{C,KG}$ in equation (6). The estimates show that, evaluated at the sample mean, the production function is characterised by increasing returns over all inputs, including public capital (i.e. $\lambda = 1.40$). The individual output elasticities of private inputs are very much in line with expectations while the obtained output elasticity of public capital ($\varepsilon_{X,KG} = 0.31$) confirms the result in Everaert and Heylen (2000).

The insignificant term $\alpha_A$ in the cost function further reveals that our proxy for knowledge accumulation ($A$) does not capture the steady decline in costs over time due to technological progress. Notice that $1/\varepsilon_{C,X}$ being larger than one implies increasing returns to private sector factors of production. Increasing returns might arise from a number of reasons, e.g. learning-
by-doing, external or internal economies of scale. Clearly, increasing returns to private inputs provide an alternative explanation – other than knowledge accumulation - for multifactor productivity growth as measured by the traditional Solow residual. The positive output elasticity of public capital mentioned above further contributes to multifactor productivity growth.

**Table 3** Characteristics of the underlying production function $^a$

<table>
<thead>
<tr>
<th>Output elasticities, cost flexibility and returns to scale</th>
<th>Shadow cost and factor biases with respect to public capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{X,E}$ 0.51(0.03)</td>
<td>$p_{KG}^s$ 0.24(0.05)</td>
</tr>
<tr>
<td>$\varepsilon_{X,M}$ 0.45(0.02)</td>
<td>$\varepsilon_{C,KG}$ -0.32(0.17)</td>
</tr>
<tr>
<td>$\varepsilon_{X,K}$ 0.13(0.01)</td>
<td>$\varepsilon_{C,X}$ -0.28(0.05)</td>
</tr>
<tr>
<td>$\varepsilon_{X,KG}$ 0.31(0.05)</td>
<td>$\varepsilon_{C,KG}$ -0.28(0.05)</td>
</tr>
<tr>
<td>$\varepsilon_{C,X}$ 0.92(0.03)</td>
<td>$\varepsilon_{m,KG}$ -0.46(0.18)</td>
</tr>
<tr>
<td>$\varepsilon_{C,K}$ 1.40(0.07)</td>
<td>$\varepsilon_{k,KG}$ 0.33(0.29)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Factor biases with respect to output</th>
<th>Factor biases with respect to technological progress</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{E,X}$ 0.19(0.09)</td>
<td>$\varepsilon_{E,A}$ 0.90(0.23)</td>
</tr>
<tr>
<td>$\varepsilon_{M,X}$ 1.77(0.10)</td>
<td>$\varepsilon_{M,A}$ -0.80(0.28)</td>
</tr>
<tr>
<td>$\varepsilon_{K,X}$ 0.69(0.24)</td>
<td>$\varepsilon_{K,A}$ -0.23(0.52)</td>
</tr>
</tbody>
</table>

$^a$ Evaluated at the sample mean, standard deviations in parentheses.

Finally, the lower half of table 3 reports the factor biases to output and to our proxy for knowledge accumulation. Although the knowledge stock was found not to contribute to multifactor productivity, it has significant effects on private sector inputs. Demand for labour appears to be increasing in knowledge while imported materials are decreasing. The private capital stock is mainly unaffected by knowledge accumulation. Both materials and the capital stock were found to be increasing in output.

The factor biases can be used to calculate the growth in real wages warranted by output growth and knowledge accumulation. Assuming that the warranted real wage is the one that, ceteris paribus, holds the labour cost share constant, the growth in this wage can be calculated as $\Delta \ln (X) - \left( \frac{\alpha_{E,X}}{\alpha_{E,E}} \right) \Delta \ln (X) - \left( \frac{\alpha_{E,A}}{\alpha_{E,E}} \right) \Delta \ln (A)$. Evaluated at the sample mean, the estimates imply a warranted yearly real wage growth of 2.4%. Important to note is that the employment share in cost does not depend on public capital ($\varepsilon_{E,KG} \approx 0$). Therefore, the productivity benefits resulting from an increase in the public capital stock can - for given costs - only be incorporated in real wages at the expense of a lower employment.
4.2. Price setting

The estimated equilibrium domestic price setting equation is specified as:

\[ P_d = \mu_{d0} + \mu_{d1}bc + \mu_{d2} \left( \frac{P_d}{P_{im}} \right)_{t-1} + \mu_{d3} oil + \frac{\partial C}{\partial X} - \mu_{d3} d_{93}^{96}. \]

Besides the business cycle (\(bc\)) and relative prices (\(P_d/P_{im}\)) as measure of competitiveness, the mark-up on marginal cost (\(\mu_d\)) in the long-run specification is also a function of the evolution of oil prices (\(oil\)). The motivation for including oil prices in the mark-up stems from the observation that firms appear to be reluctant to transmit oil price shocks fully in domestic prices. Notice that we have also included a level-shift dummy, being 1 in the period 1993-1996, in the cointegrating relationship. The motivation for including this dummy is that the price indices used were no longer available from 1993 onwards. The extension of the series over the period 1993-1996 was done at the National Bank of Belgium using a slightly less sophisticated methodology, resulting in a possible break in the series in 1993.

The estimated long-run relation and the equilibrium adjustment parameters of the domestic price equation are reported in table 4. As relative prices (\(P_d/P_{im}\)) are found to be insignificant, they are not included. Insignificant short-run variables are also dropped in the estimation process. The high significance of the error-correction term (\(\xi_{ECM} = -5.11\)) suggests that the reported long-run relation is indeed a cointegrating vector. The results show that in the long run, when the business cycle is neutral (\(bc=0\)) and oil prices are at their average value (\(oil=0\)), firms set domestic prices 46% higher than the marginal cost of production. Note that the positive coefficient on \(bc\) implies that the mark-up is pro-cyclical. The significant negative coefficient on \(oil\) confirms that oil price increases are not fully transmitted in the domestic price level but partly absorbed by firms’ profit rates.

Although graphical inspection reveals a close relation between export prices and marginal cost, we are unable to detect a stable causal relation running from marginal cost to export prices. Rather, export prices are found to exert some feedback on marginal cost. This observation suggests that – in contrast to the theory presented in section 2 - exporting firms are price takers on the international market. In order to safeguard profitability, (marginal) cost has to adjust to changes in prices on the export market (this adjustment for instance occurs

\[^7\] Oil prices have been included as deviations from the average oil price over the sample period.
Public capital and labour market performance in Belgium

3.2.4

through changes in labour productivity). Although deviations appear to be long lasting, a tentative exploration of the relation between $P_f$ and $P^*$ indeed shows a close connection in the long run. In the remainder of this paper export prices ($P_f$) are therefore assumed to be exogenously determined on the world market.

4.3. Wage bargaining

The estimated long-run cointegrating wage bargaining relation is specified as:

$$\ln(W) = \beta_0 + (1 - \beta_p) \ln(P_X) + \beta_p \ln(P_c) + \beta_q \ln(Q) - \beta_u \ln(u) + \beta_b \ln(B_{rr})$$

$$- \beta_{tbs} \ln(1 + t_{bs}) + \beta \ln(\pi) + \beta_{c} \ln(U_{bs})$$

where $U_{bs}$ measures union bargaining strength, defined as union membership as a percentage of the labour force, and $B_{rr}$ is the gross benefit replacement rate. The results are reported in table 4. The ECM statistic shows that the null hypothesis of no cointegration can be rejected at the 5% level of significance. Since the results from estimating an unrestricted wage function show that there is no significant long-run effect of taxes to be paid by the firm ($t_{bs}$) and the real profit rate ($\pi$) on the bargained wage, we only report results imposing $\beta_{tbs} = \beta_{\pi} = 0$. Note that taxes to be paid by the firm have a strong short-run impact on the bargained wage. Next, the observation that $\beta_p$ is not statistically different from 1, is consistent with wages being automatically indexed to consumer prices. However, we did not impose $\beta_p = 1$ as this slightly deteriorated the fit of the model. Further, the coefficient on labour productivity is significantly smaller than one, indicating that – at least over the sample period - productivity growth is not fully transmitted into wages. The remaining coefficients all have signs confirming theoretical predictions. Unemployment appears to exert only a moderate downward pressure on real wages, though. Union membership in contrast has a strong positive impact on the bargained real wage.

4.4. Aggregate consumption function

The results from estimating the private consumption function (table 4) show a long-run marginal propensity to consume equal to 0.82. Further, we note that in the short-run consumption falls in response to a rise in unemployment. The underlying idea is that a rise in unemployment negatively affects consumer confidence. The ECM statistic being significant at the 1% level strongly suggests cointegration.
Table 4 Parameter estimates (3SLS) and residual diagnostics for price setting, wage bargaining, aggregate consumption demand and the external sector (1965-96)

**Price Setting**

\[ P^*_d,t = (1.464 + 3.403 bc_t - 0.008 oil_t) (\partial C_t / \partial X_t) + 0.321 d_{96}^6 \]

(0.063) (0.797) (0.002) (0.048)

\[ \Delta P_{d,t} = -0.273 (P_{d,t-1} - P^*_d,t) + 0.374 \Delta P_{d,t-1} + 0.194 \Delta (\partial C_t / \partial X_t) - 0.195 \Delta (\partial C_{t-1} / \partial X_{t-1}) + 0.033 \Delta d_{96}^6 \]

(0.053) (0.138) (0.047) (0.067) (0.009)

**Wage Bargaining**

\[ \ln(W_t^*) = 2.564 + 0.708 \ln(P_{c,t}) + 0.292 \ln(P_{x,t}) + 0.743 \ln(Q_t) - 0.112 \ln(u_t) + 0.262 \ln(B_t/W_t) \]

(1.260) (0.196) (-) (0.092) (0.035) (0.068)

\[ \Delta \ln(W_t) = -0.526 (\ln(W_t) - \ln(W_{t-1})) - 0.093 \Delta \ln(W_{t-1}) + 0.407 \Delta \ln(P_{c,t}) + 0.503 \Delta \ln(Q_t) \]

(0.092) (0.110) (0.103) (0.181)

**Aggregate consumption function**

\[ C^*_p,t = 251491 + 0.822 (Y_{dis,t}/P_{c,t}) \]

(155864)(0.024)

\[ \Delta C_{p,t} = -0.305 (C_{p,t-1} - C^*_p,t-1) + 0.260 \Delta (Y_{dis}/P_{c,t}) - 4974024 \Delta u_t + 3166849 \Delta u_{t-1} - 323668 \Delta u_{t-2} \]

(0.051) (0.136) (687758) (646876) (1024098)

**The coverage rate of exports over final imports**

\[ \ln(X_f/IM)^*_t = 7.518 - 0.197 \ln(P_d/P_{IM}) - 0.425 \ln(P_f/P^*_t) - 0.413 \ln(EXP_d,t) \]

(1.099) (0.081) (0.169) (0.071)

\[ \Delta \ln(X_f/IM)_t = -0.670 (\ln(X_f/IM)_t - \ln(X_f/IM^*_t)) - 0.692 \Delta \ln(P_d/P_{IM}) + 0.978 \Delta \ln(EXP_w,t) \]

(0.116) (0.188) (0.417)

**Cointegration tests**

<table>
<thead>
<tr>
<th>( P_{d,t} )</th>
<th>( \ln(W_t) )</th>
<th>( C_{p,t} )</th>
<th>( \ln(X_f/IM)_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{ECM} )</td>
<td>-5.11</td>
<td>-5.72</td>
<td>-5.92</td>
</tr>
<tr>
<td>MacKinnon 1% critical value</td>
<td>-5.74</td>
<td>-6.16</td>
<td>-4.28</td>
</tr>
<tr>
<td>MacKinnon 5% critical value</td>
<td>-4.90</td>
<td>-5.29</td>
<td>-3.55</td>
</tr>
</tbody>
</table>

**R² and residual diagnostics**

<table>
<thead>
<tr>
<th>( \Delta P_{d,t} )</th>
<th>( \Delta \ln(W_t) )</th>
<th>( \Delta C_{p,t} )</th>
<th>( \Delta \ln(X_f/IM)_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>0.88</td>
<td>0.97</td>
<td>0.90</td>
</tr>
<tr>
<td>Normality</td>
<td>( \chi^2(2) = 0.77 [0.68] )</td>
<td>0.52 [0.77]</td>
<td>1.77 [0.41]</td>
</tr>
<tr>
<td>Serial correlation</td>
<td>( \chi^2(4) = 4.60 [0.33] )</td>
<td>3.71 [0.45]</td>
<td>2.84 [0.58]</td>
</tr>
</tbody>
</table>

---

\[ a \] Standard errors in parentheses, p-values between square brackets.

\[ b \] Jarque-Bera (JB) test for residual normality and Box-Pierce (BP) test for fourth-order serial correlation.
4.5. The coverage rate of exports over final imports

Turning to the coverage rate of exports over final imports, both higher domestic demand and higher domestic prices and export prices reduce exports relative to imports. Higher import prices and higher prices on the world market have the opposite effect. Note that we have imposed price homogeneity by including \( \ln(P_d/P_{im}) \) and \( \ln(P_f/P^*) \). Real foreign expenditures did not have any significant effect on \( Xf/IM \) and were therefore dropped from the cointegrating vector. Again the null of no cointegration is clearly rejected.

5 SYSTEM SIMULATIONS

In this section, we explore the impact of changes to the public capital stock on – among other variables - unemployment, economic growth, nominal wages, prices, productivity and the government budget deficit by simulating the model estimated in section 4 under an alternative public investment policy. More specifically, we analyse how the economy would have evolved if the strong decline in public investments during the period 1982-89 had not occurred. Taking the evolution of the exogenous variables as given, section 5.1. tests the ability of the estimated model to capture the actual evolution in the endogenous variables by dynamically simulating the model over the period 1970-1996. In section 5.2., we compare the simulation results under the alternative public investment policy with this benchmark simulation.

5.1. Benchmark simulation

Figure 2 compares the actual evolution in total cost (\( C \)), the cost share of labour (\( S_L \)), the cost share of capital (\( S_K \)), the cost share of materials (\( S_M \)), real output (\( X \)), productivity of labour (\( Q \)), the nominal gross wage (\( W \)), the consumer price level (\( P_C \)), the unemployment rate (\( \mu \)) and the government budget (\( \text{Budget} \)) with the evolution of these variables obtained from the dynamic simulation over the period 1970-96. Endogenous variables calculated from the benchmark simulation are indicated with ‘_bench’. Generally spoken, our model is able to capture the major movements in the endogenous variables. The fit is not perfect, though. The model appears to slightly overestimate economic activity in the period 1981-85 while

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8 Due to lack of data, \( \Gamma_2 \) is assumed to be equal to 1.
9 The model was simulated over the period 1970-96 since no data were available from 1965 onward for some of the variables included in the identities.
Figure 2 Benchmark simulation (1970-96)
underestimating it in the period 1990-93. As a result output, total cost, the cost share of labour and nominal wages are overestimated in the first period and underestimated in the second period. The unemployment rate, the government deficit and the cost share of materials in contrast are underestimated in the period 1981-85 while being overestimated in the period 1990-93

5.2. Alternative public investment policy

Since the beginning of the 1970s, a lot of OECD countries have witnessed a massive build-up of government debt. In Belgium for instance, the gross government debt ratio exceeded 100% of GDP in 1982, whereas ten years earlier it was less than 65%. The explosion of the government debt and deficit forced the Belgian government to adopt two long-lasting fiscal consolidation programmes, in particular in 1982-87 and 1992-96. Since it is politically easier to cut back on investment spending than on current expenditures (which include for instance transfers and government wages), tight fiscal policy very often involves strong cuts in public investment. Figure 3 shows that real public investment in Belgium fell back from 4.63% of (business sector) GDP in 1981 to less than 1.75% in 1989. Fiscal consolidation in the 1990s was realised mainly by increases in taxes, leaving current expenditures and investments largely unaffected.

**Figure 3** Real public investments (% of business sector GDP) and real public capital stock (in billions) under the alternative investment policy

In this section, we analyse how the economy would have evolved if the strong decline in public investments during the period 1982-89 had not occurred. Special attention is paid to labour market implications. As an interesting by-product, the simulations allow for a time
series test of the well-known hypothesis that in order to be successful, fiscal consolidation should not rely on cutting public investment.

The alternative public investment series ($I_g\_sim$) is obtained by fixing the share of public investment in (business sector) GDP on its 1981 level during the period 1982-89. From 1989 onward, the simulated investment rate is again allowed to vary - starting from its higher 1989 level - with the actual rate. From the alternative investment series, we then calculate the corresponding public capital stock ($K_g\_sim$) using the perpetual inventory method used by the Belgian Federal Planning Bureau in the calculation of the original capital stock series. Figure 3 compares the actual and the alternative series. Under the alternative investment policy there is clearly no strong decline in the growth rate of the public capital stock. In 1996, the hypothetical public capital stock would be 28.5% higher than observed in reality.

Inserting $I_g\_sim$ and $K_g\_sim$, the model is dynamically simulated over the period 1970-1996. Figure 4 reports the simulation results for the key endogenous variables (indicated with ‘\_sim’). Concerning the impact of higher public capital spending on private sector performance, a number of interesting observations stand out. (i) Although public capital is cost saving, a shock to public capital investment does not lower the volume of private sector cost. In contrast, 1996 costs are 2.6% higher than the benchmark simulation. (ii) The cost shares of labour and materials decrease slightly, in favour of a higher cost share of private capital. Given the small increase in total costs, the 1996 private sector capital stock is 13.2% larger, while the volume of imported materials is largely unaffected (graphs not reported). (iii) Compared to the benchmark simulation, 1996 real private sector output is 5.7% higher. (iv) The average annual growth rate of private sector labour productivity over the period 1982-1996 equals 2.5% compared to 1.6% in the benchmark simulation. (v) The average annual growth rate of nominal wages is 0.3% point higher, resulting in a 6.8% higher nominal wage in 1996. As consumer prices are largely unaffected, the increase in the nominal wage implies a similar increase in the real wage. (vi) Given the moderate increase in cost, the decrease in the cost share of labour combined with the increase in wages implies a decrease in private sector employment with 162000 units. As a result, the 1996 unemployment rate has increased from 11.8% in the benchmark simulation to 15.6%, an increase with 3.8% points. (vii) As higher output has been produced with more or less stable costs, profits have increased strongly from 12.7% of GDP in the benchmark simulation to 16% of GDP under the alternative public investment policy (graph not reported).
Figure 4 Simulation under the alternative public investment policy (1970-96)
In order to give a better insight in how the reported results are produced, figure 5 outlines the major channels of influence of a shock to public capital spending on private sector performance, with special attention to the evolution of private sector input factors. Generally speaking public capital affects private sector inputs through three different channels: (i) complementary or substitution effects for given output, (ii) effects from changes in aggregate demand and the level of output (iii) effects from changes in real wages. Each of these channels can be subdivided in a ‘direct’ effect through changes in factor cost shares (for given total costs) and an ‘indirect’ effect through changes in total costs (for given factor cost shares).

As for the first channel, the simulations of the cost function show that direct complementary or substitution effects [2] account for a 0.4% and 1.8% point decrease in the cost shares of labour and materials respectively. The cost share of private capital increases with 2.2% points. For given cost, the decrease in the labour cost share implies a decrease in private sector employment with 23000 units. Given the statistically significant positive shadow price of public capital, 1996 total cost (for given output, wages and prices) decreases with 5% [3]. For given cost shares, this reduction in total cost implies a decrease in private sector employment with 118000 units [4]. The total effect of complementary or substitution effects on the unemployment rate through the combination of relations [2], [3] and [4] - measured by the factor bias of labour with respect to public capital reported in table 3 - is plotted in the left-hand side panel of figure 6. In 1996, substitution of labour for public capital accounted for a 3.3% point increase in unemployment. Public capital and private capital were found to be complements, i.e. the 1996 private capital stock has increased (for given demand and wages) by 8.5% in response to the shock to public investment.

The second channel concerns the effect of public capital on factor inputs through changes in aggregate demand. In the first place, aggregate demand increases due to the increase in public investment [5], accounting for a 1.4% increase in 1996 real output compared to the benchmark simulation. As the effects of the increase in the public capital stock propagate through the system, output is further affected by changes in private sector investment and private consumption\(^\text{10}\). Private sector investment, calculated from changes in the private sector capital stock [8], contributed 2.9% to the increase in output. Private domestic

\(^{10}\) Note that exports are not affected by the change in public investment as they are sold on the export market at an exogenously determined price.
consumption, affected by household disposable income [11] and the aggregate domestic price level [16], contributed another 1.4%. As a result, the total increase in 1996 output amounts 5.7%. The simulations of the cost function show that this increase in aggregate demand [6] implies a 2.0% and 0.2% point reduction in the cost shares of labour and capital respectively. The cost share of imported materials increases with 2.2% points. For given cost, the decrease in the labour cost share implies a decrease in private sector employment with 148000 units. Given a cost flexibility of output of 0.92, the increase in private sector output implies an increase in 1996 total cost of about 5.2% [7]. For given cost shares, this increase in total cost implies an increase in private sector employment with 154000 units [4]. Taking into account the decrease in the labour cost share, this leaves a positive net effect on employment - as measured by the factor biases with respect to output reported in table 3 - of 6000 units or a decrease in the unemployment rate with 0.13% point (see figure 6). Although the idea that employment and output are independent in the long run is consistent with a-priori expectations, one would expect to find a (considerable) positive impact in the short run. The small short-run impact implied by the estimates is most probably due to the specification of the dynamic cost function implying a significant amount of common response of factor shares to shocks. As we have mentioned before, the resulting speed of adjustment towards the equilibrium appears to be overestimated.

**Figure 5** Impact of shocks to public investment on private sector performance
Finally, real wages are indirectly affected by an increase in the public capital stock through changes in labour productivity [9], changes in the unemployment rate [10] and changes in the aggregate domestic price level [16]. As prices are largely unaffected, they do not have a significant effect on wages.

**Figure 6** Deviation of the unemployment rate and the government budget (% of GDP) from their benchmark simulation, split-up according to explanatory factors

Due to higher output being produced with less labour, 1996 labour productivity is 13.3% higher compared to the benchmark simulation. As only 74% is transmitted into wages, the impact of the increase in productivity on wages amounts 9.8% in 1996. The question is how an increase in real wages due to productivity growth affects employment. Since in the model outlined in section 3 both workers and firms have market power, each group will attempt to obtain a particular share of the economy’s product. The unemployment rate reconciles the real wage that is claimed by wage bargainers and the real wage that is consistent with the price setters’ profit aspirations. As a rise in the bargained real wage in line with increasing labour productivity is not inconsistent with the real wage from the price setters’ profit aspirations, there is at first sight no need for unemployment to change as there are no competing claims to reconcile. In principle, the growth rate in real wages falling behind productivity growth should even bring about increasing employment. However, one should keep in mind that the rise in the real wage induces, ceteris paribus, an increase in the relative factor cost of labour. The degree to which relative factor costs affect employment depends on the elasticities of substitution between labour on the one hand and capital and imported materials on the other. In the absence of substitution between factor inputs, labour demand is unaffected. If there is scope for substitution, though, firms will respond by substituting labour for the relatively
cheaper capital and materials.\textsuperscript{11} By doing so firms are able to increase their profits by appropriating a larger part of the benefits of the initial increase in labour productivity. Given a price elasticity of labour with respect to wages of about -0.23, the 9.8\% increase in wages due to higher productivity growth accounts for – through the combined effect of relations [12], [13] and [4] - a decrease in 1996 private sector employment with 2.3\% or 50000 units.

As the increase in labour productivity and the substitution of labour for public capital have, ceteris paribus, raised the unemployment rate, the full impact of wages on employment depends on how unions react to the higher rate of unemployment. If the responsiveness of real wages to unemployment is large enough, the increase in real wages due to higher labour productivity will be overruled by the negative effect of unemployment on real wages. Given the estimated low responsiveness of real wages to unemployment, the increase in unemployment has offset only part of the rise in real wages, though, leaving a net positive effect of 6.8\%. This 6.8\% increase in wages accounts for a decrease in 1996 private sector employment with 1.4\% or 31000 units. To contribution to the change in unemployment amounts 0.7\% points in 1996 (see figure 6).

Besides the impact on private sector performance, the simulations also allow to detect the effect of higher public investment on the government budget. Figure 4 shows that the gross government budget balance worsens considerably during the years of the fixed public investment rate (1982-89), reaching a maximum increase in the deficit of 2.9\% of (business sector) GDP in 1990. From 1991 onwards the budget improves. As a result, the increase in the deficit has declined to 0.8\% of GDP in 1996. Although this is still a significant burden on the government’s financial balances, extrapolation of the evolution of the government budget suggests a decrease, relative to the benchmark simulation, in the deficit from about 1998 onwards. These results confirm the well-known hypothesis from the literature on the success and failure of fiscal consolidation that in order to be successful, i.e. leading to a permanent reduction in government debt and deficit ratios, fiscal consolidations should not rely on cuts in public investment. The observation that it takes quite some time for increases (decreases) in investment spending to feedback to the government budget through beneficial (harmful) growth effects, also suggests why governments are tempted to reduce investment spending

\textsuperscript{11} Note that the substitution of labour for capital and materials – for a given output – in the second round further increases labour productivity. As a result, the unemployment rate will increase further.
during periods of fiscal consolidation and why they might be reluctant to reinstate investment rates at their old levels once the budgetary position has improved.

The right-hand side of figure 6 graphs the break-down of the total change in the government budget (reported as a percent of business sector GDP) into its main components, i.e. (i) public investment, (ii) unemployment benefits and taxes on labour (iii) indirect taxes and taxes on profits and (iv) interest payments on government debt. Changes in these components are measured as a percentage of the benchmark business sector GDP. The contribution of the rise in business sector GDP is taken as a separate explanatory variable (labelled output growth). The break-down shows that the worsening of the gross government budget over the period 1982-89 is mainly due to the increase in the gap between realised and simulated public investments. In 1989, higher public investments imply a burden on the budget of 2.8% of GDP. In real terms, this gap no longer widens from 1989 onward. In nominal terms, there is a further increase cumulating to an increase in nominal expenditures of 3.2% of GDP in 1996 compared to the benchmark simulation. The government’s financial balance also experiences negative effects from rising unemployment due to (i) lower receipts from taxes on labour and (ii) higher expenditures on unemployment benefits. This negative effect reaches a maximum of 1.2% of GDP in 1992. Due to the positive effect of the increase in wages, the negative effect of higher unemployment has decreased to 0.5% in 1996. As economic growth accelerates with the building up of the public capital stock, indirect tax and profit tax receipts slowly increase. Starting from 0.1% of GDP in 1982, these tax gains increase to reach 4.0% of GDP in 1996. Further, for a given deficit higher economic growth implies an ‘accounting’ gain as the deficit is expressed as a percentage of GDP. This accounting effect amounts to a 0.8% point decrease in the deficit in 1993. As the government budget deficit decreases strongly from 1994 onward, this gain has largely disappeared in 1996. Finally, by increasing gross government debt, the cumulation of deficits implies additional interest payments, amounting about 1.4% of GDP in 1996.

6 Conclusion

This paper analyses the impact of public capital on private sector performance in Belgium, in particular employment. We estimate a dynamic structural model which explains – among other variables – private output, private cost, private employment, private capital formation, wage bargaining and price setting. The estimates show that services from public capital
significantly reduce private sector total cost (for given output and wages). An increase in the public capital stock with 1 EURO reduces long-run private sector cost with 0.24 EURO. The output elasticity of public capital implied by these estimates equals 0.31, confirming the value found in our earlier research (Everaert and Heylen, 2000). As for the impact on private sector inputs, the results suggest that public capital and labour are substitutes. Public capital and private capital are found to be complements.

To find out whether the negative relationship between public capital and employment is altered once the effect of a change in public capital on aggregate demand and wages is taken into account, the model is simulated for 1970-96 under the assumption that the strong decline in public investment in 1982-89 did not occur. Due to direct substitution of labour for public capital, the private sector unemployment rate would have been 3.3% points higher under this alternative investment policy. Possible positive effects on employment might derive from the increase in aggregate demand resulting from the increase in public investment. As labour demand was found to be largely insensitive to changes in aggregate demand, the benefits in terms of employment are very moderate, accounting for only a small decrease in the unemployment rate. One should be very cautious in interpreting this result, though. As suggested in section 5, the very small impact of output on labour market performance is most probably due to the estimates of the dynamic cost function implying a very fast adjustment of factor shares toward their long-run equilibrium. Further, real wage demands increased due to a strong rise in labour productivity. As the employment share in cost was found to be independent of public capital, the productivity benefits resulting from an increase in the public capital stock can - for given costs - only be incorporated in real wages at the expense of lower employment. Therefore, the increase in real wages has induced a further rise in the unemployment rate. The combined effect accounted for a 3.8% point increase in the 1996 unemployment rate.

These results suggest that the strong positive correlation between trend public capital growth and the employment rate observed in figure 1 is produced by the evolution in a third factor, affecting public investment and employment simultaneously, rather than resulting from a direct causal relation running from public capital to employment. One possible candidate is the massive build-up of government debt since the 1970s – whatever its cause – triggering fiscal responses that negatively affected the structural characteristics of the labour market (e.g. higher taxes on labour) and ‘forced’ the government to cut investment spending.
Finally, our simulations confirm the hypothesis from the literature on the success and failure of fiscal consolidation that – in order to be successful - fiscal consolidations should not rely on cuts in public investment. Although the government deficit widens considerably during the first years of the increase in investment, the budget improves as the positive growth effects of a higher public capital stock slowly increase tax receipts. In 1996, the increase in the government deficit still amounted 0.8% of GDP. As the higher public capital stock will continue to spur economic growth, the deficit will most probably decrease – relative to the benchmark simulation - from the end of the 1990s onward, though.
APPENDIX A: DATA DESCRIPTIONS

The data are taken from various sources: OECD Economic Outlook (EO), OECD Business Sector Data Base (BSDB), OECD Data Extraction Service DES, National Accounts data from the Belgian Federal Planning Bureau and foreign trade data from the National Bank of Belgium. For data taken from OECD sources, the relevant OECD code is reported between square brackets. Variables that can be calculated from the identities outlined in section 2 are not included.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_i$</td>
<td>Share of business sector profits paid to households as dividends.</td>
<td>(Source: Belgian Federal Planning Bureau)</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>Depreciation rate for private capital. In order to match the retirement pattern of assets implied by the perpetual inventory method used in the calculation of the private sector capital stock, $\delta_k$ is variable over the sample period.</td>
<td>(Source: Belgian Federal Planning Bureau)</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Knowledge stock, cumulated from data for patents granted by the US Patent and Trademark Office.</td>
<td>(Source: Everaert and Heylen, 2000)</td>
</tr>
<tr>
<td>$Bc$</td>
<td>Business cycle, calculated as deviation of business sector GDP from its Hodrick-Prescott trend.</td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>Nominal unemployment benefit.</td>
<td>(Source: RVA jaarverslag, several issues)</td>
</tr>
<tr>
<td>$B_{rr}$</td>
<td>Benefit replacement rate, overall average.</td>
<td>(Source: OECD Database on Benefit Entitlements and Gross Replacement Rates)</td>
</tr>
<tr>
<td>$C_g$</td>
<td>Real government non-wage consumption [CGNWV] (million 1990 prices).</td>
<td>(Source: OECD EO)</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Real private consumption expenditures [CPV] (million 1990 prices).</td>
<td>(Source: OECD EO)</td>
</tr>
<tr>
<td>$E$</td>
<td>Private sector dependent employment [EEP].</td>
<td>(Source: OECD BSDB)</td>
</tr>
<tr>
<td>$E_g$</td>
<td>Government employment [EG].</td>
<td>(Source: OECD EO)</td>
</tr>
<tr>
<td>$EXP_w$</td>
<td>Real world imports (1990=1), calculated as an (export-) weighted average of real imports of our 7 most important trade partners.</td>
<td>(Source: OECD EO &amp; National Bank of Belgium)</td>
</tr>
<tr>
<td>$G$</td>
<td>Total Government outlays [YPG+IGG]</td>
<td>(Source: OECD EO)</td>
</tr>
<tr>
<td>$I_g$</td>
<td>Capital formation by the government.</td>
<td>(Source: Belgian Federal Planning Bureau)</td>
</tr>
<tr>
<td>$I_h$</td>
<td>Capital formation by households [IH].</td>
<td>(Source: OECD EO)</td>
</tr>
<tr>
<td>$IM$</td>
<td>Imported aggregate volume of final goods and services (million 1990 prices).</td>
<td>(Source: National Bank of Belgium)</td>
</tr>
<tr>
<td>$I_s$</td>
<td>Stockbuilding [ISK].</td>
<td>(Source: OECD EO)</td>
</tr>
<tr>
<td>$K$</td>
<td>Private sector capital stock (million 1990 prices).</td>
<td>(Source: Belgian Federal Planning Bureau)</td>
</tr>
<tr>
<td>$KG$</td>
<td>Public sector capital stock (million 1990 prices).</td>
<td>(Source: Belgian Federal Planning Bureau)</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Labour force [LF].</td>
<td>(Source: OECD EO)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Source</td>
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<tr>
<td>--------</td>
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</tr>
<tr>
<td>$M$</td>
<td>Imported intermediate inputs (million 1990 prices).</td>
<td>National Bank of Belgium</td>
</tr>
<tr>
<td>$oil$</td>
<td>Oil prices (in US$), deviation from sample average.</td>
<td>IMF International Financial Statistics</td>
</tr>
<tr>
<td>$P^*$</td>
<td>Price of foreign final products on the world markets (1990=1), calculated as an (export-) weighted average of prices on the domestic markets of our 7 most important trade partners.</td>
<td>OECD EO &amp; National Bank of Belgium</td>
</tr>
<tr>
<td>$P_d$</td>
<td>Price of domestic production sold on the domestic market (1990=1).</td>
<td>National Bank of Belgium</td>
</tr>
<tr>
<td>$P_f$</td>
<td>Price of domestic production sold on the world market (1990=1).</td>
<td>National Bank of Belgium</td>
</tr>
<tr>
<td>$P_{im}$</td>
<td>Price of imported final products on the domestic market (1990=1).</td>
<td>National Bank of Belgium</td>
</tr>
<tr>
<td>$P_{inv}$</td>
<td>Deflator of private investments (1990=1).</td>
<td>Belgian Federal Planning Bureau</td>
</tr>
<tr>
<td>$P_M$</td>
<td>Deflator for imported intermediate inputs (1990=1).</td>
<td>National Bank of Belgium</td>
</tr>
<tr>
<td>$R_n$</td>
<td>Nominal long-term interest rate [IRL].</td>
<td>OECD EO</td>
</tr>
<tr>
<td>$R$</td>
<td>Real long-term interest rate, calculated as the nominal long-term interest rate [IRL] minus expected inflation in investment prices. The latter is calculated as the growth rate of the trend in $P_{inv}$.</td>
<td>OECD EO &amp; Belgian Federal Planning Bureau</td>
</tr>
<tr>
<td>$T$</td>
<td>Total Government receipts [YRG+CFKG+KTRRG]</td>
<td>OECD EO</td>
</tr>
<tr>
<td>$t_{bd}$</td>
<td>Tax rate on profits to be paid by the firm $[TYB/(II\Pi-\Pi_0)]$.</td>
<td>OECD EO</td>
</tr>
<tr>
<td>$t_{bs}$</td>
<td>Tax rate on labour to be paid by the firm $[TRPBSH/(WSSS-CGW-TRPBSH)]$.</td>
<td>OECD EO &amp; OECD DES</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Tax rate on households’ capital income, fixed at 15%.</td>
<td></td>
</tr>
<tr>
<td>$t_{ind}$</td>
<td>The rate of indirect taxes $[(TIND-TSUB)/(CPAA+CGNW+IT-TIND-TSUB)]$.</td>
<td>OECD EO</td>
</tr>
<tr>
<td>$t_w$</td>
<td>Tax rate on gross wages to be paid by employees, calculated by dividing total direct taxes on gross wages to be paid by employees $[TRPESH]$ through by total gross wages $[WSSS-TRPBSH-TRPGSH]$.</td>
<td>OECD EO</td>
</tr>
<tr>
<td>$u$</td>
<td>Unemployment rate [UNR].</td>
<td>OECD EO</td>
</tr>
<tr>
<td>$U_{bs}$</td>
<td>Union membership as a percentage of the labour force.</td>
<td>Ebbinghaus and Visser, 1990</td>
</tr>
<tr>
<td>$W^*$</td>
<td>Wage rate in the business sector calculated by dividing gross wages in the private sector $[WSSS-CGW-TRPBSH]$ through by private sector dependent employment [EEP]. Notice that the contribution of employers to pension funds $[TRPBPH]$ is included in $W$.</td>
<td>OECD EO, OECD BSDB &amp; OECD DES</td>
</tr>
<tr>
<td>$W_g$</td>
<td>Wage rate in the public sector calculated by dividing gross wages in the public sector $[CGW-TRPGSH]$ through by government employment [EG]. Notice that the contribution of the government to pension funds $[TRPGPH]$ is included in $W_g$.</td>
<td>OECD EO &amp; OECD DES</td>
</tr>
<tr>
<td>$X_f$</td>
<td>Domestic output sold on the world markets (million 1990 prices).</td>
<td>National Bank of Belgium</td>
</tr>
<tr>
<td>$Y_c$</td>
<td>Capital income received by households.</td>
<td>Belgian Federal Planning Bureau</td>
</tr>
<tr>
<td>$Y_{dis}$</td>
<td>Household disposable income [YDH].</td>
<td>OECD EO</td>
</tr>
</tbody>
</table>
APPENDIX B: DERIVATION OF THE EQUILIBRIUM WAGE EQUATION

As we have explained in the main text, the nominal gross wage is assumed to result from the maximisation of the following Nash-bargaining maximand:

\[ \Omega = E^{PC} \left( \frac{W(1-t_w)}{P_c} - \frac{H(1-t_w)}{P_c} \right)^z \Pi \]  

(15’)

with\(^1\): \[ H = [1 - f(u)]W + f(u)B \]  

with: \(0 < f(u) < 1\), \(\Gamma'(u) > 0\)  

(16)

Rewriting (15’) in logs, we obtain:

\[ \ln \Omega = \rho z \ln E + z \ln(W - H) + z \ln \left( \frac{1-t_w}{P_c} \right) + \ln \Pi \]  

(B1)

For a maximum of this function with respect to the wage (and noting that for individual bargaining units tax rates and aggregate prices are exogenous), it is required that:

\[ \rho z \frac{\partial \ln(E)}{\partial \ln(W)} + z \frac{\partial \ln(W - H)}{\partial \ln(W)} + \frac{\partial \ln(\Pi)}{\partial \ln(W)} = 0 \]

\[ - \rho z e_{E,W} + z \frac{W}{W - H} + \frac{\partial \Pi}{\partial W} \frac{W}{\Pi} = 0 \]  

(B2)

with: \(e_{E,W}\) the absolute value of the wage elasticity of employment.

Since \(\frac{\partial \Pi}{\partial W} = -\frac{E(1 + t_{bs})}{P_X}\), equation (B2) can be rewritten as:

\[ \frac{W}{W - H} = \rho e_{E,W} + \frac{E W (1 + t_{bs})}{z P_X \Pi} \]  

(B3)

where \(P_X\) stands for the output price (see main text). Both multiplying and dividing the second term on the right-hand side of this equation by real value added (\(Y\)) we obtain that

\[ \frac{W}{W - H} = \rho e_{E,W} + \frac{1 - S_L}{z \pi} \]  

(B4)

with: \(S_L\) the share of real labour cost in real value added,

\(\pi\) real profit per unit of real value added.

\(^{12}\) \(E_{c}\) is constant, i.e. no hiring by the public sector.
To proceed from (B4) we shall first - following Nixon and Urga (1999) - assume that in the long-run equilibrium employment is no longer an argument in the union’s utility function (i.e. \( \rho \) goes to 0). Replacing \( H \) by equation (16) and rearranging this gives:

\[
\frac{W - [1 - f(u)]W - f(u)B}{W} = \frac{z\pi}{S_L} \tag{B5}
\]

Second, we plausibly assume that all private sector bargaining units show the same optimising behaviour, implying that \( W = \tilde{W} \). The first order condition for wage setting then becomes:

\[
\frac{f(u)(W - B)}{W} = \frac{z\pi}{S_L}
\]

\[
\frac{B}{W} = 1 - \frac{z\pi}{S_L f(u)} \tag{B6}
\]

where \( B/W \) stands for the unemployment benefit replacement rate.

As a further step, one should note that for realistic values of \( B/W \) (and thus also the right-hand side of equation B6), the following close approximation exists:

\[
\ln\left( \frac{B}{W} \right) = \ln\left( 1 - \frac{z\pi}{S_L f(u)} \right) \approx a_1 \ln\left( \frac{S_L f(u)}{z\pi} \right) + a_2
\]

with: \( a_1 \) and \( a_2 \) are constants and \( a_1 > 0 \).\(^{13}\) We then obtain

\[
\ln\left( \frac{B}{W} \right) = a_1 \left[ \ln(S_L) + \ln f(u) - \ln(z) - \ln(\pi) \right] + a_2 \tag{B7}
\]

Finally, assuming that \( f(u) = u^\gamma \), with \( \gamma > 0 \), and noting that:

\[
\ln(S_L) = \ln(W) + \ln(1 + t_{\text{in}}) - \ln(Q) \cdot \ln(P_x)
\]

with \( Q \) representing the productivity of labour (\( Q = Y/E \)), equation (B7) can be rewritten as:

\[
\ln(W) = -\frac{a_2}{a_1} + \ln(P_x) + \ln(Q) - \gamma \ln(u) + \frac{1}{a_1} \ln\left( \frac{B}{W} \right) - \ln(1 + t_{\text{in}}) + \ln(\pi) + \ln(z) \tag{B8}
\]

Equation (17) in the main text is a somewhat more flexible representation of this long-run equilibrium relation. Flexibility concerns the parameters to be estimated and the fact that in Belgium nominal wages are automatically indexed to aggregate consumer prices.

\(^{13}\) To give an example, approximation is very close for \( B/W \) between 0.5 and 0.8 when \( a_1 = 0.5 \) and \( a_2 = -1 \).
REFERENCES


Infrequent large shocks to unemployment
- New evidence on alternative persistence perspectives -

GERDIE EVERAERT*

September 2000

ABSTRACT: This paper tests whether the observed high persistence of unemployment rates in most OECD-countries is due to (full) hysteresis against the alternative that it is caused by adjustment toward an increased natural rate. The analysis relies on standard univariate unit root tests. Usually such tests cannot reject the presence of a unit root in the unemployment rate, pointing to full hysteresis. This paper shows that once we allow for infrequent level-shifts, the unit root hypothesis can clearly be rejected in almost all of the 21 considered OECD-countries. The paper also suggests how to reconcile infrequent large shocks with the natural rate hypothesis.

KEYWORDS: Unemployment, persistence, unit root, infrequent large shocks, natural rate.

1. INTRODUCTION

One of the most important challenges for future macroeconomic policy in the European Union is the dismantlement of mass unemployment. While unemployment rates rise sharply in all OECD-countries during cyclical downturns, a more acute problem experienced especially in the European countries is that unemployment shows a strong tendency to remain close to the newly attained higher level despite subsequent cyclical recoveries. Starting from about 2% in the 1960s, European unemployment has risen to a rate of 10% in 1999. In the US in contrast the unemployment rate has fluctuated around a stable level of about 6% over the last 4 decades, reaching an historically low level just above 4% at the end of the 1990s.

Generally spoken, three alternative persistence perspectives have been put forward. The first hypothesis argues that the rise in unemployment rates is caused by adjustment to an underlying long-run equilibrium (natural) rate of unemployment which has increased in response to changes in structural characteristics of the labour market. This hypothesis actually dates back to models of the 1960s and 1970s, where the unemployment rate was assumed to converge to an exogenously determined constant natural rate. In more recent years, this

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natural rate has been endogenised in terms of structural characteristics of the economy affecting the interaction of wage bargaining by unions and employers and price and employment determination by firms (see e.g. Layard, Nickell and Jackman, 1991; Phelps, 1994). Possible examples of such structural factors are the generosity of the unemployment benefit system, the tightness of employment protection legislation and the wedge between real wage costs and the real after tax consumption wage - all affecting wage bargaining - and capital costs affecting the demand for labour. A second explanation that has attracted a lot of attention since the mid 1980s is the existence of hysteresis effects. Hysteresis arises when the medium-term equilibrium rate of unemployment is path-dependent, i.e. it depends on the history of unemployment. One can show that in this case also the current level of unemployment will depend on its own history. Hysteresis effects may stem from a variety of microeconomic foundations: insider behaviour, outsider ineffectiveness and negative duration effects, capacity scrapping (see e.g. Lindbeck and Snower, 1988; Carlin and Soskice, 1990). In this interpretation, by increasing actual unemployment cutbacks in aggregate demand are responsible for an increase in the medium-term equilibrium rate of unemployment, heralding a period of persistent higher unemployment. In the case where the medium-term equilibrium rate eventually converts back toward the long-run natural rate of unemployment, there is only partial hysteresis. In the extreme case where each change in the actual rate of unemployment completely feeds through into the long-run equilibrium rate, the traditional concept of the natural rate disappears. This case is generally known as full or pure hysteresis. This is our third hypothesis.

The policy implications of the alternative hypotheses are fundamentally different especially concerning the effects of demand side policies. Since changes in the unemployment rate are permanent if the labour market is characterised by full hysteresis, stabilisation policy is not only very important - to avoid increases in the actual rate of unemployment - but also very effective for it has permanent effects. If the unemployment rate converges toward a natural rate in contrast, the prime concern of the government should be to keep this natural rate as low as possible by affecting the structural characteristics of the economy. Depending on the observed degree of medium-term persistence, demand side policies will only have temporary effects in this case.
As noted by Elmeskov and Macfarlan (1993), a direct methodology for assessing the nature of unemployment persistence is analysing the time-series properties of the unemployment rate. In standard (Dickey-Fuller) unit root tests, the null hypothesis of a unit root corresponds to full hysteresis, while the alternative hypothesis implies reversion to a natural rate with the observed degree of hysteresis depending on the speed of the convergence process. Implementing these tests for 23 OECD-countries, Elmeskov and Macfarlan show that a unit root cannot be rejected in any of the considered countries if stationarity is the alternative hypothesis. When this alternative hypothesis includes a linear trend - which should capture a slowly increasing natural rate - there is weak evidence of trend-stationarity only in three countries. These results clearly favour the full hysteresis hypothesis in most countries. Roughly the same conclusion is drawn by - among others - Mitchell (1993), Leslie et al. (1995) and Song and Wu (1998).

These results should be interpreted with great care though. Leslie et al. (1995) show that in small samples, standard unit root tests generally have low power against stationary alternatives even if this alternative has a root far from unity. Since the null hypothesis is the existence of a unit root, rejecting non-stationarity needs strong evidence against it. Given the relatively small samples usually available, this might be an arduous thing to do. In order to avoid the problem of low power, Kwiatkowski et al. (1992) have developed an alternative test, which takes (trend-) stationarity as the null hypothesis. However, using this alternative test, Leslie et al. (1995) are still not able to provide convincing evidence against the unit root hypothesis. In an alternative attempt to raise power, Song and Wu (1998) proceed to use panel-based unit root tests. The increased power of the test is mainly due to cross-equation restrictions imposing an equal root in all countries. In contrast to univariate tests, a unit root can be rejected once the data are pooled over countries. However, the degree of persistence remains very high - i.e. higher than 95% in all specifications. From a policy perspective, this result has highly similar implications as the finding of a unit root for it takes about 45 years to reverse 90% of a shock to unemployment.

Besides the low power of standard univariate unit root tests, the incapacity to reject the null of a unit root might be caused by mis-specification of the alternative hypothesis (Elmeskov and Macfarlan, 1993). Perron (1989) has pointed out that standard tests of the unit root hypothesis against (trend-) stationary alternatives are biased toward non-rejection of the unit root if the
true data generating process includes breaks in its deterministic components. This implies that if the actual rate of unemployment reverts to a variable natural rate, traditional unit root tests will be biased toward non-rejection of the full hysteresis hypothesis.

The possibility of a single break in the deterministic component of unemployment rates in the OECD countries was first tested by Mitchell (1993), who allowed for a segmented linear trend. Still, the unit root hypothesis cannot be rejected in almost all countries. However, as noted by the author, these results should be interpreted with great care for the determination of the timing of the break points was the result of an unsatisfactory ‘eye-balling’ exercise. Endogenising the timing of the break date, Arestis and Mariscal (2000) are able to reject the unit root hypothesis in 9 out of 22 countries.

The results in both Mitchell (1993) and Arestis and Mariscal (2000) rely on the assumption that only a single break has occurred. In this paper, I allow for multiple breaks in the mean of the unemployment rate. The actual specification of the alternative hypothesis is data-dependent, relying on Tsay’s (1988) outlier detection algorithm, which is designed to identify significant outliers in the unemployment series. Especially level-shifting outliers are of particular importance for they imply infrequent permanent shocks to the mean of the unemployment rate. Balke and Fomby (1991) show that although measures of persistence are the same regardless of the frequency of permanent shocks, this frequency has important implications for how one should interpret the long-run persistence in time series. If level-shifting innovations occur with high frequency - i.e. in every period - and with ‘low’ variance, unemployment contains a unit root. Since a unit root implies that all individual innovations are permanent, unemployment persistence is caused by full hysteresis. If the unemployment rate is characterised by infrequent permanent shocks with ‘high’ variance, only a small number of significant economic events have permanent effects. Since most innovations have only temporary effects, the full hysteresis hypothesis must be rejected in this case.

The results show that infrequent large shocks are indeed responsible for the apparent unit root in unemployment rates. Once these infrequent large shocks are controlled for, there is good evidence against the unit root hypothesis in most countries. A similar argument, albeit completely different in methodology, has been made by Belke and Göcke (1996), who argue that the rejection of a cointegrating relationship between employment and real wages - among
some other variables - is not due to ‘unit root persistence’ in unemployment - implying a degeneration of the adjustment toward the equilibrium - but must be attributed to structural breaks in the cointegrating relationship due to serious economic shocks.

The remainder of the paper is organised as follows. Section 2 reproduces the results from traditional Augmented Dickey-Fuller (ADF) unit root test. To make sure that the finding of full hysteresis is not due to the lack of power of ADF-tests in small samples, I also run two alternative tests taking stationarity as the null hypothesis. Section 3 outlines the procedure for detecting infrequent large shocks and reports the results for the unemployment rate in the considered 21 OECD-countries. The identified shocks are then used to estimate intervention models from which the unit root hypothesis can be tested. Section 4 concludes by discussing how one can interpret infrequent large shocks in terms of the alternative persistence hypotheses.

2. STANDARD UNIT ROOT TESTS

Before estimating intervention models allowing for infrequent large shocks, some results from standard unit root tests are reported for comparison. Columns 2 and 3 of table 1 report the results from running Augmented Dickey-Fuller tests on the unemployment rate (\( u_t \)) with the following standard specification:

\[
\Delta u_t = u_0 + \rho u_{t-1} + \sum_{i=1}^{\rho} \alpha_i \Delta u_{t-i} + \epsilon_t
\]  

under the null hypothesis (\( \rho = 0 \)) we have a unit root corresponding to full hysteresis while the alternative hypothesis (\( \rho < 0 \)) implies adjustment to a constant natural rate (\(-u_0/\rho\)) with the degree of hysteresis depending on the persistence of deviations from this natural rate.

At the 5% level, the ADF test cannot reject the hypothesis of a unit root in any of the considered countries. If we reject with 90% confidence, a unit root can be rejected in only two countries, i.e. Portugal and the US. Consistent with the conclusion of Elmeskov and MacFarlan (1993), the results do not support the hypothesis of mean reversion to a constant natural rate. Dickey-Fuller tests including a linear trend in the alternative hypothesis yield similar results. They are not included in table 1, though. The main reason is that I’m not convinced that trend-stationarity is a relevant alternative hypothesis. Not only is trend-
stationarity impossible for a bounded series like the unemployment rate\(^1\), a linear trend would also imply a fixed yearly increase in the natural rate, which is hard to reconcile with the true evolution of structural characteristics of the economy.

Table 1  Testing stationarity of unemployment rates (1960-99) \(^a\)

<table>
<thead>
<tr>
<th>Countries</th>
<th>Lags</th>
<th>(\tau_p)</th>
<th>Augmented Dickey-Fuller test(^b)</th>
<th>Kwiatkowski et al. test(^c)</th>
<th>Leybourne and McCabe test(^c)</th>
<th>Persistence(^d)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td>Lag truncation parameter (q)</td>
<td></td>
<td></td>
<td>Lags</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Australia</td>
<td>(p = 0)</td>
<td>-1.31</td>
<td>1.26**</td>
<td>0.80**</td>
<td>0.61**</td>
<td>0.50**</td>
</tr>
<tr>
<td>Austria</td>
<td>(p = 1)</td>
<td>-0.33</td>
<td>1.19**</td>
<td>0.76**</td>
<td>0.57**</td>
<td>0.47**</td>
</tr>
<tr>
<td>Belgium</td>
<td>(p = 2)</td>
<td>-1.11</td>
<td>1.23**</td>
<td>0.78**</td>
<td>0.59**</td>
<td>0.50**</td>
</tr>
<tr>
<td>Canada</td>
<td>(p = 1)</td>
<td>-1.84</td>
<td>0.97**</td>
<td>0.65**</td>
<td>0.51**</td>
<td>0.43*</td>
</tr>
<tr>
<td>Denmark</td>
<td>(p = 1)</td>
<td>-1.57</td>
<td>1.13**</td>
<td>0.72**</td>
<td>0.55**</td>
<td>0.46*</td>
</tr>
<tr>
<td>Finland</td>
<td>(p = 2)</td>
<td>-0.96</td>
<td>0.97**</td>
<td>0.65**</td>
<td>0.53**</td>
<td>0.48**</td>
</tr>
<tr>
<td>France</td>
<td>(p = 1)</td>
<td>-0.92</td>
<td>1.38**</td>
<td>0.87**</td>
<td>0.65**</td>
<td>0.54**</td>
</tr>
<tr>
<td>Germany</td>
<td>(p = 1)</td>
<td>-0.78</td>
<td>1.33**</td>
<td>0.86**</td>
<td>0.66**</td>
<td>0.55**</td>
</tr>
<tr>
<td>Greece</td>
<td>(p = 1)</td>
<td>-1.31</td>
<td>0.86**</td>
<td>0.56**</td>
<td>0.43*</td>
<td>0.37*</td>
</tr>
<tr>
<td>Ireland</td>
<td>(p = 1)</td>
<td>-1.52</td>
<td>1.01**</td>
<td>0.64**</td>
<td>0.49**</td>
<td>0.41*</td>
</tr>
<tr>
<td>Italy</td>
<td>(p = 1)</td>
<td>-0.64</td>
<td>1.33**</td>
<td>0.85**</td>
<td>0.65**</td>
<td>0.54**</td>
</tr>
<tr>
<td>Japan</td>
<td>(p = 1)</td>
<td>-0.46</td>
<td>1.22**</td>
<td>0.80**</td>
<td>0.63**</td>
<td>0.53**</td>
</tr>
<tr>
<td>Netherlands</td>
<td>(p = 1)</td>
<td>-2.13</td>
<td>1.00**</td>
<td>0.65**</td>
<td>0.50**</td>
<td>0.42*</td>
</tr>
<tr>
<td>New Zealand</td>
<td>(p = 1)</td>
<td>-1.14</td>
<td>1.21**</td>
<td>0.77**</td>
<td>0.59**</td>
<td>0.49**</td>
</tr>
<tr>
<td>Norway</td>
<td>(p = 1)</td>
<td>-1.86</td>
<td>0.99**</td>
<td>0.64**</td>
<td>0.50**</td>
<td>0.43*</td>
</tr>
<tr>
<td>Portugal</td>
<td>(p = 1)</td>
<td>-2.63*</td>
<td>0.74**</td>
<td>0.52**</td>
<td>0.43*</td>
<td>0.39*</td>
</tr>
<tr>
<td>Spain</td>
<td>(p = 1)</td>
<td>-1.51</td>
<td>1.27**</td>
<td>0.80**</td>
<td>0.61**</td>
<td>0.50**</td>
</tr>
<tr>
<td>Sweden</td>
<td>(p = 1)</td>
<td>-1.83</td>
<td>0.84**</td>
<td>0.58**</td>
<td>0.48**</td>
<td>0.44*</td>
</tr>
<tr>
<td>Switzerland</td>
<td>(p = 1)</td>
<td>-1.83</td>
<td>0.94**</td>
<td>0.63**</td>
<td>0.50**</td>
<td>0.45*</td>
</tr>
<tr>
<td>UK</td>
<td>(p = 2)</td>
<td>-1.22</td>
<td>1.04**</td>
<td>0.68**</td>
<td>0.52**</td>
<td>0.44*</td>
</tr>
<tr>
<td>US</td>
<td>(p = 1)</td>
<td>-2.60*</td>
<td>0.30</td>
<td>0.22</td>
<td>0.19</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Notes: \(^a\) Unemployment rates (commonly used definitions) are taken from the OECD Statistical compendium 1998/2. Data for 1998 and 1999 are based on estimates and projections.
\(^b\) The critical values are equal to 3.61, 2.94 and 2.61 at the 1%, 5% and 10% level of significance respectively (MacKinnon, 1991).
\(^c\) The critical values are equal to 0.739, 0.463 and 0.347 at the 1%, 5% and 10% level of significance respectively (Kwiatkowski et al., 1992).
\(^d\) The persistence measure equals the estimated AR(1) coefficient in an ARMA(1,\(q\)) model for unemployment, with the number of MA-components \(q\) determined such that any significant serial correlation in the residuals was removed (Barro, 1988). Standard errors are reported in parentheses.
**(*) rejection of the null hypothesis at the 5% (10%) level of significance.

In order to deal with the low power of the Dickey-Fuller test, Kwiatkowski et al. (1992) have developed an alternative test which takes (trend-) stationarity as the null hypothesis. The

\(^1\) Leslie et al. (1995) argue that while the unemployment rate cannot permanently have a linear trend, a time trend is still possible as long as the time domain is bounded.
A second test which takes (trend-) stationarity as the null hypothesis has been proposed by Leybourne and McCabe (1994). The main difference with the test suggested by Kwiatkowski et al. lies in the correction for serial correlation in the error term. While Kwiatkowski et al. apply the Newey-West correction, Leybourne and McCabe proceed along the same lines as the ADF test, including a specific parametric autoregressive structure, i.e. an ARIMA\((p,1,1)\) representation, in the model. Both simulation and theoretical evidence (Leybourne and McCabe, 1994) reveal two important advantages of the latter approach. First, the outcome of the test is found to be less sensitive to fitting redundant AR components, making it more robust. Second, the test seems to have greater power to reject a false null hypothesis of (trend-) stationarity.

Columns 4 to 9 of table 1 report the results from running both alternative testing procedures, with stationarity as the relevant null hypothesis. The Leybourne and McCabe test clearly rejects the hypothesis of stationarity in all countries. The evidence from the Kwiatkowski et al. test is less straightforward. The main problem is that the results are highly sensitive to the choice of the truncation lag, included to pick up autocorrelation in the error term (Newey-West correction). Following the suggestions of Kwiatkowski et al. (1992), we consider values for the lag truncation parameter up to eight, which is a trade-off between size and power of the test\(^2\). In 10 out of the 21 considered countries, the hypothesis of stationarity can only be rejected at the 10% level for \(q\) equal to 8. In Greece and Portugal, the same conclusion holds for \(q\) equal to 6. In all other specifications, the null hypothesis is rejected except for the US where the test results clearly point to stationarity in all specifications. Note that the fact that stationarity cannot be rejected (with 90% confidence) for ‘large’ values of \(q\) might be due to the decreasing power of the test as \(q\) grows.

\(^2\) A large value for \(q\), needed to avoid size distortions, significantly reduces the power of the test in small samples. In a sample of about 50 observations, the test has reasonable power for \(q=4\), while in order for the
The overall picture that stems from the three different testing procedures is that there is considerable evidence in favour of the hypothesis of a unit root in the unemployment rate of all considered countries except the US where the results point - although the evidence is weak - in the direction of stationarity. The fact that the results from the Dickey-Fuller tests are confirmed by the alternative Leybourne and McCabe test - and to a lesser degree by the Kwiatkowski et al. test - suggests that the lack of power of standard unit root tests is not the main reason for the non-rejection of the unit root hypothesis. The results from the unit root tests are confirmed by estimating the persistence in unemployment rates from simple ARMA models. Measured persistence is close to - and not significantly different from - one in all countries except the US (and maybe Portugal) where we have clear mean reversion.

However, these results do not allow to conclude that the major part of the OECD-countries are plagued with full hysteresis in their labour markets. Although Dickey-Fuller tests are asymptotically robust in uncovering I(1) behaviour, they are unable to distinguish between two alternative sources of this non-stationarity, i.e. whether shocks occur each period or infrequently (Balke and Fomby, 1991)\(^3\). This frequency of shocks is crucial for the interpretation of long-run persistence in time series, though. Non-rejection of a unit root is evidence in favour of full hysteresis only if permanent innovations to the unemployment rate occur every period. If the non-stationarity is caused by infrequent shifts in the deterministic component of the unemployment rate, full hysteresis must be rejected. In the next section, we check whether infrequent large shocks are responsible for the apparent non-stationarity of unemployment rates.

3. INFREQUENT LARGE SHOCKS AND THE UNIT ROOT HYPOTHESIS

The major problem with the tests in section 2 is that they cannot identify the source of non-stationarity in the unemployment rate, making them unable to distinguish between the three alternative persistence perspectives outlined in the introduction. In this section, we use Tsay’s (1988) outlier detection algorithm to identify infrequent large shocks to the unemployment rate. Three alternative shocks are considered: additive outliers (AO), innovational outliers (IO) and level-shifting (LS) outliers. With level data, both innovational and additive outliers test to have more or less correct size, \(q\) should be raised to about 12. The unemployment rate is I(1) regardless of the frequency of permanent shocks.
imply temporary changes in unemployment, while level-shifting outliers represent permanent shocks to the level of unemployment. Innovational and additive outliers are of interest mainly because their occurrence can cause serious inference problems in standard unit root tests while level-shifts are of particular interest for they may be responsible for the apparent I(1) behaviour of unemployment rates. To check whether a unit root remains present in the data once we control for infrequent large shocks, we estimate intervention models including intervention dummies to capture the identified outliers.

3.1. Identifying infrequent large shocks

An iterative procedure for detecting outliers in univariate time series was developed by Tsay (1988). His methodology is based on an unobserved components model in which the regular series $Z_t$ is disturbed by an outlier component $f(t)$

$$Y_t = Z_t + f(t).$$

with $Z_t$ being described by a autoregressive moving average (ARMA) model

$$\Phi(L)Z_t = \theta_0 + \theta(L)e_t,$$

with $\Phi(L)$ and $\theta(L)$ polynomials in $L$ of degrees $p$ and $q$ respectively and $e_t$ an independent Gaussian variate with mean zero and variance $\sigma^2$. Tsay considers different types of outliers nested in the following general specification for $f(t)$

$$f(t) = \omega_0 B(L)Tb_d$$

with $\omega_0$ a constant denoting the initial impact of the disturbance, $B(L)$ a polynomial in $L$ representing the dynamic effect of the outlier on $Y_t$ and $Tb_d$ indicating the timing of the disturbance (i.e. $Tb_d = 1$ if $t = d$ and zero otherwise). This paper considers three alternative classes of outliers:

(i) **Additive Outliers (AO):** $B(L)=1 \Rightarrow$ Only $Y_d$ is affected by the outlier. An additive outlier occurs when the observation for a particular year is extreme but subsequent observations remain unaffected. The standard example is a measurement error.

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4 The model in (2)-(4) can easily be extended to allow for other types of disturbances.
Infrequent large shocks to unemployment

(ii) **Innovational Outliers** (IO): \( B(L) = \theta(L)/\Phi(L) \Rightarrow \) Dynamic effect on \( Y_t \) from time \( d \) onward. An innovational outlier occurs when a large innovation, e.g. an oil price shock, propagates in the unemployment rate through the dynamics of the model.

(iii) **Level-shifting Outliers** (LS): \( B(L) = 1/(1 - L) \Rightarrow \) Permanent effect on \( Y_t \) from time \( d \) onward. A level-shift occurs when a large innovation induces a permanent increase in the unemployment rate.

Defining \( y_t = \Phi(L)/\theta(L) Y_t, \pi(t) = \Phi(L)/\theta(L) \) and \( \eta(t) = \pi(L)/(1 - L) \), Tsay proposes to use the following test statistics to identify the three alternative outliers:

\[
\begin{align*}
\lambda_{IO,t} &= \frac{y_t}{\sigma_a} = \frac{\omega_{IO,t}}{\sigma_a} \\
\lambda_{AO,t} &= \frac{\rho^2_{AO,t}}{\rho_{AO,t}^2} \left( y_t - \sum_{i=1}^{T-t} \pi_i y_{t+i} \right) / \rho_{AO,t}^2 \sigma_a = \frac{\omega_{AO,t}}{\rho_{AO,t}^2} \sigma_a \\
\lambda_{LS,t} &= \frac{\rho^2_{LS,t}}{\rho_{LS,t}^2} \left( y_t - \sum_{i=1}^{T-t} \eta_i y_{t+i} \right) / \rho_{LS,t}^2 \sigma_a = \frac{\omega_{LS,t}}{\rho_{LS,t}^2} \sigma_a
\end{align*}
\]

with \( \rho^2_{AO,t} = \left( 1 + \sum_{i=1}^{T-t} \pi_i^2 \right)^{-1} \) and \( \rho^2_{LS,t} = \left( 1 + \sum_{i=1}^{T-t} \eta_i^2 \right)^{-1} \). The numerator of these tests reflects the size of the outlier while the denominator captures its standard error. The critical values for these test statistics are based on simulation results of Chang (1982), suggesting a critical value in the range of 4.0 to 3.0. Consistent with other applications, we set the critical value equal to 3.0. By using the lower end of the range, a larger number of outliers will be identified. The significance of these outliers will be tested in the intervention models estimated in the second part of this section.

In practice, outliers are identified through running a sequential detection algorithm, consisting of an outer and an inner iteration. In the outer iteration an ARMA\((p,q)\) model is estimated, extracting the residuals and computing their variance. The results from the outer iteration are then used in the inner iteration in which outliers are identified using the test statistics described in equation (5). If an outlier is detected, it is removed from the series, recalculating the residuals and their variance using the parameters of the ARMA\((p,q)\) model estimated in the outer iteration. In the same manner, identify and remove new outliers in the adjusted residuals until no more significant ones are found. Next return to the outer iteration in which the ARMA\((p,q)\) model is re-estimated and start the inner iteration again. Proceed iterating until no more significant outliers are found.

\[\text{Intuitively, this means that errors are labelled outliers only if their value exceeds three standard deviations.}\]
Following Balke (1993), we use a small modification of this algorithm starting the outlier search process with an ARMA(0,0) model in the first outer iteration and proceeding with the standard procedure in subsequent outer iterations. The justification of the approach lies in the fact that series containing level-shifts are to some degree observationally equivalent with series characterised by a high degree of persistence. In this case, estimating the initial ARMA\((p,q)\) model in the way proposed by Tsay might imply that the residuals will not reflect the true nature of the level-shift outlier, i.e. level-shift outliers can be misidentified as innovational outliers or might not be identified at all.

Table 2 presents results from this outlier detection algorithm applied to the unemployment rates of 21 OECD-countries. Three important conclusions stand out. First, positive level-shifts are detected in all countries except the US. Second, the timing of the level-shifts over the considered countries is clustered in three periods: 1975-77, 1980-83 and 1992-94 (i.e. 39 out of the 50 identified level-shifts occur in these three periods). Third, in only two countries - Ireland and Norway - a negative level-shift is detected, occurring in the late 1990s.

### 3.2. Estimating intervention models

The outliers identified in section 3.1 can now be used to allow for a more sophisticated specification of the deterministic alternative in standard unit root tests, embedded in the following general intervention model.

\[
\Delta u_t = \alpha + \rho u_{t-1} + \sum_{i=1}^{P} \beta_i \Delta u_{t-i} + \sum_{j=S}^{S} \delta(j) LS(j) + \sum_{j=1}^{I} \gamma(j) IO(j) + \sum_{j=1}^{A} \lambda(j) AO_{t-j} + \epsilon_t \quad (6)
\]

with \(S\), \(I\) and \(A\) denoting the number of level-shifting, innovational and additive outliers respectively. \(LS\) is equal to zero before the level-shift and one afterwards. Both \(IO\) and \(AO\) have a value of one at the time of the outlier and are zero otherwise.

The results from estimating the intervention models are reported in table 3. Initially all outliers identified in the previous section were included. Subsequently, the intervention models were pared down by removing insignificant outliers. Also notice some small adjustments to the timing of some of the outliers identified by the algorithm in the previous section. Although the modified outlier detection algorithm is much better capable of
### Table 2: Outlier detection analysis for unemployment rates, 1960-1999\(^a\)

| Iteration | Model     | Parameters \(^b\) | Outliers \(^c,d\) | Date | Type | \(\omega_i\) | \(|\lambda_i|\) | \(\sigma_a\) |
|-----------|-----------|-------------------|-------------------|------|------|--------------|-------------|-----------|
| **Australia** |          |                   |                   |      |      |              |             |           |
| I         | ARMA(0,0) | 5.53 (0.49)       |                   |      |      |              |             | 3.12      |
|           | 1         |                   |                   |      |      |              |             |           |
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| II        | ARMA(1,0) | 3.62 (0.61) 0.76 (0.10) |                   |      |      |              |             | 0.90      |
| **Austria** |          |                   |                   |      |      |              |             |           |
| I         | ARMA(0,0) | 3.21 (0.29)       |                   |      |      |              |             | 1.83      |
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| II        | ARMA(1,0) | 1.82 (0.17) 0.64 (0.12) |                   |      |      |              |             | 0.37      |
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| III       | ARMA(1,0) | 1.89 (0.26) 0.82 (0.09) |                   |      |      |              |             | 0.28      |
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| IV        | ARMA(2,0) | 2.84 (0.28) 1.33 (0.14) -0.55 (0.14) |       |      |      |              |             | 0.37      |
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a A RATS-procedure implementing Tsay’s algorithm is available from the author on request.
b Standard errors are reported in parentheses.
c Level-shifts identified in the first year of the sample are not reported for they cannot be considered real outliers as they imply an adjustment in the mean of the entire series. They are handled by re-estimating the mean of the series.
d \(i = IO, AO, LS\).
identifying level-shifts compared to the algorithm initially proposed by Tsay, it appears to have some trouble picking the exact date of the shift. This problem becomes apparent if one analyses the timing of innovational and especially additive outliers identified in the second outer iteration. In a large number of cases these outliers are clustered around the timing of the level-shifts identified in the first outer iteration, indicating that the timing of the level-shift is suboptimal\(^6\). A clear example is Germany. In the first outer iteration, a level-shift is identified in 1983 to which a 1982 additive outlier - of more or less comparable size - is added in the second outer iteration. This suggests that the level-shift has occurred in 1982 in stead of 1983. A similar argument applies to level-shifts in Belgium, Finland, Greece and Portugal.

In three cases, we also exogenously intervened in the number of level-shifts included in the intervention model. The first case is again Germany where the positive level-shift in 1993 combined with the negative innovational outlier in the same year suggests that an additional level-shift has occurred later on in the 1990s. Adding a shift in 1996 indeed contributed greatly to the fit of the intervention model. In Denmark and Italy a level-shift was added in 1997 and 1975 respectively. Although such a shift is identified by the outlier algorithm, it is not reported in table 2 for both test statistics were lower than the critical value of three. They are nevertheless included in the intervention models as both turn out to be highly significant and contributing to the significance of the other outliers.

As the specification of the deterministic term has changed, the test statistic for the null hypothesis of a unit root \((\rho = 0)\) in the intervention model obviously has a different distribution than the one based on standard Dickey-Fuller tests. Therefore, table 3 includes for each country critical values generated by Monte Carlo simulations. Inspired by Bradley and Jansen (1995) we generate two different sets of critical values. The first set relies on the procedure found in Balke and Fomby (B-F) (1991), generating 20 000 replications of a random walk with i.i.d. \(N(0,1)\) innovations\(^7\) and estimating for each replication the specific intervention model used for the country under consideration. The only difference with the B-F procedure is that the number of lagged first-differences \((p)\) in each estimation is not exogenously set equal to zero but chosen endogenously using a \(t\)-test on the last coefficient

---
\(^6\) The problem is most probably caused by the lack of dynamics in the ARMA(0,0) model estimated in the first outer iteration.

\(^7\) Although the finite sample distribution of the test statistic depends on the correlation structure in the data, Perron and Vogelsang (1992) argue that this specification of the sequence of innovations implies no loss in
Infrequent large shocks to unemployment

(see also Perron and Vogelsang (1992))\(^8\). This small modification raises critical values with about 0.3 on average. The second set uses a procedure similar to the one outlined by Bradley and Jansen (B-J) (1995) generating 10 000 replications of a time series containing a unit root and matching the sample characteristics (i.e. error variance and number of lagged first-differences) of the unemployment rate of the country under consideration. Subsequently Tsay’s (modified) outlier detection algorithm is applied to each replication in order to identify the outliers to be included in the intervention model. Both procedures clearly have different implications. Under the first method, the critical values for a specific country depend on the number and timing of the outliers identified in section 2. Under the second method, critical values only depend on the characteristics of the original series.

The most important conclusion that emerges from table 3 is that the unit root hypothesis can strongly be rejected in most countries once we allow for level-shifts in the deterministic component of the data generating process. For a lot of countries, we can even reject with more than 99% confidence. In Canada the evidence is weak, though. A unit root can be rejected - at the 10% level - only if the B-F critical values are used. In the US, the unit root hypothesis can be rejected at the 5% level if we use the B-F critical values but it cannot be rejected using the B-J simulation results. Since only innovational outliers and no level-shifts have occurred, the B-J critical values might be too restrictive though.

The overall picture that emerges from table 3 is that the apparent I(1) behaviour of unemployment rates is not due to a unit root but is caused by a limited number of permanent shocks to the mean of unemployment rates. Once these infrequent shocks are controlled for, measured persistence in unemployment rates drops significantly in most countries, ranging from to 0.25 in Switzerland to 0.85 in Spain. These results imply that 90% of a shock to unemployment in Switzerland is reverted in less than 2 years, while in Spain it still takes about 14 years. Together with Greece, Spain is a clear outlier though. In most countries, reversion takes about 3 to 6 years.

\(^8\) The maximum value of \(p\) was set equal to five.
### Table 3  
Intervention models for unemployment rates, 1960-1999

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<th>t-stat.</th>
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<tr>
<td></td>
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<td>0.48(0.14)</td>
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| **Austria** | | | |
| | c | 0.62 | 4.68 |
| | UNR(-1) | -0.38 | -5.23 |
| | LS1982 | 1.24 | 6.21 |
| | LS1993 | 0.50 | 2.82 |

| **Belgium** | | | |
| | c | 0.87 | 5.65 |
| | UNR(-1) | -0.39 | -7.64 |
| | LS1975 | 1.74 | 6.27 |
| | LS1981 | 1.71 | 5.14 |
| | LS1993 | 0.42 | 2.05 |

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Critical values

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<th>B-F</th>
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Persistence

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|                | B-F | B-J | B-F | B-J | B-F | B-J |
| critical values | -4.35 | -4.51 | -4.52 | -4.68 | -3.95 | -5.36 |
| 1%              | -3.55 | -3.65 | -3.75 | -3.82 | -3.16 | -4.27 |

|                | 0.25(0.10) | 0.71(0.10) | 0.64(0.11) |
| persistence     | 0.25(0.10) | 0.71(0.10) | 0.64(0.11) |

\(^a\) A RATS-procedure calculating critical values is available from the author on request.

\(^b\) Similar to the calculation in section 2, the persistence measure equals the estimated AR(1) coefficient in an ARMA(1,\(q\)). The only difference is that the regressions now also include the identified outliers.

4. CONCLUDING DISCUSSION

This paper tests three competing explanations for the large persistence in most OECD unemployment rates. The first hypothesis states that the observed persistence is caused by adjustment toward an underlying natural rate of unemployment which has increased in response to changes in the structural characteristics of the labour market. The other two hypotheses argue that unemployment persistence is due to hysteresis effects, meaning that present structural unemployment depends on past actual unemployment. When this dependence eventually decays as time elapses, the unemployment rate converges toward a fixed natural rate, implying only partial hysteresis. When the increase in actual unemployment permanently feeds through in structural unemployment, the labour market is characterised by full hysteresis.

One line of research has tested the relevance of the last two hypotheses using standard univariate unit root tests. The results clearly point to full hysteresis in most OECD-countries. The hypothesis of adjustment to an increased natural rate has not yet been properly included in these tests, though. In this paper I extend standard unit root tests by allowing for level shifts under the alternative hypothesis of stationarity. Since level shifts allow the unemployment rate to be stationary around a variable mean, this alternative hypothesis should be able to deal with variation in the natural rate of unemployment. The identification of level-shifts is based
on the outlier detection algorithm developed by Tsay (1988). Once level-shifts are allowed for, univariate unit root tests strongly reject the null of a unit root in almost all OECD-countries.

In terms of the alternative persistence hypotheses outlined in the introduction, this result is in favour of the hypothesis that the rise in OECD unemployment rates is due to adjustment to an increased natural rate. Nevertheless, it would be hard to advocate that the identified level-shifts in the actual unemployment rate coincide with shifts in the natural rate. Two observations underpin this reluctance. (i) As noted in the previous section, the timing of most of the level-shifts in the considered countries is clustered in three time periods. It would at least be surprising to find that shifts in the natural rate of unemployment - caused by changes in structural characteristics of the economy – have occurred at more or less the same time in all countries. (ii) Each of these periods can be linked to large temporary demand or supply shocks. The level-shifts in the periods 1975-77 and 1980-83 reflect the upshot in unemployment in the aftermath of the two oil price shocks while the period 1992-94 coincides with a recession in most European countries. Without full hysteresis - which has been rejected in the previous section - such ‘temporary’ shocks generally do not have the power to shift the long-run equilibrium (natural) rate of unemployment. Rather than reflecting shifts in the natural rate, the infrequent large shocks must in my opinion be interpreted as jumps - caused by extreme adverse shocks to the unemployment rate - in the direction of a natural rate, which might have increased years before the upshot in actual unemployment.

A theoretical model that is able to account for this feature can be found in Ljungqvist and Sargent (1998). They develop a general equilibrium search model in which workers accumulate skills on the job and lose skills during unemployment. Since unemployment benefits are determined by workers’ past earnings, unemployed workers face increasing difficulties of finding a job that they prefer to their unemployment compensation as their skills deteriorate. The main implication of this model is that even with such unfavourable labour market characteristics, low unemployment is sustainable as long as the economy is not subject to any major adverse shocks. The intuition behind this result is that the availability of a lot of ‘good jobs’ counteracts the adverse effects of generous unemployment benefits. When the economy is hit by a severe adverse shock, generous benefits erode the ability of the labour

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9 Although lasting longer than the average business cycle, the increase in the oil prices during the oil shocks of the 1970s was largely undone by the reverse shock in 1986.
market to adjust, though. Ljungqvist and Sargent (1998) argue that in this interpretation, the smooth performance of the European economies in the 1950s and 1960s concealed an inherent instability. Their model implies that the gradual build-up of the welfare state – implying unfavourable labour market characteristics like higher direct taxes on labour and generous unemployment benefits – was a virtual ‘time bomb’ waiting to explode.

The conclusion that the adjustment of actual unemployment toward its natural rate is mainly driven by infrequent large shocks can also explain why a lot of empirical research has troubles detecting a strong causal relation between changes in labour market characteristics and the actual unemployment rate. The OECD (1994) for instance argues that a generous unemployment benefit system does have a negative impact on employment but only with a lag ranging from 5-20 years. In the interpretation of Ljungqvist and Sargent, these lags are purely coincidental as they are driven by the timing of adverse shocks hitting the economy.

Although these results take the edge off the hypothesis of full hysteresis, a fairly large amount of persistence remains in some countries, implying not only slow convergence to the variable natural rate but also partial hysteresis after shocks to aggregate demand. In Spain for instance, it takes about 14 years to revert 90% of a shock to unemployment. In most countries however, this reversion takes about 3 to 6 years, implying only moderate hysteresis.

The main policy implication of these results is that the battle against high and persistent unemployment can only be won if governments focus on structural labour market reform. The moderate amount of medium-term persistence which remains in the unemployment rate of most OECD-countries after controlling for the evolution of the natural rate reduces the scope for demand side policies, although they might be very useful as attendant measure with structural reforms, for they can speed up the convergence to a lower natural rate.
REFERENCES


Appendix A: A note on the construction of capital stock data

The measurement of capital is one of the nastiest jobs that economists have set to statisticians. (Hicks 1981, p. 204)

A.1. INTRODUCTION

A lot of critical assumptions underpin the construction of capital stock data. Unfortunately, the validity of some of them is not immediately clear. Therefore, this appendix will try to identify the most crucial assumptions and analyse to what extent small modifications affect the estimated size of the public capital stock.

The remainder is organised as follows. Section A.2. first outlines an important difference between two alternative capital stock measures, i.e. gross versus net valuation. Section A.3. sets out the Perpetual Inventory Method (PIM), a methodology developed to compile investment data into capital stock measures. Since most of the work in this thesis is based on Belgian data only, we will restrict our attention to the methodology used by the Belgian Federal Planning Bureau. Section A.4. reports the results from a sensitivity analysis estimating the size of the Belgian public capital stock under a set of different assumptions.

A.2. GROSS VERSUS NET VALUATION

Generally spoken, there are two alternative aggregate capital stock measures. Gross capital stocks on the one hand are defined as the cumulative value of past investment less discarded investments. This calculation relies on the assumption that the efficiency of the included assets remains at about the same level regardless of their age. Maintenance is assumed to be sufficient to keep performance of assets steady until the moment they are liquidated. Net capital stocks on the other hand are constructed assuming some steady decline in the value of the assets, reflecting the steady decay in discounted future income streams.

While net capital stocks are mostly used to measure value to asset holders, gross capital stocks are more appropriate as indicators of the value of the capital stock as a factor of
production (OECD, 1993). Since our focus is on production analysis, we will concentrate on the construction of gross capital stock data.

A.3. THE PERPETUAL INVENTORY METHOD

A.3.1. General functional form

Given the definition in the previous section, the gross capital stock ($GCS$) can be calculated as (see Gilot and Floridor, 1993):

$$ GCS_t = \sum_{i=0}^{m} s_i I_{t-i} $$  \hspace{1cm} (A.1)

with

- $I_{t-i}$: investment in period $t-i$, in prices of a selected base year
- $s_i$: share of assets installed in period $t-i$ that are still productive at time $t$
- $i$: age of the asset
- $m$: maximum economic lifetime of assets, set equal to $2 \times T$, with $T$ the average lifetime

Equation (A.1) states that the gross public capital stock at a given moment is simply the sum of all investments that are still productive at that time. Calculation of (A.1) requires information on nominal investment, the evolution in asset prices and the lifetime during which these assets are productive. Recent data on capital formation and asset prices are readily available and thought to be quite accurate\(^1\). However, equation (A.1) reveals that the construction of long time series requires information - depending on the assumed lifetime of the considered asset - on even longer series on capital formation. An average lifetime of 60 years, which is commonly used for buildings for instance, implies historical data on investment and investment prices dating back 120 years. Such data are generally less easy to collect and often of low quality\(^2\). However, the impact on the accuracy of capital stock measures of these ‘low quality’ data declines over time by the retirement of assets (OECD, 1993).

\(^1\) Reliable data are available in the National Accounts from 1953 onwards.

\(^2\) For public investments over the period 1896-1929, De Biolley and Gilot (1987) have to fix the annual growth rate at 2%, with zero growth during world war I. Prior to 1896, the growth rate was assumed to equal 1.5%.
Much more crucial for accuracy over the whole time interval is the assumption on the period during which assets are kept in the capital stock. Usually such information is scanty available (OECD, 1993), though. The choice of the lifetime of assets can be subdivided in the average service live and the distribution of liquidations around this average. The concrete implementation of these assumptions in Belgian is the subject of the following two subsections.

A.3.2. Average service lives

Average economic lifetimes used in Belgium are fixed over the whole period but vary across sectors and assets (see Gilot and Floridor, 1993). Assets are split up in three different types: equipment, vehicles and buildings. In principal, information on lifetimes is based on depreciation rates specified by tax authorities. However, in some cases lifetime was increased in order to match the values calculated by Paccoud (1983) for a number of European countries. For public capital investment, this results in average service lifetimes equal to 15, 7 and 60 years for equipments, vehicles and buildings respectively.

A simple comparison of service lives used in a number of OECD countries (see OECD, 1993) reveals a considerable variability, raising doubts about the reliability of the assumed lifetimes. Concerning public investments in equipment, there are generally spoken two groups. The first, including Belgium, assumes an average service life of 15 years. The second sets service lives equal to 20 years. For buildings and other constructions works of the public administration, the average service life ranges between about 50 and 80 years. The lifetime of vehicles finally, lies between 7 and 14 years. An analysis of the sensitivity to alternative assumptions about average lifetimes is provided in section A.4.

A.3.3. Distribution of retirements around the average: mortality functions

Apart from being a crude approximation, the lifetimes reported above are only averages. In reality, some assets will remain productive far beyond their average lifetime while others may be withdrawn much earlier. This problem is usually tackled by simulating retirement patterns

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3 This was for instance the case for equipment for which the lifetime was increased from 10 to 15 years
based on some kind of mortality function. A wide range of straightforward calculable retirement patterns is available;

(i) **Linear retirement** assumes equal amounts to be liquidated from the first year after installation onwards.

(ii) **Delayed linear retirement** is conceptual very similar to linear functions but assumes retirements over a shorter interval, constructed symmetrically around the average lifetime.

(iii) **Declining-balance functions** use a constant depreciation rate, implying the largest retirements in the early lifetime of the assets.

(iv) **Simultaneous exit** assumes all assets to be retarded at their average lifetime.

(v) **Bell-shaped mortality functions** allow for a smooth retirement pattern with the largest discards concentrated around the average lifetime. In addition, the degree of skewness and kurtosis can be adjusted in order to match observed retirement patterns more closely.

Figure A.1 plots a number of the suggested retirement patterns for an asset with an average lifetime of 15 years. The delayed linear retirement pattern assumes assets to be discarded over the period -25%/+25% of the average lifetime. The declining balance methodology uses the double declining balance formula which sets the yearly depreciation rate equal to $2/T$ (Hulten, 1990). The bell-shaped mortality function is taken from Gilot and Floridor (1993), who use the following quasi-logistic specification:

$$s_i = \frac{1}{1 + \exp \left( \frac{c}{m} \left( \frac{ma}{m+1-i} + \frac{bm}{1-i} \right) \right)}$$

(A.2)

with $c$, parameter which determines the degree of kurtosis. $a$ and $b$ parameters determining the degree of skewness.

Figure A.1 shows huge differences between the alternative retirement patterns. Linear and declining-balance functions for instance produce very divergent retirement patterns. The OECD (1993) has argued however that “... the bell-shaped function is really the only
A note on the construction of capital stock data

Especially linear and declining-balance functions suffer some important drawbacks. Contrary to common sense, both methods imply large retirements in the early lifetime of the assets. Under certain conditions, simultaneous exit and delayed linear functions are relevant approximations to the bell-shaped function and therefore also plausible candidates (OECD, 1993).

Figure A.1 Alternative retirement patterns

A.4. SENSITIVITY ANALYSIS

In this section, we assess the impact on public capital stock estimates if some small changes are made to the assumptions underlying the calculation of official data.

A.4.1. Sensitivity to changes in assumed service lives

In order to check the sensitivity of public capital stock measures to changes in assumed average service lives, we construct two alternative measures. In the first one, the lifetime of buildings is reduced from 60 to 50 years. Given that Belgium is at the low end of the range of assumed lifetimes for vehicles and equipment, these values are not reduced. The second measure extends the lifetime of buildings from 60 to 70 years. For vehicles and equipment,
average service lives are fixed at 20 and 14 years respectively. The results are shown in figure A.2.

Figure A.2 Sensitivity of the public capital stock (billion 1985 BEF) to alternative average lifetimes

Changing average lifetimes clearly has important implications for the estimated size of the capital stock. In 1996, the estimate based on longer lifetimes lies about 6% higher than the official number. The proposed reduction in service lives even implies an estimate which lies 15% below the official number, a difference worth more than 800 billiards of Belgian francs.

A.4.2. Sensitivity to changes in mortality functions

Figure A.3 plots the total gross public capital stock in Belgium constructed using five alternative mortality functions. Delayed linear, linear and declining balance retirement patterns are as specified in section A.3.3. The exact specification of the quasi-logistic mortality function is taken from Gilot and Floridor (1993), who set the parameter determining the degree of kurtosis (c) equal to $2.2\times m$. The parameters determining the degree of skewness ($a$ and $b$) are both fixed at 0.5 for vehicles and equipment, yielding a symmetric mortality function. For buildings, $a=0.4$ and $b=0.6$, implying left skewness or a larger part of investments that is retired after their assumed average lifetime.
Figure A.3 Sensitivity of the public capital stock (billion 1985 BEF) to alternative mortality functions

Figure A.3 reveals the sensitivity of capital stock data to changes in assumed retirement patterns. The declining balance methodology - an inventory method frequently encountered in the literature - and linear retirement clearly yield much smaller estimates of the public capital stock. Given the objections raised above, one should be very cautious with results based on these methodologies. The bell-shaped mortality function, simultaneous exit and delayed linear retirement give very similar results. Note that the similarity between simultaneous exit and bell-shaped mortality functions also indicates that the public capital stock is fairly insensitive to possible measurement errors in historical data.

The results from a final sensitivity exercise are shown in figure A.4, reporting public capital stock series based on three alternative specifications of the quasi-logistic mortality function. The first, which is the same as the one reported in figure A.3, is based on the official specification, fitting a left-skewed mortality function for buildings. The second assumes a symmetric mortality function for the three different types of assets. The third measure is constructed by increasing the parameter determining the degree of kurtosis from $2.2 \times m$ to $3.5 \times m$, implying a larger fraction of assets to be liquidated at about their assumed average lifetime.

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4 By assuming that some assets are liquidated long after they have reached their average lifetime, bell-shaped mortality functions rely much more on historical data than simultaneous exit.
lifetime. The results show that small changes to the specification of the quasi-logistic mortality function do not have a large impact on the estimated size of the capital stock.

Figure A.4 Sensitivity of the public capital stock (billion 1985 BEF) to alternative specifications of the quasi-logistic mortality function

Sources: Own calculations on investment data provided by the Belgian Federal Planning Bureau.

A.5. CONCLUSION

Calculation of gross capital stock data requires information on capital formation, the evolution in asset prices and the lifetime during which these assets are thought to be productive. Unfortunately, reliable information concerning the period during which assets are kept in the capital stock is scantly available. In order to check the sensitivity of the estimates to the choice of service lives, alternative public capital stock measures were calculated under alternative assumptions about average assets’ lifetime and retirement patterns, describing the distribution of liquidations around the average. On the one hand, the size of capital stock estimates is found to be fairly insensitive to plausible alternative retirement patterns. On the other hand, the results vary considerably when using different average lifetimes. Further, linear retirement and declining-balance functions are shown to yield unreliable results.
REFERENCES


